Work

Wednesday, May 20, 2020 10:13 AM

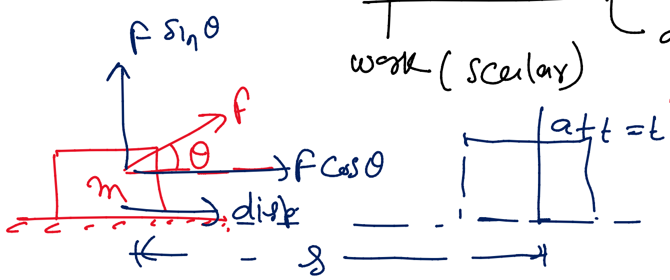
Work done :-

$$\boxed{dW = \int \vec{F} \cdot d\vec{s}}$$

force

dW: work (scalar)

ds: displacement of pt. in dirn. of force



$$\boxed{dW = F \cdot s \cos\theta}$$

work is dot product of force & displacement

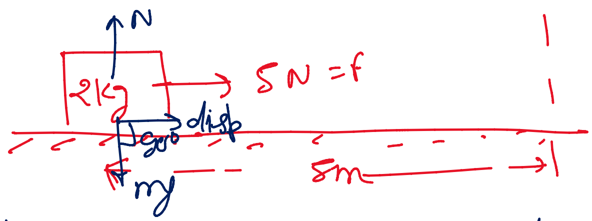
θ = angle b/w force & displacement.

Unit Joule (SI), erg (CGS)

$$\boxed{1\,J = 10^{7}\,erg}$$

Dim $[W] = [ML^2T^{-2}]$

Q.



find $W_N = ?$

$W_{mg} = ?$

$W_F = ?$

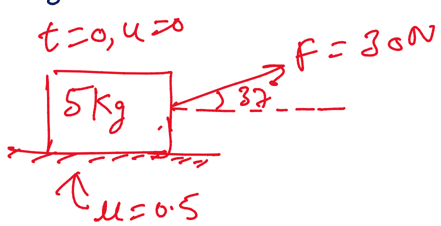
$W_N = 0$; $W_{mg} = 0$; $W_F = ?$

∵ θ = 90° ; θ = 90° ; θ = 0°

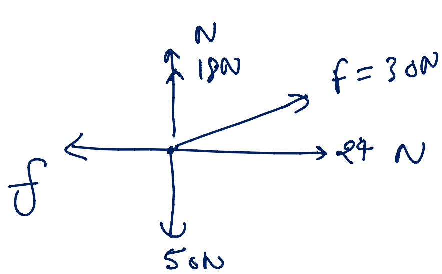
cos 90 = 0

$$\boxed{W_F = 25\,J}$$

Q.



find $W_{friction} = ?$, $W_{force} = ?$ at t = 3 sec



i) $N + 18 = 50$

$N = 50 - 18$

$$\boxed{N = 32\,N}$$

$f = \mu N = \frac{1}{2} \times 32$

$$\boxed{f = 16\,N}$$

ii) $24 - f = ma$

$24 - 16 = 5a$

$$\boxed{a = \frac{8}{5}\ m/s^2}$$

iii) $s\big|_{at\ t=3} = \cancel{ut}^{0} + \frac{1}{2}at^2$

iv) $W_F = 24 \times 7.2$

$\boxed{f = 16\,N}$

iv) $W_F = 24\times 7.2$

$= 172.8\,J$ Ans.

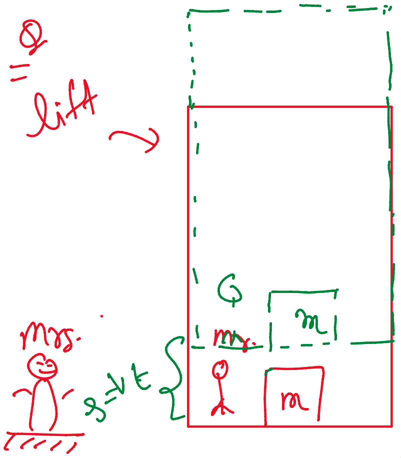
$W_f = -16\times 7.2$

$\boxed{W_f = -115.2\,J}$ Ans. $\theta = 180^\circ$

$s = 0 + \frac{1}{\cancel{2}}\times\frac{\cancel{8}^{\,4}}{5}\times 9$

$\boxed{s = 7.2\,m}$

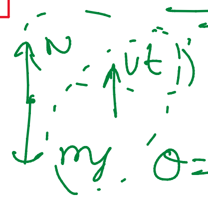
Q. lift



write work done w.r.t. mr. & mrs. after time 't'.

