Rotational Motion Cont...

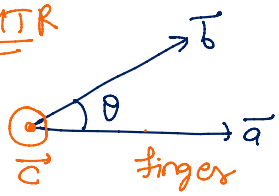
Tuesday, May 19, 2020 5:15 PM

CROSS product:

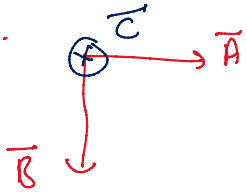
$$\vec{c} = \vec{a} \times \vec{b}$$

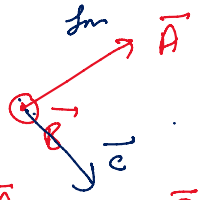
$$\Rightarrow |\vec{c}| = ab \sin\theta \, \hat{n}$$

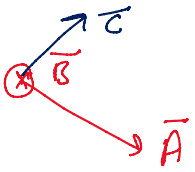
RHTR

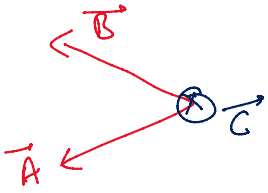


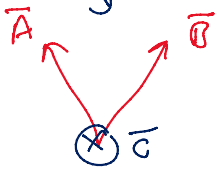
$\otimes$ { $\odot$ outward, $\otimes$ Inward }

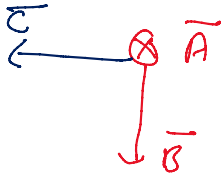
Q. 

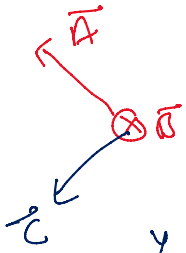
Q) 

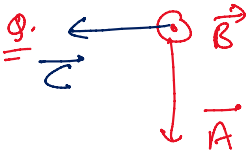
Q) 

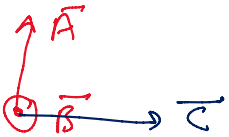
Q. 

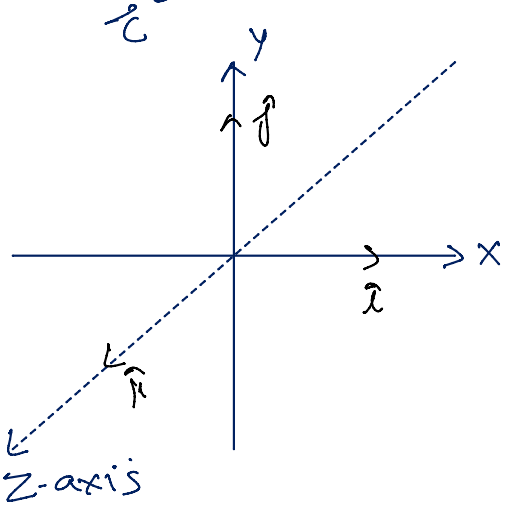
Q. 

Q 

Q. 

Q. 

Q. 



a) $\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0 \, \hat{n}$

$= 1 \cdot 1 \cdot 0 \cdot \hat{n}$

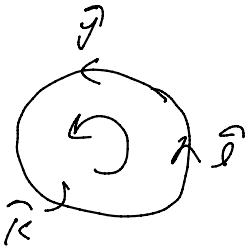
$\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$

b) $\hat{i} \times \hat{j} = |\hat{i}| \cdot |\hat{j}| \sin 90 \cdot \hat{k}$

$\hat{i} \times \hat{j} = \hat{k}$

Similarly, $\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$



Q. $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + \hat{j} - \hat{k}$

find $\vec{A} \times \vec{B} = ?$

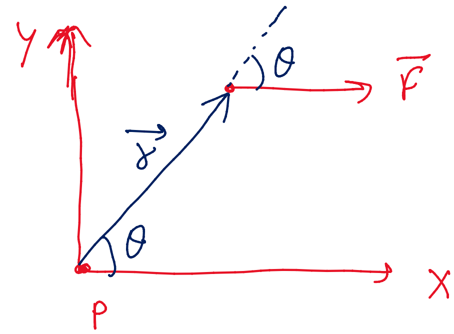
Sol^n $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i}(-1-1) - \hat{j}(-2-1) + \hat{k}(2-1)$

$= -2\hat{i} + 3\hat{j} + \hat{k}$ Ans.

