

Electrical Resistance Strain Gage Circuits

In the preceding sections we have examined the various construction schemes and application procedures for electrical resistance strain gages. By these techniques and devices, then, we have supposedly succeeded in transferring the extensional strain of the surface of a specimen into a proportional (or equal) strain in the element(s) of a resistive strain gage. The problem at this juncture is to develop a method suitable for the conversion of resistance changes in the gage element into a more readily observable quantity. We must, in other words, transduce resistance changes.

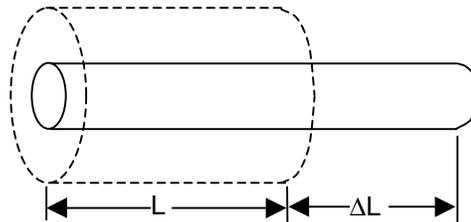
At first consideration, resistance measure appears to be a straightforward operation. It is a basic electrical parameter and ohmmeters for its measure are a common electronic instruments. In the present application, however, the accuracy and resolution needed for the measure of strain at engineering levels is beyond the capabilities of all but the most refined of these instruments. As a result, special circuitry and wiring procedures have been developed to directly and accurately convert the resistance changes into proportional voltage changes. Several techniques will be discussed in the following sections.

Electrical Properties of the Resistance Gage

The fundamental formula for the resistance of a wire with uniform cross section, A , and resistivity, ρ , can be expressed as:

$$R = \rho \frac{L}{A} \quad (1)$$

where L is the wire length. This relation is generally accurate for common metals and many



nonmetals at room temperature when subjected to direct or low frequency currents*. We consider the gage to be formed from a length of uniform wire and subjected to an elongation as shown:

The change in resistance can be expressed from Eq. 1 as

$$\Delta R = \rho \frac{L}{A} - (\rho + \Delta\rho) \frac{L + \Delta L}{A + \Delta A}$$

where Δ signifies a change in the quantity. This is a complicated expression in its present form, however, it should be clear that for metallic wires subjected to engineering strain levels that $\Delta L \ll L$ and $\Delta A \ll A$. If $\Delta\rho \ll \rho$ as well, then we can simplify the expression by approximating Δ with the infinitesimal differential change, $d()$:

* Resistivity can also be affected by electromagnetic, nuclear, barometric optical and surface effects.

$$\Delta R \cong dR = d\left(\rho \frac{L}{A}\right)$$

The differential expression on the right side is tedious to compute directly but can be easily determined using “log derivatives” as follows. First take the natural log (ln) of the equation yielding:

$$\ln(R) = \ln(\rho) + \ln(L) - \ln(A)$$

and now take the differential of this recalling that $d(\ln(x))=dx/x$ to get the simpler result:

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

In general we may write

$$A = C D^2$$

where D is a cross section dimension and C is some constant (e.g., $D=R$ and $C=\pi$ for a circle). Using the “log derivative” method, it follows that:

$$\frac{dA}{A} = 2 \frac{dD}{D}$$

At this point we note that the longitudinal strain can be written in differential form as:

$$\varepsilon = \frac{dL}{L}$$

and the transverse or lateral strain as:

$$\varepsilon_D = \frac{dD}{D}$$

Also for linearly elastic and isotropic behavior of the wire:

$$\varepsilon_D = -\nu \varepsilon$$

Then using these results:

$$\frac{dA}{A} = 2 \varepsilon_D = -2\nu \varepsilon = -2\nu \frac{dL}{L}$$

Finally, the resistance change per unit resistance ($\Delta R/R$) can then be written:

$$\frac{dR}{R} = \frac{d\rho}{\rho} + (1 + 2\nu) \varepsilon \quad (2)$$

This expresses the basic proportionality between resistance and strain in the gage element material.

A measure of the sensitivity of the material (or its resistance change per unit applied strain) is defined as the Gage Factor:

$$GAGE \ FACTOR = GF = \frac{dR/R}{\varepsilon} \quad (3)$$

From the above resistance calculations (Eq. 2) the Gage Factor can then be determined as

$$GF = 1 + 2\nu + \frac{d\rho/\rho}{\epsilon}$$

The Gage Factor as expressed above includes effects from two sources. The first term on the right represents directly the Poisson effect, i.e., the tendency in an elastic material to contract laterally in response to axial stretching. The second term represents the contribution due to changes in resistivity of the material in response to applied strain. In the absence of a direct resistivity change, then, the maximum and minimum values expected for the Gage Factor would be

$$1 \leq GF \leq 2$$

corresponding to the theoretically allowable range $0 \leq \nu \leq 1/2$ for Poisson's Ratio.

In practice the Gage Factor is not nearly so well behaved. This is amply evident in Figure 1 which is a plot of fractional resistance strain against strain for common gage materials. In this figure the Gage Factor is simply the slope of the curve. The 10% Rhodium/Platinum alloy exhibits a desirably high GF but this changes abruptly at about 0.4% strain - an undesirable behavior in all but the most special cases. Pure nickel is also poorly behaved and exhibits a negative GF for small strain! This material is seldom used alone but is often employed as an element in other alloys. The most common material for static strain measure at room temperatures is the relatively well behaved constantan alloy. Table 1 presents a summary of the Gage Factor and ultimate elongation for several materials.

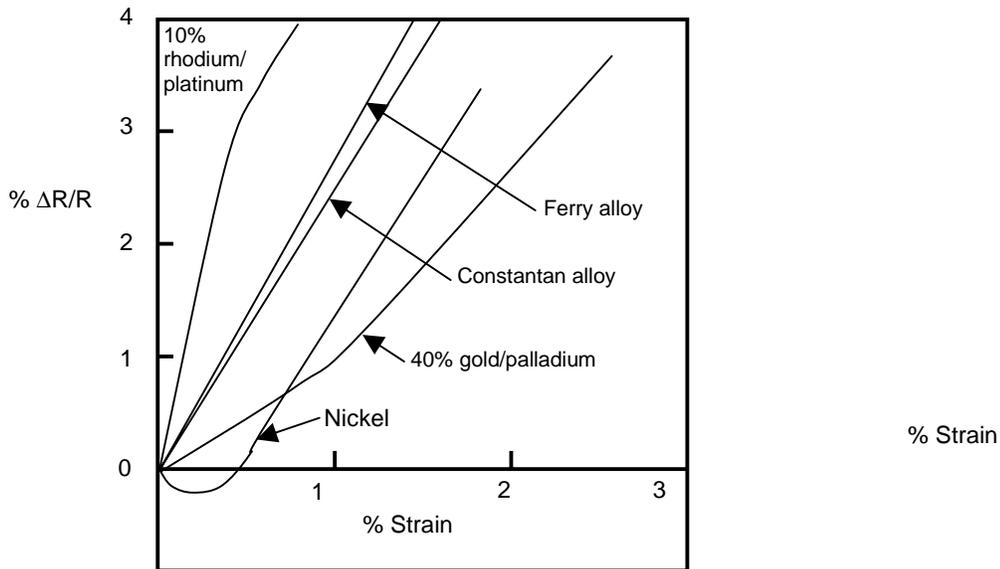


Figure 1. Change in Resistance with Strain for Various Strain Gage Element Materials

At this point a basic difficulty has appeared. The Gage Factor is only of order unity and therefore the resistance changes in the gage must be of the same order as the strain changes! In engineering materials this strain level is typically from 2 to 10,000 microstrain or 0.000002 to 0.01. Thus, changes in resistance in the gage of no more than 1% must be detected! Herein lies the challenge in the design of measuring circuitry and a problem in practical installations. (It might be noted with satisfaction, however, that our original assumption of small $\Delta R/R$ has been verified and thus the development is sound.) Two examples illustrate the point:

Table 1. Gage Factors for Various Grid Materials

Material	Gage Factor (GF)		Ultimate Elongation (%)
	Low Strain	High Strain	
Copper	2.6	2.2	0.5
Constantan*	2.1	1.9	1.0
Nickel	-12	2.7	--
Platinum	6.1	2.4	0.4
Silver	2.9	2.4	0.8
40% gold/palladium	0.9	1.9	0.8
Semiconductor**	~100	~600	--

* similar to "Ferry" and "Advance" and "Copel" alloys.