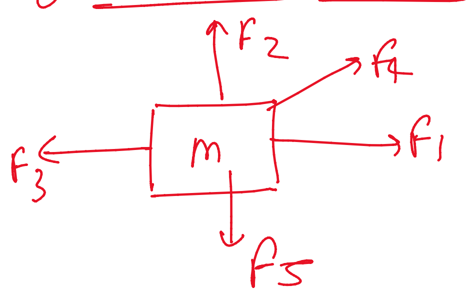
Sol^n w.r.t mr. $W_{mg} = 0$, $W_N = 0$

$\theta = 0^\circ$ w.r.t mrs. displacement $= vt$.



$W_{mg} = -mg\,vt$; $W_N = mg\,vt$

Work done by multiple forces:-



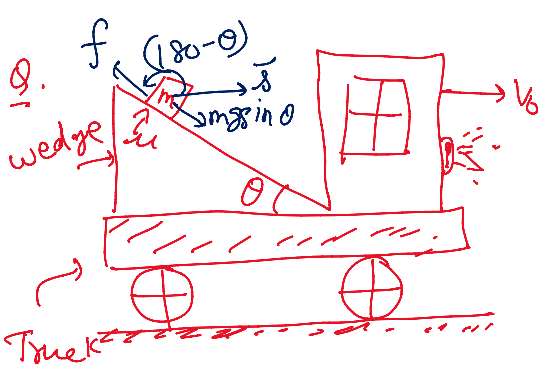
$W_{F_1} = \vec{F_1}\cdot d\vec{s}$

$W_{F_2} = \vec{F_2}\cdot d\vec{s}$

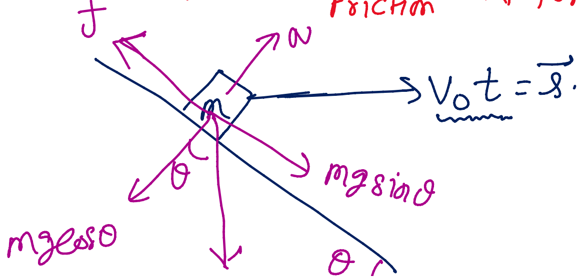
$W_{F_3} = \vec{F_3}\cdot d\vec{s}$

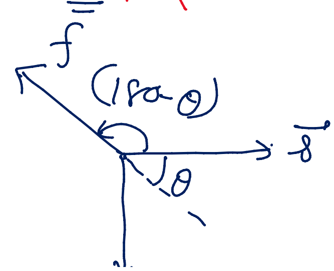
$\boxed{W_{net} = W_{F_1} + W_{F_2} + W_{F_3} + \dots W_{F_n}}$

Q.

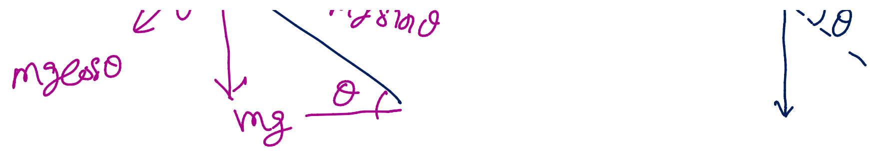


Block is at Rest w.r.t wedge. find $W_{friction}$ after time 't', ?





Truck



i) $N = mg\cos\theta$

$f_{max} = \mu mg\cos\theta \longrightarrow (1)$

ii) $W_f = \vec{f}\cdot\vec{s}$

N: If block is at Rest then friction force balanced by $mg\sin\theta$.