N. $\hat{c} = \frac{\vec{c}}{|\vec{c}|}$

N. $\hat{C} = \dfrac{\vec{C}}{|\vec{C}|}$

Moment of force (Torque) (τ)



Torque about point 'P' is given by, $\vec{\tau} = \vec{r} \times \vec{F}$ ⊗

$\Rightarrow$ $\tau = rF\sin\theta$ ⊗

Unit N-m Dim $[MLT^{-2}][L]$

$[\tau] = [ML^2T^{-2}] = [W] = [E] = [K.E]$

N. Dirn is given by RHTR.

N. The head of $\vec{r}$ is towards the point where force is applied and tail at the point about to which torque to be calculated.

N. Torque depends on the point of application of force.

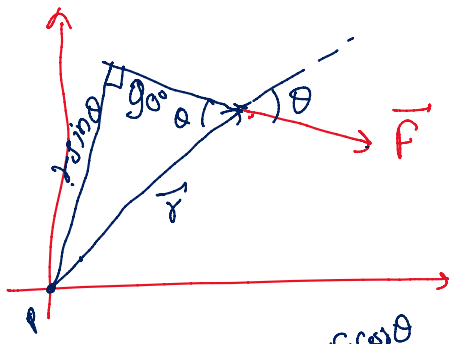
F →
- magnitude
- Direction
- Point of application.

N.
- $s \to \theta$
- $v \to \omega$
- $a \to \alpha$
- $m \to I$
- $F \to \tau$

N. i) $\longrightarrow \bar{v}$, $\longrightarrow \bar{a}$ } $v\uparrow$ ses ω α ω↑

ii) $\longrightarrow \bar{v}$, $\longleftarrow \bar{a}$ } $v\downarrow$ ω α ω↓sing

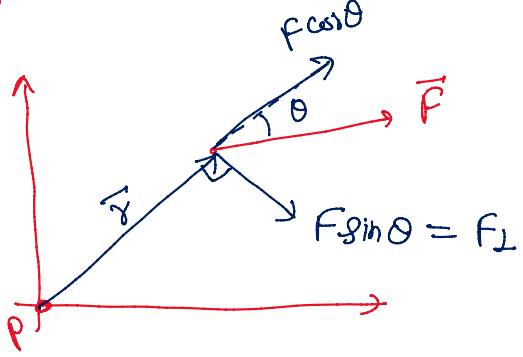
N:



$\tau = F \cdot (r \sin\theta)$

$\boxed{\tau = F r_\perp}$

N:



$\tau = \vec{r} \times \vec{F} = r F \sin\theta$

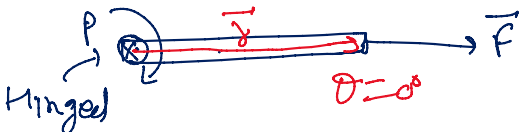
$\tau = r F \sin\theta$

$\boxed{\tau = r F_\perp}$ *

N: $\tau = r F \sin\theta$

if $\theta = 0°$, $\theta = 180°$ $\Rightarrow$ $\boxed{\tau = 0}$

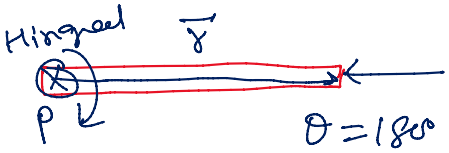
exp:



$\vec{\tau} = r F \sin\theta$

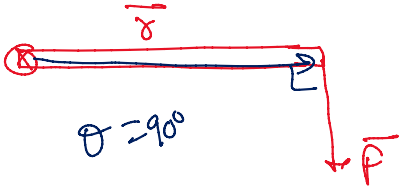
$\boxed{\tau = 0}$

exp:



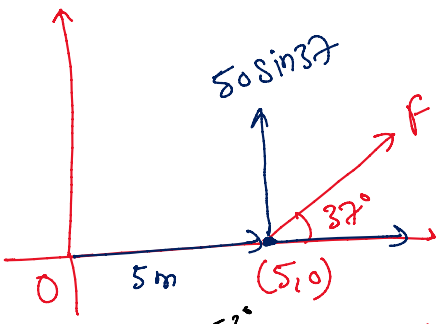
$\boxed{\tau = 0}$

exp:



$\boxed{\tau_{max} = rF}$ ⊗

Q:



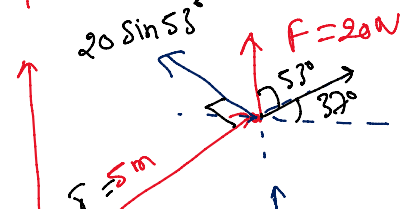
Find torque about Point 'O'?

Soln $\tau = 5 \times 50 \sin 37$ ⊙

$= 5 \times 50 \times \frac{3}{5}$

$\boxed{\tau = 150 \text{ N·m}}$ ⊙

Q:



$\tau_O = ?$

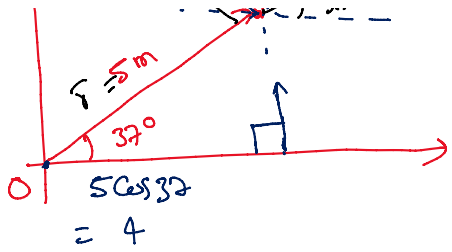
1st method $\boxed{\tau_O = r_\perp F}$

$\tau_O = 20 \times 4$

2nd method

$\tau = r F_\perp$

$= 5 \times 20 \sin 53$



$\tau_0 = 20 \times 4$

$\tau_0 = 80$ Nm

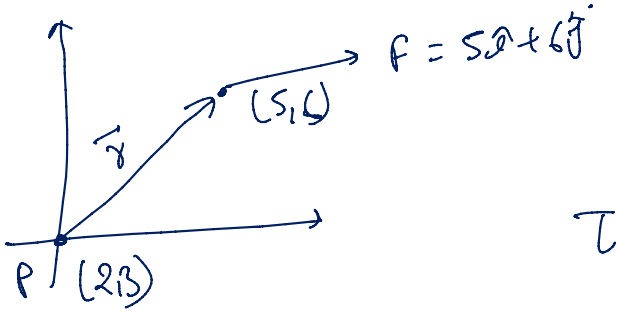
$\vec{\tau} = \vec{r} \times \vec{F}_2$

$= 5 \times 20 \sin 53$

$= 80$ Nm ⊙

Q. $\tau = ?$ about point (2,3), if a $\vec{F} = 5\hat{i} + 6\hat{j}$ is applied on a point (5,6).

Soln



$\vec{r} = (5-2)\hat{i} + (6-3)\hat{j}$

$\vec{r} = 3\hat{i} + 3\hat{j}$ ——— (1)

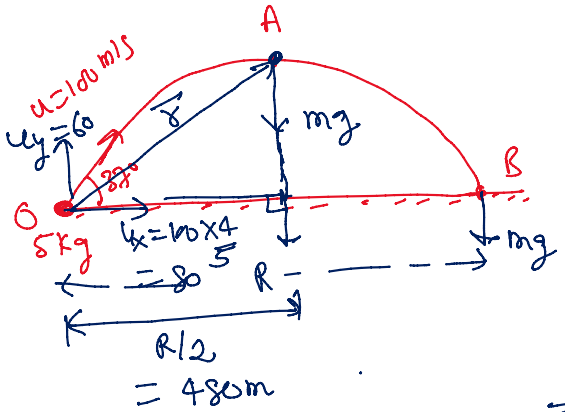
$\vec{\tau} = \vec{r} \times \vec{F} = (3\hat{i} + 3\hat{j}) \times (5\hat{i} + 6\hat{j})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 5 & 6 & 0 \end{vmatrix}$$