** semiconductor gage factors depend highly on the level and kind of doping used.

Example 1

Assume a gage with $GF = 2.0$ and resistance 120 Ohms. It is subjected to a strain of 5 microstrain (equivalent to about 50 psi in aluminum). Then

$$\begin{aligned}\Delta R &= GF \varepsilon R \\ &= 2(5e - 6)(120) \\ &= 0.0012 \text{ Ohms} \\ &= 0.001\% \text{ change!}\end{aligned}$$

Example 2

Now assume the same gage is subjected to 5000 microstrain or about 50,000 psi in aluminum:

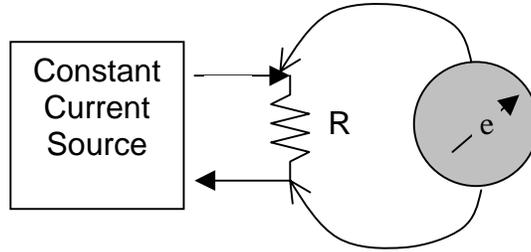
$$\begin{aligned}\Delta R &= GF \varepsilon R \\ &= 2(5000e - 6)(120) \\ &= 1.2 \text{ Ohms} \\ &= 1\% \text{ change}\end{aligned}$$

Resistance Measuring Circuits

It is apparent thus far that quite small resistance changes must be measured if resistance strain gages are to be used. Direct measurement of 0.001 Ohm out of 120 Ohms would require a meter with a resolution of better than one part in 100,000, in other words, a 6 digit ohmmeter! Several measuring techniques are available for this purpose but the Wheatstone Bridge circuit has proven the most useful for a number of important reasons. This circuit will be described shortly but first several fundamental techniques will be discussed.

Current Injection

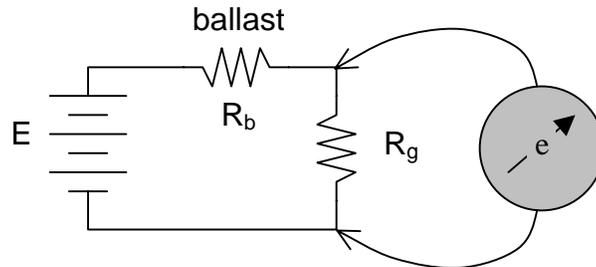
This is a simple and common technique used to make measurements of resistance accurate to about 0.1% at best. A constant known current is forced through the unknown resistance, and according to Ohm's Law the resulting voltage drop across it is directly proportional to the resistance.



This technique is generally used in portable ohmmeters and combination volt-ohm-ammeters. A major drawback for the present application is that this circuit indicates the total resistance and not the change in resistance, which is what is really desired.

Ballast Circuit

This circuit shown below is similar to the current injection technique but avoids the need for a constant current source. Instead, a simple voltage source is used and the gage is placed in series with a ballast resistor. This makes the voltage source resemble a constant current source (note that if $R_b \rightarrow \infty$ the output is always equal to e , independently of the value of R_g).



The voltage output, e , is given as:

$$e = \frac{R_g}{R_g + R_b} E$$

When used with strain gages, only very small changes in resistance, ΔR_g , are developed and it is appropriate to make the assumption that $\Delta R_g \approx dR_g$ so that the change in output voltage, de , can be determined in terms of differentials as:

$$\begin{aligned} de &= \left[\frac{dR_g}{R_g + R_b} - \frac{R_g dR_g}{(R_g + R_b)^2} \right] E \\ &= \frac{R_b R_g E}{(R_b + R_g)^2} \frac{dR_g}{R_g} \\ &= \frac{R_b R_g E}{(R_b + R_g)^2} GF \epsilon \end{aligned} \quad (4)$$

where use has been made of previous results for dR_g/R_g (Eq. 3). The only remaining question is to determine an appropriate value for the ballast resistance, R_b , and this can be done by asking for a value that will maximize the "sensitivity" of the circuit, or in other words, provide the greatest output per unit change in strain. The sensitivity of this circuit can be defined as:

$$S = \frac{de}{\epsilon} = \frac{R_b R_g E GF}{(R_g + R_b)^2}$$

and the maximum sensitivity is attained when R_b is adjusted so that:

$$0 = \frac{dS}{dR_b} = \frac{R_g - R_b}{(R_b + R_g)^3} R_g E GF$$

This occurs when:

$$R_b = R_g$$

Then

$$de = \frac{GF}{4} \epsilon E$$

and the total output voltage is the sum of the initial (e) and the change (de) as:

$$e + de = E/2 + GF/4 \epsilon E$$

As a point of interest it should be noted that these results are valid only for infinitesimally small changes in resistance, dR_g , induced by the applied strain, ϵ . If we were to consider finite ΔR_g values, the ballast circuit output, $e + \Delta e$, is not linear with ΔR_g as indicated by Eq. 4 above. Rather, the relationship is nonlinear as shown in Figure 2 below. Note that the sensitivity calculations above indicate that the middle curve in Figure 2 (with $R_b = R_g$) provides the greatest sensitivity (largest slope) and also results in $e/E = 0.5$ when $\Delta R_g = 0$. But of course, the above results are only for infinitesimal dR_g and so represent the behavior around the zero point on the $\Delta R_g/R_g$ scale in Figure 2 (e.g., linearized behavior).

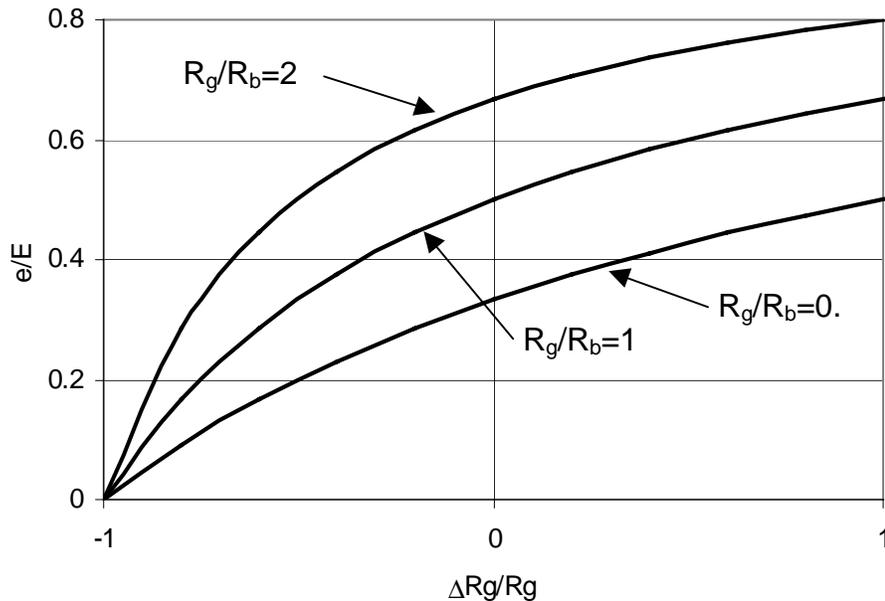


Figure 2. Ballast Circuit Nonlinear Behavior for Finite Changes in Gage Resistance, R_g

Again, as with the previous circuit, the major drawback is that the strain produces a relatively small change in the output which is nominally $E/2$. As an example, consider a typical configuration with $GF=2.0$, $R_g=120$ Ohms and $\epsilon=5$ microstrain

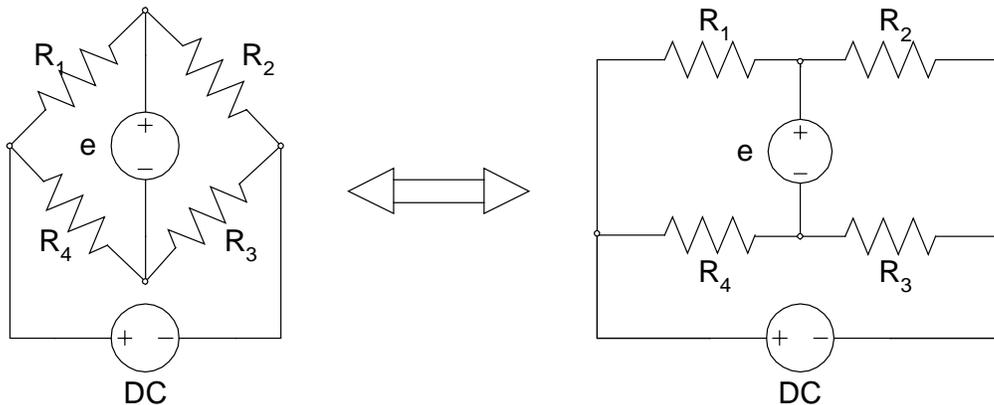
$$de = \frac{2}{4} \times (5 \times 10^{-6}) \times E \text{ volts} \quad :$$

$$e + de = (1 + 0.0000025) E \text{ volts}$$

It should be clear that such a circuit would require voltage measurement with a resolution of at least 5 parts per million so a suitable digital voltmeter would require 6 decades of precision! These difficulties can be rectified if we can devise a technique for eliminating the steady state, $E/2$, output. This is effectively accomplished in the well-known Wheatstone Bridge which is next discussed.