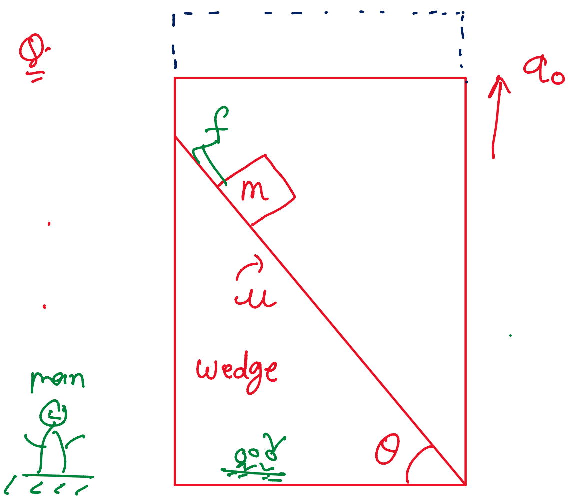
$$\boxed{f = mg\sin\theta}$$

iii) $W_{friction} = \vec{f}\cdot\vec{s}$

$= mg\sin\theta \cdot v_0 t \cos(180-\theta)$

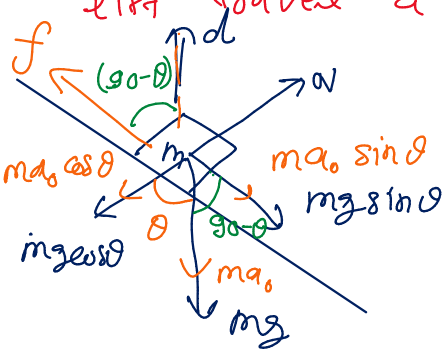
$$\boxed{W_f = -mg\cdot v_0 t\sin\theta\cdot\cos\theta}$$

Q.



$$\boxed{W_{china} = 0}$$

Block is at rest w.r.t wedge. find work done by the friction when lift travel a distance 'd'. ?



i) $f = mg\sin\theta + ma_0\sin\theta \longrightarrow (1)$

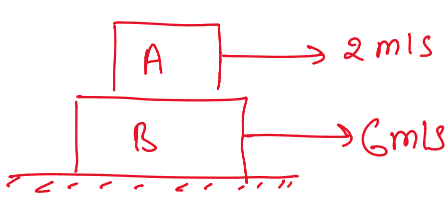
(ii) $W_f = f\cdot d\cos(90-\theta)$

$= (mg\sin\theta + ma_0\sin\theta)\, d\sin\theta$

$$\boxed{W_f = md(g+a_0)\sin^2\theta}$$

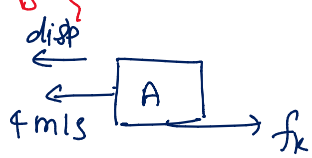
Work Done in Static friction:-

Q.



find sign of work done on block A & B ?

Sol

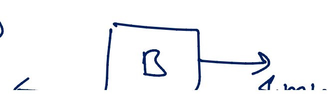


$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

$= 2-6 = -4$ m/s

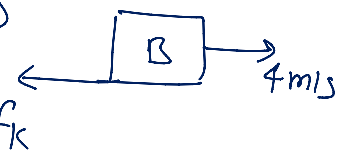
$\theta = 180° \Rightarrow W = -ve$

(ii)



$\vec{V}_{BA} = 4$ m/s

$= (ii)$

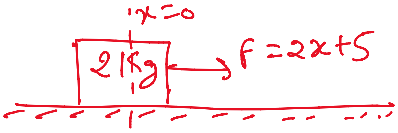


$\vec{V}_{BA} = 4\,m/s$

$\theta = 180^\circ \Rightarrow \boxed{W = -W}$

Work done of varriable force :-

Q.