$= \hat{i}(0) - \hat{j}(0) + \hat{k}(18-15)$

$\boxed{\vec{\tau} = 3\hat{k}}$

Projectile Problems:-



Torque about 'O' →

i) When particle its highest point (A)

ii) When Particle just hitting the ground.? (B)

Range $R = \frac{2u_x u_y}{g}$

$= \frac{2 \times 80 \times 60}{10} \Rightarrow \boxed{R = 960\text{ m}}$

(i) $\tau_0 = mg \cdot 480$

$= 50 \times 480$

$\boxed{\tau_0 = 24000}$ Nm ⊗

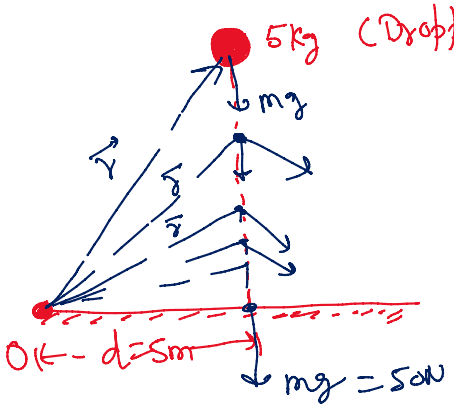
(ii) $\tau_0 = mg \times 960 = 50 \times 960$

$\boxed{\tau_0 = 48000\text{ Nm}}$ ⊗

Q. (Dabbed)

$\tau_O = 48000\ \text{Nm}$ ⊗

Q.



Comment on Torque about point 'O'

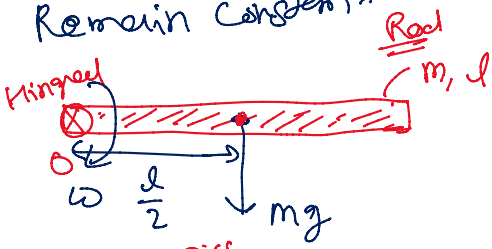
i) Always ↑
ii) Always ↓
iii) If ↑ then ↓ge
iv) If ↓ then ↑e
~~v)~~ Remain constant
vi) always zero

$\tau_O = 50 \times 5$

$\tau_O = 250\ \text{Nm}$ ⊗

Remain constant

Q.

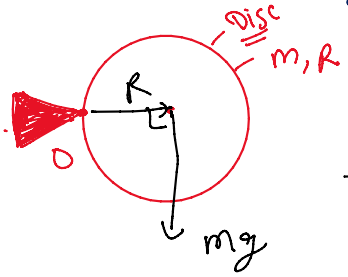


find $\tau_O = ?$

$\tau_O = \frac{mgl}{2}$ ⊗

N. force mg always act at com of the body.

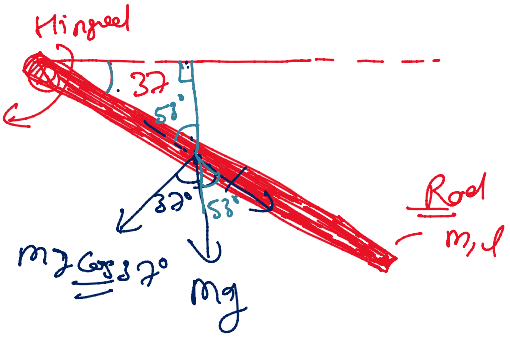
Q.



find $\tau_O = ?$

$\tau_O = mgR$ ⊗

Q.



$\tau_O = ?$

$\tau = mg\cos 37 \cdot \frac{l}{2}$

$= mg \times \frac{4}{5} \times \frac{l}{2}$

$\tau_O = \frac{2mgl}{5}$ ⊗

N.