Wheatstone Bridge

The Wheatstone Bridge is the most basic of a number of useful electrical bridge circuits that may be used to measure resistance, capacitance or inductance. It also finds applications in a number of circuits designed to indicate resistance changes in transducers such as resistance thermometers and moisture gages. In the circuit shown below it is apparent that the bridge can be imagined as two ballast circuits (composed of R_1, R_2 and R_3, R_4) connected so that the initial steady state voltages are cancelled in the measurement of e .



The output voltage can be written as the difference between two ballast circuits as:

$$e = \left[\frac{R_2}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right] E = \frac{R_2 R_4 - R_1 R_3}{(R_1 + R_2)(R_3 + R_4)} E \quad (5)$$

Clearly, an initial steady state voltage exists unless the numerator above is zero. Such a configuration with zero output voltage is termed a “Balanced Bridge” and is provided when:

$$R_2 R_4 = R_1 R_3$$

This relationship is not of direct concern here but it is interesting to note that if any three of the four resistances are known, the fourth can be determined by ratioing the values obtained at balance. For the present, however, we are concerned with the output produced by small changes in the resistance of the bridge arms. If we consider infinitesimal changes in each resistor then $R_i \Rightarrow R_i + dR_i$ and we can compute the differential change in the output voltage, e , as:

$$de = \frac{\partial e}{\partial R_1} dR_1 + \frac{\partial e}{\partial R_2} dR_2 + \frac{\partial e}{\partial R_3} dR_3 + \frac{\partial e}{\partial R_4} dR_4$$

or

$$de = \left[\frac{R_1 R_2}{(R_1 + R_2)^2} \left(\frac{dR_1}{R_1} - \frac{dR_2}{R_2} \right) + \frac{R_3 R_4}{(R_3 + R_4)^2} \left(\frac{dR_3}{R_3} - \frac{dR_4}{R_4} \right) \right] E$$

If the bridge is balanced so that:

$$R = R_1 = R_2 = R_3 = R_4$$

then using this in the above equation for de yields:

$$de = \frac{1}{4} \left[\frac{dR_1}{R_1} - \frac{dR_2}{R_2} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4} \right] E \quad (6)$$

We can use Eq. 3 to express de in terms of the strains as:

$$de = \frac{GF}{4} [\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4] E$$

where ε_i is the strain in the gage placed in the i -th arm of the bridge. Since the bridge is initially balanced, this is the only output and we have the desired result that:

$$e = \frac{GF}{4} [\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4] E \quad (7)$$

This is the basic equation relating the Wheatstone Bridge output voltage to strain in gages placed in each arm. Several remarks are in order:

- The equation identifies the first order (differential) effects only, and so this is the “linearized” form. It is valid only for small (infinitesimal) resistance changes. Large resistance changes produce nonlinear effects and these are shown in Figure 3 where finite changes in R (ΔR) in a single arm are considered for an initially balanced bridge.
- Output is directly proportional to the excitation voltage and to the Gage Factor. Increasing either will improve measurement sensitivity.
- Equal strain in gages in adjacent arms in the circuit produce no output. Equal strain in all gages produces no output either.
- Fixed resistors rather than strain gages may be used as bridge arms. In this case the strain contribution is zero and the element is referred to as a “dummy” element or gage.

Equations 6 and 7 along with the above remarks thus serve to describe the electrical behavior of a Wheatstone Bridge. The major intent here is not to suggest electrical measurement techniques but rather to describe the behavior in enough detail so that the effect of changes in any parameter can be directly assessed. While there are cases when the entire bridge circuit must be custom assembled, as for example in a special transducer, a majority of typical strain gage applications in structural testing involve measurement of strain in single gages, one at a time, with commercial equipment. Since all presently available strain measuring instruments employ the Wheatstone Bridge circuitry, our discussion on the relative effects of Gage Factor and strain changes in individual arms on the output indication is directly applicable.

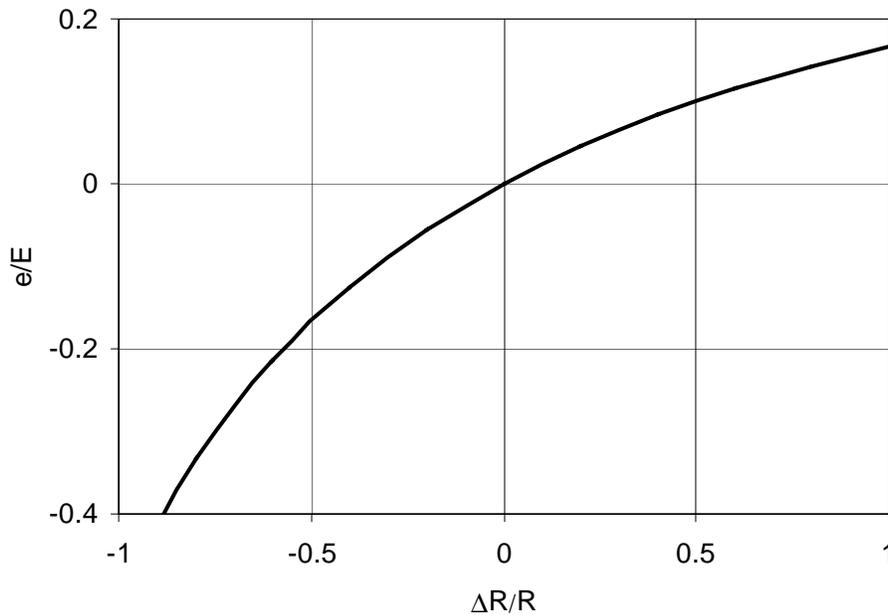


Figure 3. Wheatstone Bridge Output for Initially Balanced Bridge with Single Active Arm

Wheatstone Bridge Circuit Considerations

The equation describing the basic electrical response of the Wheatstone Bridge circuit has been developed in the preceding section. In the following discussions we will examine the effects of changes in various parameters and will consider several wiring configurations that find application in strain measurement.

Temperature Effects in the Gage

Fluctuations in ambient and in operating temperatures produce the most severe effects generally dealt with in strain measuring circuitry. The problems arise primarily from two mechanisms: (1) changes in the gage resistivity with temperature and (2) temperature induced strain in the gage element. Additionally, for certain bridge circuits in which the elements are widely separated (~ 20-100 feet), the thermally induced resistance changes in the lead wires may also be significant.

Typical values for the fractional change in resistivity per degree for common strain gage element materials is presented in Table 2. Figures are expressed as temperature coefficients in parts per million (ppm) rather than fractions or percentages to better illustrate the magnitude of the effect. For example, the value, 260, for isoelastic material is equivalent (using the definition of GF from Eq. 3 and GF=3.5) to an apparent strain:

$$\epsilon = \frac{dR/R}{GF} = \frac{260}{3.5} = 74 \text{ microstrain}/^{\circ}F$$

It is obvious from this that changes in gage temperature of only a few degrees will produce apparent indications of hundreds of microstrain. Several alloys listed in Table 2 have obviously

been chosen for their very low temperature coefficient of resistivity. The “constantan” alloy is probably the most common material for general static applications “Isoelastic” alloy is frequently used for gages which are subjected to dynamic strains but when interest is in the measurement of peak-to-peak values only. In this case the higher gage factor is attractive while the thermally induced change in resistance appears as a steady state offset and is not recorded in peak-to-peak (e.g. AC) measurements.

Table 2. Properties of Various Strain Gage Grid Materials

Material	Composition	Use	GF	Resistivity (Ohm/mil-ft)	Temp. Coef. of Resistance (ppm/F)	Temp. Coef. of Expansion (ppm/F)	Max Operating Temp. (F)
Constantan	45% Ni, 55% Cu	Strain Gage	2.0	290	6	8	900
Isoelastic	36% Ni, 8% Cr, 0.5% Mo, 55.5% Fe	Strain gage (dynamic)	3.5	680	260		800
Manganin	84% Cu, 12% Mn, 4% Ni	Strain gage (shock)	0.5	260	6		
Nichrome	80% Ni, 20% Cu	Thermometer	2.0	640	220	5	2000
Iridium-Platinum	95% Pt, 5% Ir	Thermometer	5.1	135	700	5	2000
Monel	67% Ni, 33% Cu		1.9	240	1100		
Nickel			-12	45	2400	8	
Karma	74% Ni, 20% Cr, 3% Al, 3% Fe	Strain Gage (hi temp)	2.4	800	10		1500

Erroneous indications of strain can also occur in gage installations where there is a difference in the coefficients of thermal expansion of the gage and the substrate material. Precise calculation of this effect is difficult due to the variety of mechanisms which come into play. For example, if the cross-sectional area of the gage element transverse to its measuring axis is very small compared to the similar cross-section of the substrate material, then essentially all of the substrate thermal strain is transferred to the gage (a coupled-stress problem). This is generally the case in practice and from Table 2 it appears that for a “constantan” gage on an aluminum substrate with a thermal expansion coefficient of 13 ppm/ F the contribution from this effect is:

$$\varepsilon = 13 - 8 = 5 \text{ microstrain}/^{\circ}F$$

Thus the net apparent strain due to resistive as well as expansion effects would be roughly:

$$\varepsilon = 3 + 5 = 8 \text{ microstrain}/^{\circ}F$$

In the usual practice the strain gage manufacturer cold works or otherwise metallurgically treats the gage element material so as to equalize the thermal effects over a reasonable temperature span when the gage is mounted on a specified substrate. This is marked on the package and a plot of error as a function of temperature, or equivalently, a plot of gage factor correction as a function of temperature is provided. Gages typically are available compensated for use on materials with thermal expansion coefficients shown in Table 3.