Find WD when block is displaced by 5m.

$$\int dW = \int \vec{F}\cdot d\vec{s}$$

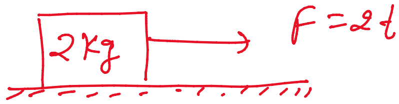
$$\int_0^W dW = \int_0^5 (2x+5)\,dx$$

$$W = \left[\frac{2x^2}{2} + 5x\right]_0^5$$

$$= (25+25)$$

$$\boxed{W = 50\,J}$$

Q. $t=0;\ u=0$



WD after $t = 3$ sec.

$$W = \int F\cdot \underbrace{ds \times \frac{dt}{dt}}_{=v}$$

$$= \int F\cdot v\,dt$$

$$= \int_0^3 2t\cdot\frac{t^2}{2}\,dt$$

$$= \left[\frac{t^4}{4}\right]_0^3 = \frac{81}{4}\,J$$

$\because F = 2t$

$ma = 2t$

$2a = 2t$

$$\int_0^v dv = \int_0^t t\,dt$$

$$\boxed{v = \frac{t^2}{2}}$$

Dot product :-

$\vec{A} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{B} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\boxed{\vec{A}\cdot\vec{B} = x_1x_2 + y_1y_2 + z_1z_2}$$ scalar

Q. $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ & $\vec{B} = \hat{i} - \hat{j} - \hat{k}$ find $\vec{A}\cdot\vec{B} = ?$

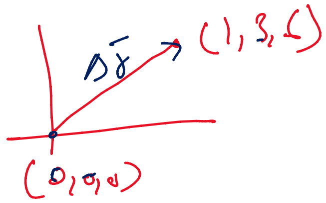
Sol^n^ $\vec{A}\cdot\vec{B} = 2 - 3 - 1$

$$\boxed{\vec{A}\cdot\vec{B} = -2}$$

find WD = ?

A·N

Q. $\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ find WD = ?



$\Delta\vec{r} = \hat{i} + 3\hat{j} + 6\hat{k}$

$W = \vec{F} \cdot \Delta\vec{r}$

$= (2\hat{i} + 3\hat{j} + 4\hat{k})(\hat{i} + 3\hat{j} + 6\hat{k})$

$\boxed{W = 35\ J}$

Note

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ (x_1, y_1, z_1)

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ $\downarrow$ (x_2, y_2, z_2)

find WD = ?

$$\boxed{W = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz}$$

Q. $F = x\hat{i} + y\hat{j}$ $P_1(0, 0)$ to $P_2(1, 1)$

$$W = \int_0^1 x\,dx + \int_0^1 y\,dy$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[\frac{y^2}{2}\right]_0^1 = \frac{1}{2} + \frac{1}{2} = \underline{1\ J}$$

Q. $\vec{F} = x^2\hat{i} + y^2\hat{j} + 2z\hat{k}$ find WD from (1, 2, 3) to (4, 5, 6).

Soln $W = \int_1^4 x^2\,dx + \int_2^5 y^2\,dy + \int_3^6 2z\,dz$

$$= \left[\frac{x^3}{3}\right]_1^4 + \left[\frac{y^3}{3}\right]_2^5 + \left[\frac{2z^2}{2}\right]_3^6 = \underline{87\ J}$$

HW

Q. $\vec{F} = 3x^2\hat{i} + 4y^2\hat{j}$. $P_1(1, 1)$ to $P_2(3, 4)$ find work done?

Soln $W = \int_1^3 3x^2\,dx + \int_1^4 4y^2\,dy = \underline{110\ J}$ Ans.

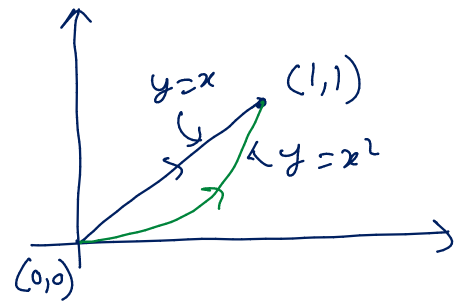
Q. $\vec{F} = y\hat{i}$ find WD from (0, 0) to (1, 1)?

Q. $\vec{F} = y\hat{i}$ find WD from (0,0) to (1,1)?

Soln Case I if $y = x$ Then,

$WD = \int y\,dx$

$= \int_0^1 x\,dx = \frac{1}{2}$ J



Case II if $y = x^2$; Then,

$WD = \int y\,dx = \int_0^1 x^2\,dx = \frac{1}{3}$ J

from the above example, we can say that —

i) Conservative force

ii) Non-conservative force.