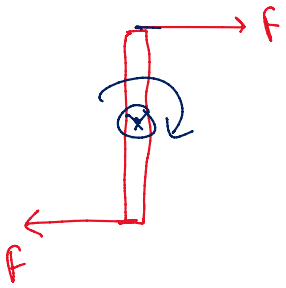
$F_{net} = 2F - F = F$

$F_{net} = F$ $\quad \theta = 180°$

$\tau = 0$ means $F_{net} \neq 0$, $\tau = 0$

$\tau = 0$ means $F_{net} \neq 0$, $\tau = 0$

N.



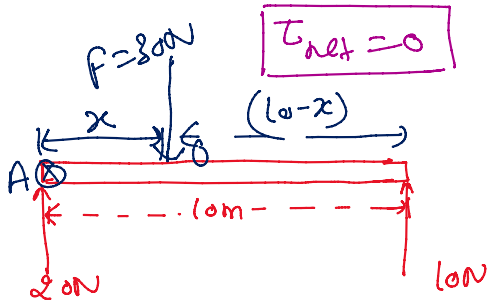
$F_{net} = 0$

$\tau_{net} \neq 0$

:- Equilibrium problems :-

$\tau_{net} = 0$ $\quad$ $F_{net} = 0$

Q.



Find the value of force, which should be applied on the rod & keep it in equilibrium.
Also find location where force is applied from 20N force.

Sol

$\tau_A = 0$

$80x + 10 \times 10 = 0$

$-80x + 100 = 0$

$x = \frac{10}{8}$ m

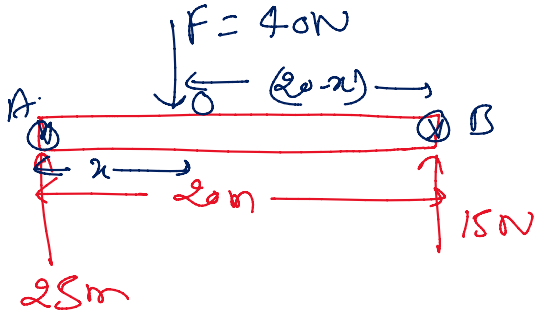
2nd method τ_0 $\quad \tau_{left} = \tau_{right}$

$20x = 80(10-x)$

$3x = 10 \Rightarrow$

$x = \frac{10}{3}$ m

Q.



about 'O'

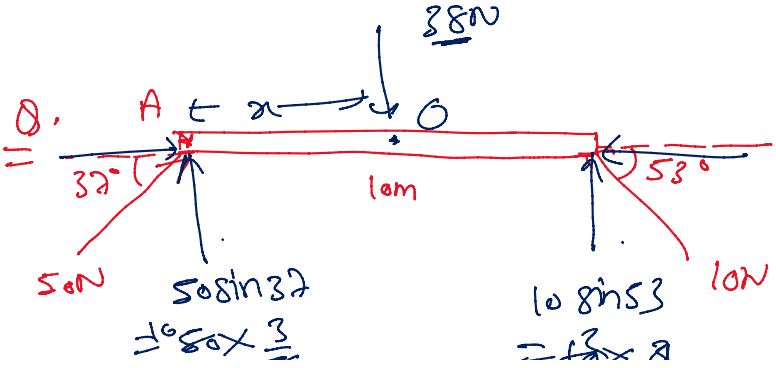
$5 \cancel{25} x = \cancel{3} 15 (20-x)$

$5x = 60 - 3x$

$8x = 60$

$x = \frac{60}{8_2} 15 \Rightarrow x = 7.5$ m

Q.



Find location of force from point 'A' so system is in equilibrium

50N $50\sin 37$ $10\sin 53$ 10N Point 'A' so system is in equilibrium?

$= 50 \times \frac{3}{5}$ $= 10 \times \frac{4}{5}$

$= 30N$ $= 8N$

about O,

$30x = 8(10-x)$

$15x = 40 - 4x$

$19x = 40 \Rightarrow \boxed{x = 40/19\ m}$ Ans