Table 3. Materials for which Strain Gages can be Compensated (typical)

PPM/°F	Material
3	Molybdenum
6	Steel; titanium
9	Stainless steel, Copper
13	Aluminum
15	Magnesium
40	Plastics

It should be pointed out here that temperature induced resistance changes in the lead wires, especially if they are over 10 feet long, can also produce substantial error. Gage modification is of no use but compensation can be made in the Wheatstone Bridge circuitry as will be illustrated later.

Temperature changes in the gages will obviously be produced by changes in ambient temperature. In addition significant temperature changes may be produced directly in the gage by resistive heating due to the electrical current flow. The magnitude of this effect is a direct function of the power applied to the gage and the ability of the gage to dissipate the resultant heat. Gages mounted on thin materials with poor thermal conductivity (such as plastics or fiberglass or thin sheet metal) are most affected by this problem, while conversely, gages attached to thick (compared to gage dimensions) material of high thermal conductivity (such as copper or aluminum) are least susceptible.

Experience has proven that power densities of from 1 to 10 watts per square inch over the gage element area can be tolerated depending on the substrate material and accuracy required. Particular recommendations are shown in Table 4.

Table 4. Strain Gage Power Densities for Specified Accuracies on Different Substrates

Accuracy	Substrate Conductivity	Substrate Thickness	Power Density (r) (Watts/sq. in.)
High	Good	thick	2-5
High	Good	thin	1-2
High	Poor	thick	0.5-1
High	Poor	thin	0.05-0.2
Average	Good	thick	5-10
Average	Good	thin	2-5
Average	Poor	thick	1-2
Average	Poor	thin	0.1-0.5

Table Notes:

Good = aluminum, copper

Poor = steels, plastics, fiberglass

Thick = thickness greater than gage element length

Thin = thickness less than gage element length

The above dissipation figures must be translated into allowable excitation levels (E) for the bridge circuit according to the usual electrical procedures. Power generated within a gage in a Wheatstone Bridge is given as:

$$P_g = E_g i_g \quad (8)$$

where E_g and i_g are voltage across and current through a given gage (arm). Due to the symmetry

of the Wheatstone Bridge it is easy to show that one half of the total current flows through any arm and that one half of the power supply voltage develops across it. Thus with E and i defined as the Wheatstone Bridge supply voltage and total bridge current:

$$P_g = \frac{E}{2} \frac{i}{2} = \frac{1}{4} E i \quad (9)$$

but since the net bridge resistance as seen by the power supply is R_g (e.g., $2xR_g$ in one half in parallel with $2xR_g$ in the other half) it follows (using Eq. 8 for the complete bridge) that:

$$i = \frac{E}{R_{bridge}} = \frac{E}{R_g} \quad (10)$$

Finally, using Eq. 10 in Eq. 9::

$$P_g = \frac{E^2}{4R_g}$$

Power density is given by

$$\rho = \frac{P_g}{A_g} = \frac{P_g}{L_g W_g} \quad (11)$$

where L_g =gage grid length, W_g =gage grid width, and P_g =gage power dissipation. Thus the maximum excitation voltage is given by:

$$E_{max} = 2\sqrt{\rho L_g W_g R_g} \quad (12)$$

In this case, ρ must be selected from the previous table and L_g , W_g , and R_g are determined by the particular choice of gage. One should note that higher electric output per unit strain (e.g., sensitivity) can be achieved by using higher excitation levels, subject to the above power limitations. Thus bridge output is increased, then, by increasing:

- Strain (ϵ)
- Gage Factor (GF)
- Gage resistance (R_g)
- Gage element area ($L_g W_g$)

Temperature Compensation in the Bridge Circuit

Temperature compensation of the strain gage alone does not generally eliminate thermal problems entirely. Such compensation is rarely exact and the differences must usually be eliminated by careful configuration of the Wheatstone Bridge circuit. The ability to make such compensation is, in fact, one of the more desirable features of this circuit. This is accomplished as follows.

The extraneous effects of temperature and other factors inducing a resistance change in the gage can effectively be considered as an additional strain, ϵ_i^T so that the strain in the i -th arm becomes

$$\epsilon_i + \epsilon_i^T$$

Then the bridge equation can be written:

$$e = \frac{GF}{4}[\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4]E + \frac{GF}{4}[\epsilon_1^T - \epsilon_2^T + \epsilon_3^T - \epsilon_4^T]E \quad (13)$$

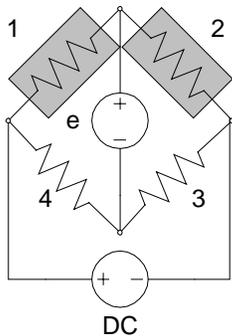
where the rightmost term represents the extraneous temperature effects. It should be clear from this result that if the extraneous temperature effects, ϵ_i^T , are all equal, they will cancel out; in fact they will cancel out so long as equal effects occur in any pair of adjacent arms (e.g., arms 1 and 2 or arms 2 and 3, etc). On the other hand, it should be noted that if these same effects occurred in two opposite arms only (e.g., arm 1 and arm 3), the effect would be additive and would not cancel out (not desirable!).

When similar strain gages are used in all four arms of the bridge and when they are mounted so that each experiences the same temperature change, then the bridge output voltage will be a function of the material strain only. In this case the temperature induced resistance changes will be the same for each gage and their contribution to the bridge output will cancel. But at the same time, it also follows that an output will only be produced when unequal changes in strain are produced in the gages. This may be difficult to arrange in some applications and as a result two or three gages in the bridge must be replaced with “dummy” fixed resistances. Then, the strain term for these arms vanishes and the only contribution is the temperature effect. One way to overcome these problems is to use as the “dummy” gages actual strain gages affixed to a piece of the same kind of metal as that to which the 4th or “active” gage is applied. If these dummy gages experience no strain but do experience the same temperature changes as the active gage (e.g., by locating them in close proximity), they will properly compensate the bridge for temperature

There are a number of variations in the bridge wiring and the configuration of active and dummy gages that can provide suitable performance for different applications. These are described as follows:

Half Bridge Configuration:

In this case two dummy resistances are provided in adjacent arms (#3 and #4) as shown and the bridge output is then given from Eq. 13 by:



$$e = \frac{GF}{4}(\epsilon_1 - \epsilon_2)E + \frac{GF}{4}(\epsilon_1^T - \epsilon_2^T)E$$

since gages #3 and #4 are dummy resistances and therefore experience no strain or temperature changes (if located together).

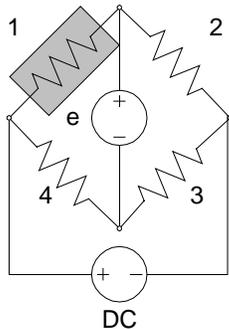
The bridge output is not sensitive to temperature so long as any temperature changes occur equally in the two active arms #1 and #2 and provided the dummy elements are temperature insensitive. An output is produced only when unequal resistance changes are produced in gages #1 and #2. The most useful application of this circuit is in the measure of bending strain in a thin plate or beam. In this case the two gages are mounted opposite each other on opposite (top/bottom) surfaces so that a compressive strain ($-\epsilon_b$) is introduced in one and an equal tensile strain (ϵ_b) in the other. Then:

$$e = \frac{GF}{4}(\epsilon_b - (-\epsilon_b))E = \frac{GF}{2}\epsilon_b E$$

The output is twice that for a single active gage and if the two gages are close to each other the temperature changes will be nearly identical and no temperature sensitivity will be observed. Moreover, if a component of uniform stretching (or compression) is present in addition to the bending, its effect would cancel since it would be the same for each gage!

Quarter Bridge Configuration:

In this case three dummy resistances are provided as shown below and the output is determined from Eq. 13 as:



$$e = \frac{GF}{4}(\epsilon_1)E + \frac{GF}{4}(\epsilon_1^T)E$$

since now gages #2, #3 and #4 are dummy resistances and therefore experience no strain or temperature changes (if located together).