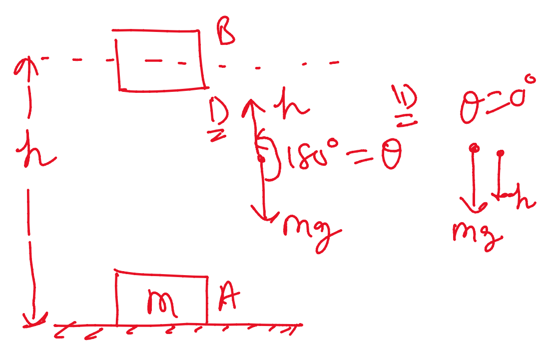
Conservative force

i) Path independent but depends on the initial & final position.

ii) work done in a complete cycle is zero.

eg Gravitation force.

eg



i) when block going upward.

$W_{mg\,A\to B} = -mgh \longrightarrow$ (I)

ii) when block fall down.

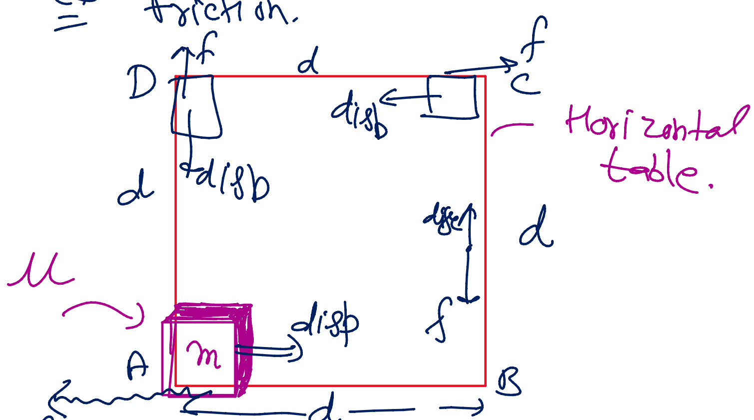
$W_{mg\,B\to A} = +mgh \longrightarrow$ (II)

Non-conservative force

Path dependent and also depends on initial & final Position.

ii) WD in complete cycle is non-zero.

eg friction.



$f = \mu mg$ ⊗ mg ⊙ N $\boxed{N = mg}$

WD by friction

$W_{AB} = -\mu mgd$; $W_{BC} = -\mu mgd$

$W_{CD} = -\mu mgd$; $W_{DA} = -\mu mgd$

$= W_{mg\,(B\to A)} = +mgh \longrightarrow (II)$

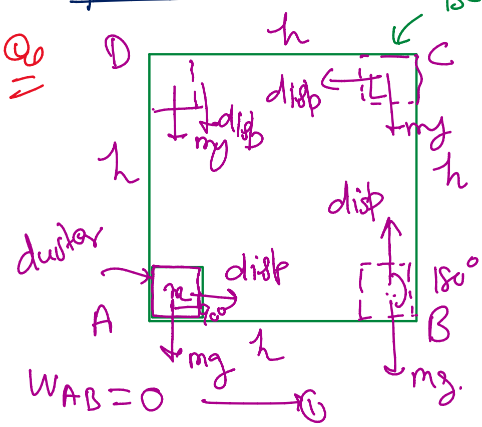
$\therefore W_{ABA} = W_{AB} + W_{BA}$

$= -mgh + mgh$

$\boxed{W_{ABA} = 0}$

white Board (h)

Q6



$W_{AB} = 0 \longrightarrow (i)$

$W_{BC} = -mgh \longrightarrow (ii)$

$W_{CD} = 0 \longrightarrow (iii)$

$W_{DA} = mgh \longrightarrow (iv)$

$\boxed{W_{ABCDA} = 0}$ in one complete cycle.