For this arrangement the output will be temperature sensitive unless the active gage (#1) has a zero temperature coefficient. There is no inherent compensation in the bridge circuit except for arms #3, #4 and #2, #3. This circuit is often used when accuracy requirements are not severe (~10 microstrain) because it requires only a single active gage. It also may be the only arrangement possible if differential strain measurement cannot be employed. Temperature compensation may be made, however, if resistance #2 is replaced by a “dummy” strain gage bonded to the same type of material as the active gage (#1) and located as near to it as possible so as to maintain equal temperatures in each. This is essentially a half bridge but with only one active gage and is generally used for high accuracy (2 microstrain) single gage measurements.

Leadwire Temperature/Resistance Compensation

When strain gage installations involve running signal lines of more than about 20 feet through areas of temperature change, a significant error may be introduced by the resistance of the lines or by the temperature induced changes in this resistance. To illustrate this point, a 100 foot length of two-conductor 26 AWG copper wire is used to connect a single strain gage into a quarter-bridge circuit as shown above. The wire has a resistance at 68°F (20°C) of 40.8 Ohms/thousand feet, so for the two leads:

$$R = 40.8 (100/1000) 2 = 8.16 \text{ Ohms}$$

This is a significant resistance compared to the nominal 120 Ohm resistance of a typical strain gage and in addition the wires are also subject to temperature effects!

The most common method for overcoming the effects of leadwire resistance is to introduce the lead resistances into the Wheatstone Bridge in such a way that their effects do not unbalance the circuit. This is accomplished by arranging the connection of the strain gage to the bridge so that one leadwire appears in one arm and the other leadwire appears in an adjacent arm. In this way the initial leadwire resistance appearing in adjacent arms does not unbalance the bridge (e.g., is not noticed). Furthermore, if the temperature changes in each leadwire are the same

(which can reasonably be assumed for the commonly used twisted shielded pair cables) then the corresponding resistance change will appear equally in adjacent bridge arms and therefore not produce an unbalance.

The particular circuitry used for this compensation involves making a three-wire hookup to the strain gage. An example is shown in Figure 4. It should be noted that an apparently superfluous lead wire has been soldered to one of the gage terminals. Closer examination of the circuit should reveal, however, that this junction (A) actually forms one of the four "corner" connections for the Wheatstone Bridge. Two of the wires (AB & A'E) are part of the bridge wiring while the third is either a signal or power lead.

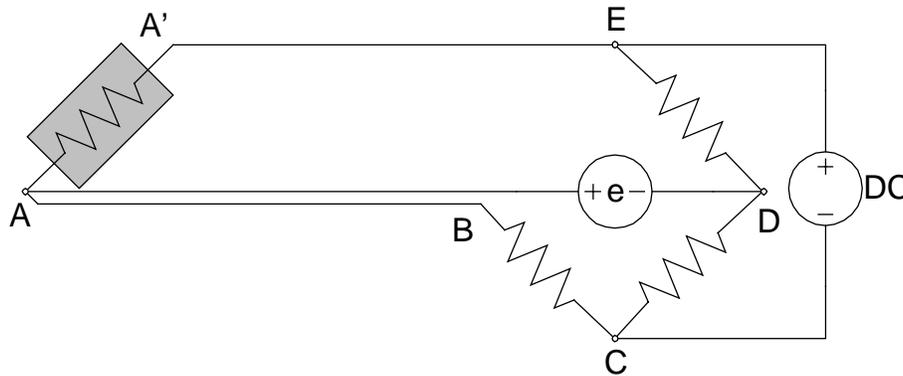


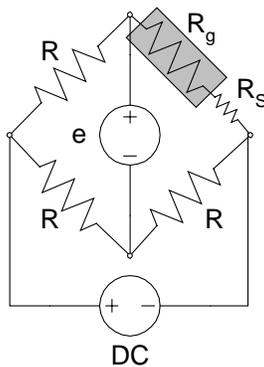
Figure 4. Leadwire Compensation Using a Three-Wire Hookup

Provision is generally made for this type of connection to commercial strain indicating instruments. It is often referred to as a "three-wire hookup."

If the leadwires in the above examples are on the order of 100 feet or more, the added resistance in each arm while not unbalancing the bridge may amount to a substantial fraction of the gage resistance. If this occurs, the effective gage strain sensitivity is reduced because the same ΔR produced by an ϵ is now a smaller fraction of the arm resistance. In other words:

$$\frac{\Delta R_g}{R_g} \geq \frac{\Delta R_g}{R_g + R_{wires}}$$

This situation can be illustrated by considering the effect of a lead-wire resistance, R_s , added to R_g for a single active arm (quarter-bridge circuit) assumed to be initially balanced.



R_s = total resistance of leadwires to gage

The bridge output can be expressed using Eq. 6 for a quarter-bridge configuration as:

$$e = \frac{1}{4} \frac{\Delta R_g}{R_g + R_s} E = \frac{1}{4} \frac{\Delta R_g}{R_g (1 + \beta)} E$$

where

$$\beta = \frac{R_s}{R_g} = \text{leadwire resistance as a fraction of the gage resistance}$$

Then using the definition of Gage Factor from Eq. 3:

$$e = \frac{1}{1 + \beta} \frac{1}{4} \frac{\Delta R_g}{R_g} E = \frac{1}{1 + \beta} \frac{1}{4} GF \epsilon E$$

or

$$e = \frac{1}{4} \frac{GF}{1 + \beta} \epsilon E = \frac{1}{4} GF^* \epsilon E$$

where

$$GF^* = \frac{GF}{1 + \beta} \approx GF (1 - \beta) \quad (14)$$

if it can be assumed that $\beta \ll 1$.

The result demonstrates that leadwire resistance will have the effect of desensitizing the bridge circuit by effectively reducing the gage factor. This suggests a compensation scheme. To correct the readings, one can simply use a new gage factor, GF^* , in calculating strains from voltages. If commercial measuring equipment is used which indicates the strain directly, then the corrected gage factor, GF^* , should be used rather than the value listed for the strain gage. Note that to find β one must either measure (or compute from wire resistance tables) the leadwire resistance, R_s . Also, it should be noted that the effect of R_s is decreased for larger gage resistances (i.e., the effect for a 350 Ohm gage is about 1/3 that for a 120 Ohm gage for the same leadwire length).

Example:

We would like to connect a 120 Ohm gage with $GF=2.0$ to a measuring instrument located 300 feet away. The wire used is 28 AWG copper (28 AWG has a resistance of 64.9 ohms per thousand feet). The corrected gage factor is computed as follows:

$$R_s = 2 \times 300 \times \frac{64.9}{1000} = 38.9 \text{ Ohms}$$

$$\beta = 38.9 / 120 = 0.324$$

$$GF^* = \frac{GF}{1 + \beta} = \frac{2.0}{1.324} = 1.51$$

For a 350 Ohm gage the corrected value would be $GF^* = 1.80$.

BRIDGE BALANCING

In the previous discussions it has been assumed that the Wheatstone Bridge circuit was initially balanced with all resistances equal. In reality, this is not the case due to inherent

irregularities between even the most accurate of strain gages or dummy resistances. As a result, the bridge output voltage, e , is not zero but instead may show an initial unbalance of as much as 0.1% of E , (i.e. up to 10 mV for $E=10V$). This may actually surpass the true strain-induced signal in many cases.

Fortunately; there is a relatively simple and straightforward method for eliminating this unbalance without adversely affecting the basic bridge circuit. It consists simply of some method for adding or subtracting from the resistance in any one arm so as to satisfy the balance requirement:

$$R_1 * R_3 = R_2 * R_4$$

The adjustment may be made singly in one arm or relatively (differentially) in two or, more arms. Figure 5 shows several methods for accomplishing this. In Fig. 5(a) a small resistance is shown added in series with R_1 to increase the arm resistance. The problem here is that this approach can only be used to increase the lefthand side of the above balance equation; thus one must know in advance which arm(s) have the lowest resistance.

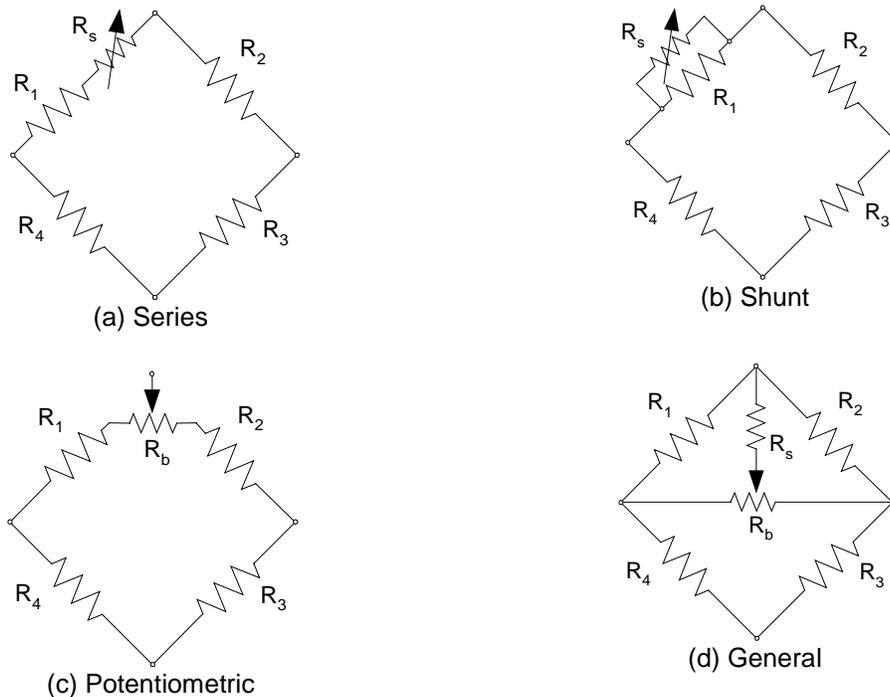


Figure 5. Strain Gage Balancing Circuits

A second problem is that the added balance resistance must be extremely small in order not to unbalance the bridge in the opposite direction. The magnitude of this added balance resistance can be determined from the Wheatstone Bridge equation. In order to cancel an initial unbalance voltage, a resistance change must be produced in one arm that is equal to the equivalent strain-induced resistance change in a gage that would produce the same unbalance voltage. Using Eq. 6 for a single active arm (quarter bridge configuration) it follows that:

$$\Delta R_1 = 4 \frac{e_0}{E} R_1$$

In typical situations, the unbalance (output) voltage is a few millivolts for an excitation voltage of five or ten volts. For example, given, $e_o = 0.005$ V and $E=10$ V:

$$\frac{\Delta R_1}{R_1} = 4 \times \frac{0.005}{10} = 0.002 = 0.2\%$$

or for a 120 Ohm gage:

$$\Delta R_1 = 0.002 \times 120 = 0.24 \text{ Ohms}$$

This is a very small resistance and is; in fact, comparable in magnitude to the contact resistance that develops between mating connectors in a circuit cable. In strain gage bridges, this approach is not practical. On the other hand, a much larger resistance can be added in parallel with an arm to reduce its effective resistance and therefore to balance the bridge (e.g., recall that the resistance, R_p , of two parallel resistors is given by $R_p=R_1R_2/(R_1+R_2)$). This is shown in Fig. 5(b). While this solves the second problem above, one must still know in advance which arm has the largest initial resistance.

These problems can be remedied by employing the differential arrangements shown in Figures 5(c) and 5(d) where the adjustment is made simultaneously in adjacent arms. Since adjacent arm resistances appear in opposite sides of the balance equation above, this method is capable of handling either positive or negative initial unbalance voltages. Again, however, as with the circuit in Fig. 5(a), the arrangement in Fig. 5(c) is not practical because the potentiometer resistance must be extremely small. Large values, even a few Ohms, while maintaining a balanced state will effectively increase the nominal arm resistance and thus desensitize the bridge. This is exactly the same phenomenon noted in the previous section where the use of long leadwires added extra (balanced) resistance but desensitized or reduced the bridge output for a given strain.

The arrangement in Fig. 5(d) offers the most practical and effective way to initially balance a Wheatstone Bridge. Several features are noteworthy. The basic structure of the bridge is preserved with no balancing components inserted directly into any of the legs. This avoids problems with junction contact resistances and desensitization due to added arm resistance. Potentiometer R_b provides a means for differentially adding parallel resistance to both R_1 and R_2 . Assuming for the moment that $R_s=0$, the maximum balancing effect will occur when the wiper is at an extreme position, in which case R_b is in parallel with one arm and the other arm (R_1 or R_2) is shorted. At the midpoint position, $R_b/2$ is in parallel with both R_1 and R_2 . In order not to desensitize the bridge, this parallel combination should not change the bridge arm resistance by more than about 1%. Using the parallel resistance formula, one can show that:

$$R_b \approx 2 \frac{R_i^2}{\Delta R_i} \quad (15)$$

where R_i is a typical arm resistance and ΔR_i is the maximum change allowed. Using $\Delta R_i = 1\%$ for a 120 Ohm gage yields $R_b = 24$ Ohms. Resistance R_s is included in the circuit simply to limit the maximum balance action which occurs at the extreme potentiometer settings. In this case, R_s is in parallel with one arm and R_b is simply connected between opposite bridge corners. Typical values for R_s range between 1 and 5 kOhms. The particular arrangement of R_s and R_b within the bridge is unimportant; R_b can be connected between signal corners or excitation corners with no difference in performance. For best stability and minimum drift, these resistances should be

metal film or wirewound designs (NOT carbon composition) with a temperature coefficient of less than 5 ppm/F.

CALIBRATION

The output from a strain gage bridge is proportional to changes in resistance of all of the arms. In most situations, only one or two arms are active and it is desirable to be able to provide some means of assurance that the circuit is working properly. The Wheatstone Bridge circuit is ideally suited for this purpose because it is relatively easy to affect a change in resistance in one or more arms that is proportional to a known physical parameter. In order to change the resistance in one arm, two approaches are possible as shown in Fig. 6. A resistance may be added in series to increase the arm resistance or else a resistance may be added in parallel to reduce the arm resistance. Of these two, the parallel or shunt connection is preferred because it requires the least modification of the bridge and utilizes a more practical resistance value. The series connection requires very high quality switch contacts in order to maintain bridge continuity and thus preserve accuracy. It also requires use of very small calibration resistances which are not easily fabricated. On the other hand, the parallel connection requires a relatively large resistance and the switch contacts are not a direct (internal) part of the bridge circuit.

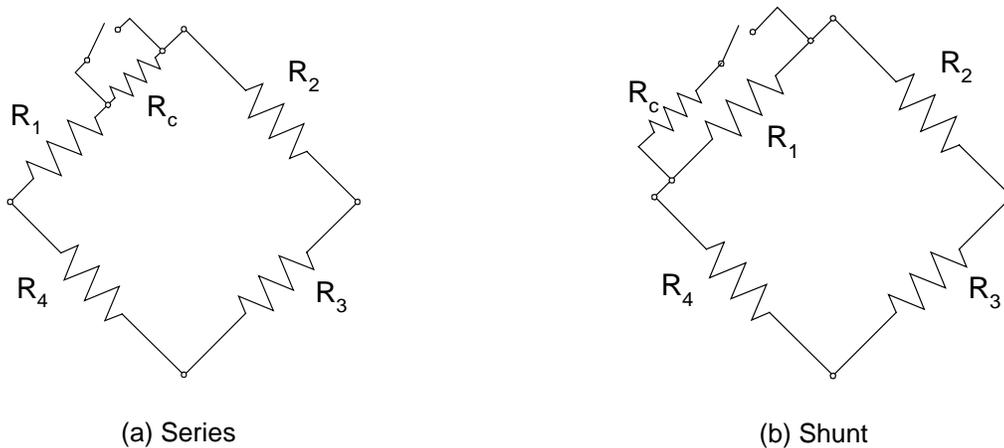


Figure 6. Strain Gage Bridge Calibration Circuits

An analysis of the calibration circuit in Fig. 6(b) can easily be carried out by use of the basic bridge equation (Eq. 6) and the parallel resistance equation. The new or altered arm resistance in Fig. 6(b) is given as:

$$R_1' = \frac{R_1 R_c}{R_1 + R_c}$$

and thus:

$$\Delta R_1 = R_1 - R_1' = R_1 - \frac{R_1 R_c}{R_1 + R_c} = R_1 \left(1 - \frac{R_1 R_c}{R_1 + R_c} \right) = \frac{R_1^2}{R_1 + R_c}$$

The change in resistance is equivalent to an imaginary strain whose magnitude can be computed from the equation defining the gage factor (Eq. 3) as:

$$\varepsilon = \frac{\Delta R_1 / R_1}{GF} = \frac{1}{GF} \frac{R_1}{R_1 + R_c} \quad (16)$$

Normally, this is solved to yield the value of R_c required to produce an equivalent strain output:

$$R_c = R_1 \left(\frac{1}{GF \varepsilon} - 1 \right) \approx \frac{R_1}{GF \varepsilon} \quad (17)$$

where the simplification is based on the fact that ε is a very small value (microstrain).