$$\boxed{\oint F\cdot ds = 0}$$

$W_{CD} = -\mu mgd$; $W_{DA} = -\mu mgd$

$\theta = 180°$

$\boxed{W_{ABCDA} = -4\mu mgd}$

$$\boxed{\oint \vec{F}\cdot\vec{ds} \neq 0}$$

↑ cyclic integral

Perfect Differential formate:-

i) $d(xy) = y\,dx + x\,dy$

ii) $d(x^2y^2) = 2xy^2\,dx + 2x^2y\,dy$

iii) $d(x^2y^3z^2) = 2xy^3z^2\,dx + 3x^2y^2z^2\,dy + 2x^2y^3z\,dz$

iv) $d(x^4y^3z^2) = 4x^3y^3z^2\,dx + 3x^4y^2z^2\,dy + 2x^4y^3z\,dz$

v) $d(\sin x\cos y) = \cos x\cos y\,dx - \sin x\sin y\,dy$.

v) $d(\sin x \cos y) = \cos x \cos y \, dx - \sin x \sin y \, dy$.

v) $d(\sin^2 x \cos^2 y \tan z) = 2 \sin x \cos x \cos^2 y \tan z \, dx +$
$-2 \sin^2 x \cos y \sin y \tan z \, dy +$
$\sin^2 x \cos^2 y \sec^2 z \, dz$.

Q. $\vec{F} = y\hat{i} + x\hat{j}$ find WD from $(1,1)$ to $(2,3)$.

Soln $W = \int y\,dx + \int x\,dy$

$$= \int_{(1,1)}^{(2,3)} d(xy) = (2\times3 - 1\times1)$$
$$= 6-1$$
$$\boxed{W = 5\,J}$$

Q. $\vec{F} = 3x^2y^2z^2\hat{i} + 2x^3yz^2\hat{j} + 2x^3y^2z\hat{k}$

Find WD $(1,1,1)$ to $(2,2,2)$.

Soln $W = \int_{(1,1,1)}^{(2,2,2)} d(x^3y^2z^2) = (8\times4\times4 - 1)$

$$\boxed{W = 127\,J}$$

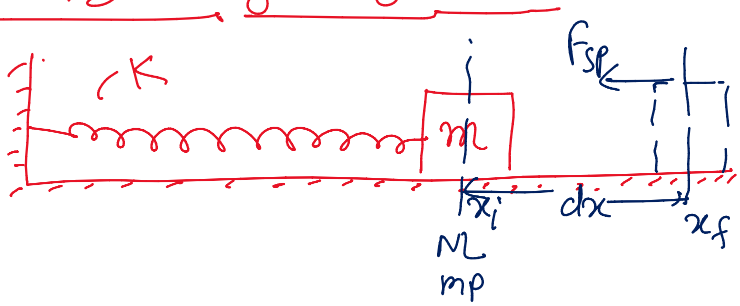
Q. $\vec{F} = 2xy^3z^2\hat{i} + 3x^2y^2z^2\hat{j} + 2x^2y^3z\hat{k}$

WD from $(0,0,0)$ to $(1,1,1)$.

Ans $\boxed{W = 1\,J}$

Work Done By spring force :-

Hook's Law $\boxed{\vec{F}_{sp} = -k\vec{x}}$



$\vec{F} = -kx\hat{i} + 0\hat{j} + 0\hat{k}$

$$W_{sp} = \int \vec{F}\cdot d\vec{s}$$
$$= \int_{x_i}^{x_f} -kx \cdot dx$$
$$= -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