Example:

Suppose it is desired to connect a shunt calibration resistance across a 120 Ohm gage with $GF=2.09$ to simulate a 2000 microstrain reading. The value of R_c is then given by Eq. 17 as:

$$R_c = 120 \times \left(\frac{1}{2.09 \times 0.002} - 1 \right) = 28,588 \text{ Ohms}$$

This particular value of resistance is not commonly available. Suppose instead that a value for R_c is selected in advance to be $R_c=20$ kOhms. Now, from Eq. 16 the indicated strain will be:

$$\varepsilon = \frac{1}{GF} \frac{R_1}{R_1 + R_c} = 2,854 \text{ microstrain}$$

As a final note, it should be pointed out that at least one strain gage manufacturer makes precision shunt calibration resistors especially for this purpose. The resistances are available for specific combinations of gage resistances, GF values, and apparent or induced “calibration” strains.

Strain Gage Switching Circuits

Strain gages are almost always used in multiples since a common application is to measure the strain at various locations on a part or within a system. It is impractical to use separate strain gage indicator circuits for each strain gage used in such situations, and as a result, some kind of switching circuitry is generally employed. And depending on whether the application is for quarter bridge, half bridge or full bridge gage wiring, the switching circuitry be somewhat different. The following sections discuss some of the alternatives and make note of special problems to anticipate.

Interbridge Switching

The simplest approach is interbridge switching which involves switching complete full bridge circuits as shown in Figure 7. The most common approach is to supply a common excitation voltage to all of the bridges concurrently and then switch the outputs to the strain indicator using a two-pole, multi-position switch. No switching is done within the Wheatstone Bridge itself and the signal switching involves negligibly small currents, the demands on the switch are minimal and modest quality switches may be used without difficulty. If the power is also switched, there will be modest current flowing in the bridge and the switch must be designed accordingly. Switching the power supply is normally only done if the gages must be operated for short periods just to take a reading (this may be done to minimize thermal drift from resistance heating in the gages if very high voltages are used to get the highest possible sensitivities). The key advantage for interbridge switching is that it maintains the full integrity of

the Wheatstone Bridge and can also include balancing circuitry for each switched bridge. Of course the disadvantage is the need to work entirely with full bridges which is often difficult to do. If quarter bridge configurations are used (typically for individual strain measurements), then this interbridge switching circuit will require use of 3 dummy gages for each single active strain gage and this can be costly and complex.

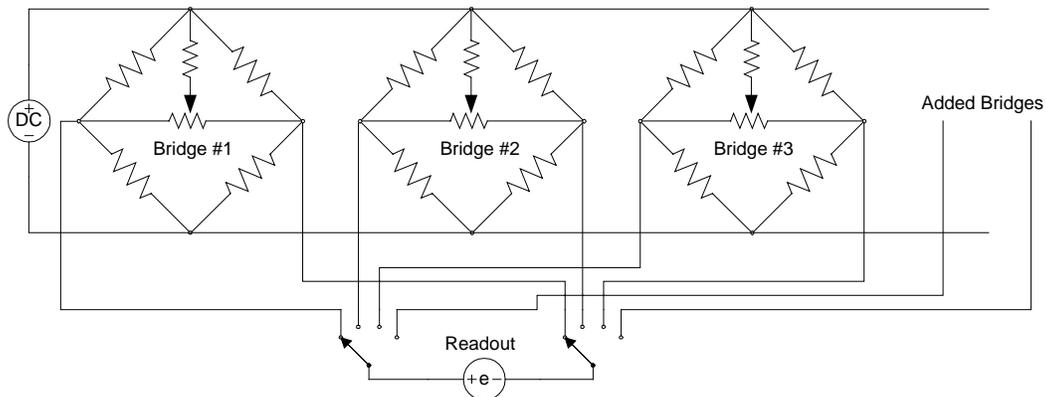


Figure 7. Strain Gage Interbridge Switching

Intrabridge Switching – Quarter Bridge

Intrabridge switching involves switching some of the arms within the bridge while maintaining only a single Wheatstone Bridge circuit. This is the approach most commonly used when working with quarter bridge configurations. Figure 8a shows a typical quarter bridge configuration in which a single pole, multiposition switch is used to complete the fourth arm of the Wheatstone Bridge that contains the active strain gage. This is perhaps the simplest of all the switching configurations in terms of the number of components but it introduces the following problems:

- The added resistance of the switch appears along with the gage resistance for the switched arm and this may be a substantial factor. Switch resistance can also vary considerably between switch operations and this adds a precision uncertainty. High quality switches with gold plated wiping contacts can often overcome these problems but at a cost.
- There is no easy way to balance out any residual strain in individual switched gages without adding additional switch poles.

Intrabridge Switching – Half Bridge

Intrabridge switching can also involve switching two adjoining arms of the Wheatstone Bridge (half-bridge switching) as shown in Figure 8b. In this configuration, two arms (A, B) form one half of the bridge and are common for all of the switched gages. In this particular configuration, all switched gages are powered continuously and only the bridge outputs are switched (the unswitched half of the bridge simply defines the reference for the bridge output circuit). Balancing can also be added to each of the external half bridges using a circuit similar to that shown in Figure 5d. This bridge configuration provides perhaps the best versatility and overall performance. If quarter bridge (single active gage) configurations must be used, this circuit will require addition of a single dummy gage for each active gage.

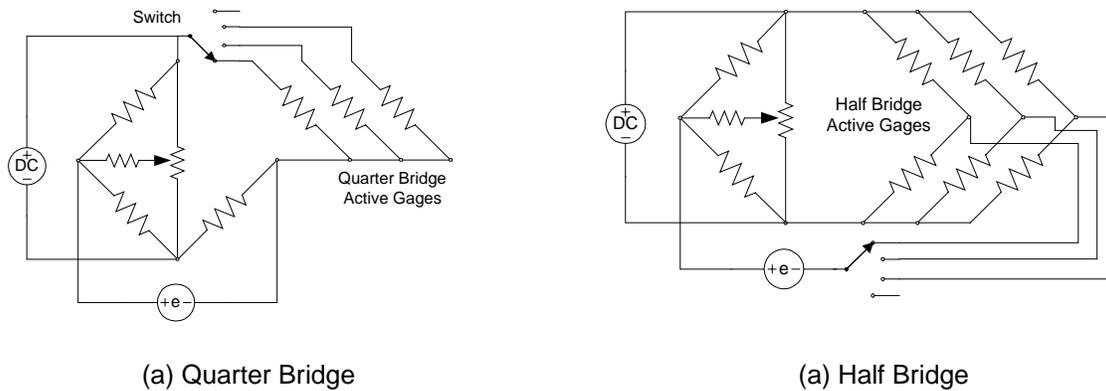


Figure 8. Strain Gage Intrabridge Switching Configurations

Commercial Switching and Balancing Units

When commercial strain indicators are used to manually measure multiple strain gages, a matching commercial switching and balancing unit (SBU) is often employed. These SBU's typically contain provisions for full, half and quarter bridge switching and include separate balancing controls for each channel. For the highest performance and accuracy, the switches and connectors in these SBU's are usually gold plated and designed to minimize spurious contact resistance. The wiring between the SBU and the companion strain indication is usually indicated on the instruments and may seem to bear little resemblance to the circuitry described in these notes. Nonetheless, with a little persistence in studying the schematic diagrams, it should become obvious that simple variations of the same circuitry are involved in all cases!

Strain Gage Applications

One of the most appealing characteristics of strain gages is their versatility, particularly when incorporated into a Wheatstone Bridge. On first exposure, the complexity of the Wheatstone Bridge circuit and the need to provide either active or dummy gages in all 4 arms seems almost overwhelming. It seems downright inefficient to have to provide 3 additional bridge arms just to be able to sense the strain in a single active strain gage! However, on closer inspection and after more extensive experimental work, the versatility of the bridge may become more evident.

For example, while it is often true that only a single strain gage measurement is needed, it is usually also true that the gage is expected to respond to a particular type of loading the designer of the experiment may have gone to great length to assure this. On the other hand, it may be possible to install two strain gages in such a way that the undesired response is automatically rejected by the bridge, even though it has been sensed by the gages themselves! This is particularly important when making measurements of 2D strain or stress states where a rosette gage can be used very effectively to measure the shear strain which cannot even be directly detected by the strain gages themselves.

In this section, a number of different strain gage applications will be described to further illustrate the versatility of these devices and to reinforce the basic and underlying strength of materials theory. Perhaps other applications will suggest themselves.