N: $\boxed{\text{'}x\text{' is always measured from NL}}$

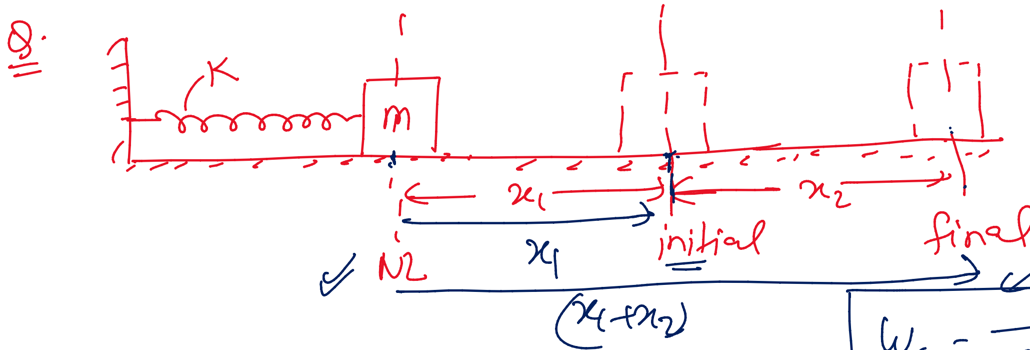
$W_{sp} = -k\,(x_f^2 - x_i^2)$

$$\boxed{W_{sp} = -\frac{k}{2}\left(x_f^2 - x_i^2\right)}$$

Generalization

if $x_i = 0$ then,

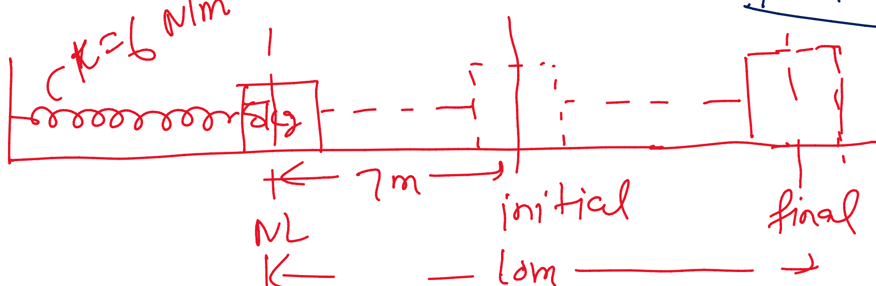
$$* \quad \boxed{W_{spring} = -\frac{1}{2}kx_f^2}$$

Q. 

find WD=?

$$\boxed{W_{sp} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)}$$

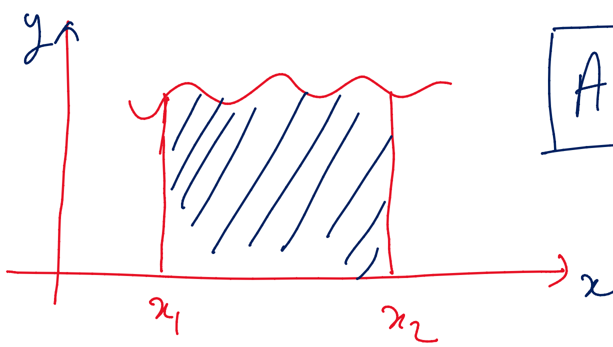
$$\boxed{W_{sp} = -\frac{1}{2}k\left((x_1+x_2)^2 - x_1^2\right)}$$

Q. 

find WD by spring?

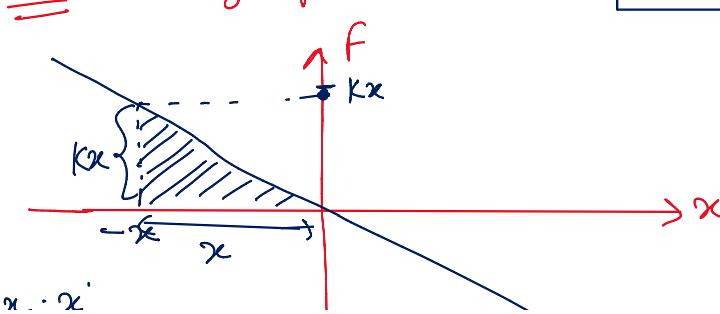
$$\boxed{WD = -153\,J}$$

Area under force-displacement curve:-



$$\boxed{A = \int y\,dx} = WD$$

Q. find WD by $F = -kx \Rightarrow$ $\boxed{y = -mx}$ $\boxed{\theta = -k}$



$W = \frac{1}{2}kx \cdot x$

$-x \qquad x$

$W = \frac{1}{2}kx \cdot x$

$\boxed{W = \frac{1}{2}kx^2}$

$F = -kx$