Bridge Equation

The basic Wheatstone Bridge strain gage circuit can be described by the simple relation developed in Eq. (7):

$$e = \frac{GF}{4}(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4)E \quad (7\text{-repeated})$$

where GF=gage factor, E=bridge excitation voltage and ε_i are the strains sensed by the individual bridge gages. This relation is accurate for most common situations but it does not apply to the following cases: (a) large strains (in excess of 10,000 microstrain), (b) gages with different values for GF in different arms, (d) bridges in which the sensing device is current sensitive rather than voltage sensitive. It is doubtful that any of these limitations will pertain to most applications but they are stated for completeness.

One of the most useful aspects of Eq. 7 is that the strains are additive with alternating signs as one moves around the bridge. As a result, it is possible to deduce these important “bridge rules”:

- equal changes in strain in adjacent arms produces no bridge output,
- equal changes in strain in opposite arms produces bridge output at double the sensitivity of a single active arm,
- opposite changes in strain in adjacent arms produces bridge output at double the sensitivity of a single active arm,
- opposite changes in strain in opposite arms produces no bridge output.
- equal changes in strain in all arms produces no bridge output.

These relatively simple rules are easily confirmed from Eq. 1 and are the source for the different measurement configurations discussed below.

In order to apply these rules and to correctly assess the results, it will be useful to define a nomenclature as follows. Assume that the reference bridge performance is that associated with a quarter-bridge circuit, that is a bridge with only a single active gage. In this case Eq. 7 indicates that the output will be:

$$e = \frac{GF}{4} \varepsilon_1 E$$

To reflect the changes in bridge performance by adding one or more additional active arms, a bridge factor, K, will be added so that the new bridge equation is:

$$e = K \frac{GF}{4} \varepsilon_1 E \quad (2)$$

For example, a quarter-bridge has a value, K=1, while a bridge in which all arms contribute equally and additively, K=4. Practical values lie somewhere in between.

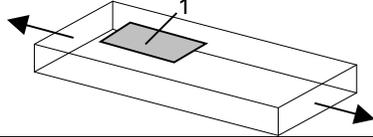
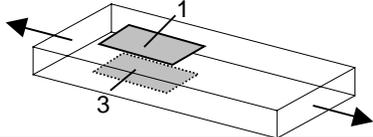
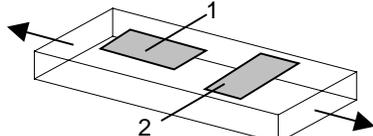
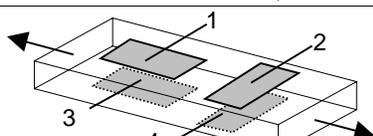
Sensing Strain in a Member Under Uniaxial Load

It is often necessary to measure the strain in a prismatic member (a bar) subjected to an axial load. A common example might be a truss element which is designed to carry axial load. In this case application of a single strain gage oriented in the axial direction on the bar would appear to be sufficient. However, several problems arise. First, there is the problem of what to do about the other 3 arms of the Wheatstone Bridge, but second, it is not always so easy to assume that a single gage will correctly sense the axial strain in the bar. For example, while the stress state

may be uniaxial (consist of only a single nonzero stress), the strain state is not, and there are significant lateral strains that the gage might sense, especially if it is not accurately aligned. Moreover, any slight bending in the member (due to initial eccentricities, for example) or other irregularities might cause the axial strain to vary across the cross section of the bar.

Table 5 below summarizes some of the most common situations that might be encountered and provides wiring notes and K values.

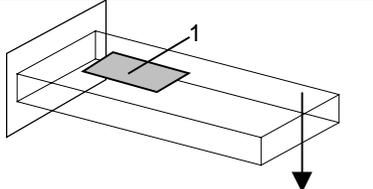
Table 5. Bridge Configurations for Uniaxial Members

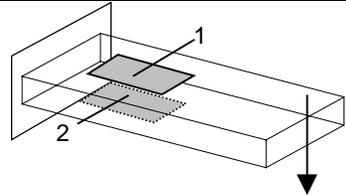
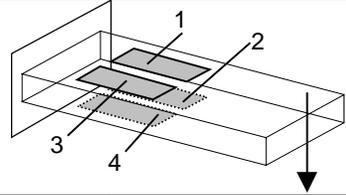
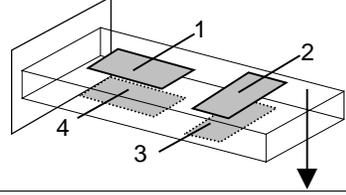
No.	K	Configuration	Notes
A-1	1		Must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation
A-2	2		Rejects bending strain but not temperature compensated; must add dummy gages in arms 2 & 4 to compensate for temperature.
A-3	(1+v)		Temperature compensated but sensitive to bending strains
A-4	2(1+v)		Best: compensates for temperature and rejects bending strain.

Sensing Strain in a Member Under Bending Load

Quite the opposite to what was considered in the previous section can also be true. That is it may be necessary to sense the bending induced strains in a prismatic member and NOT the strains due to axial loading. This is usually the case when dealing with beams or other so-called flexural elements. For these cases, the bridge can be wired so that the equal and opposite strains that are induced on the upper and lower surfaces of a simple beam will appear in adjacent arms where the strains will be combined. Even when the beam cross section is such that the centroid is not equal distances from the top and bottom surfaces (e.g., a ‘tee’ section), the strains will still be of opposite sign and will add constructively in the bridge equation. Table 6 below summarizes the most common configurations for flexural applications.

Table 6. Bridge Configurations for Flexural Members

No.	K	Configuration	Notes
F-1	1		Also responds equally to axial strains; must use dummy gage in an adjacent arm (2 or 4) to achieve temperature compensation

F-2	2		Half-bridge; rejects axial strain and is temperature compensated; dummy resistors in arms 3 & 4 can be in strain indicator.
F-3	4		Best: Max sensitivity to bending; rejects axial strains; temperature compensated.
F-4	$2(1+\nu)$		Adequate, but not as good as F-3; compensates for temperature and rejects axial strain.

It should be noted that in Table 6, no consideration is given to the relative locations of the strain gages along the length of the beam. In practice the bending moment will vary and so will the strains. For example, case F-4 will be less sensitive if gages 2 and 3 experience less strain as a result of experiencing less bending moment (e.g., because they are closer to the load point in a cantilever configuration as illustrated).

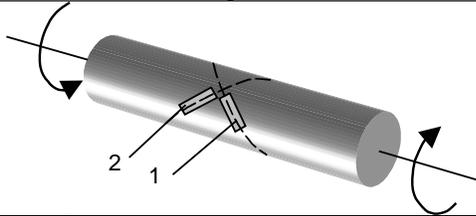
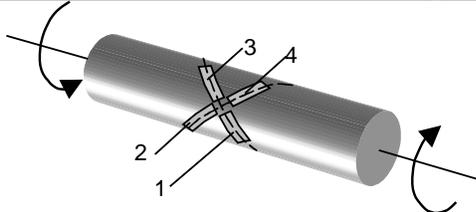
One area of interest for flexural gage installations is in sensing the flexural deformation that accompanies buckling of a slender column. Mounting back-to-back gages near the midpoint of a column in a bridge configuration as shown in F-3, can result in a sensor that responds quite sensitively to small flexural movements as the column is loaded. In this case, a graphical extrapolation procedure such as that due to R.V. Southwell can be effective in indicating the buckling load well before it is reached and damage may occur.

Also it should be pointed out that the configurations for use on flexural elements may apply directly to thin plates when the objective is to sense the bending deformation and not the inplane stretching. Again, back-to-back mounted strain gages when properly wired into a Wheatstone Bridge can often serve to effectively reject the inplane strains while responding to bending strains.

Sensing Torsion Strains in a Circular Shaft

Torsion presents an interesting challenge because the dominant stress is a shear stress, $\tau_{x\theta}$, and therefore it is usually necessary to measure the resulting shear strain, $\gamma_{x\theta}$. We have seen earlier that while strain gages can directly sense only an extensional strain, a rosette can be used to measure the strain state at a point and to thereby infer any particular 2D strain component. In the general case as treated earlier, it is necessary to apply at least 3 gages in a rosette to determine the 2D strain state. However, if some information is already known, it is usually possible to use fewer than 3 gages. This is the case when sensing the torsion strain in a circular shaft where the directions of principal strain are known in advance to be at 45° to the axial direction on the surface of the shaft. In view of this knowledge, a simple two-element “tee” or 90° rosette can be mounted such that the two individual gages are aligned in the principal strain directions. Table 7 below summarizes some of the possible configurations.

Table 7. Bridge Configuration for Torsion Members

No.	K	Configuration	Notes
T-1	2		Half Bridge: Gages at $\pm 45^\circ$ to centerline sense principal strains which are equal & opposite for pure torsion; bending or axial force induces equal strains and is rejected; arms are temperature compensated.
T-2	4		Best: full-bridge version of T-1; rejects axial and bending strain and is temperature compensated.

It probably is obvious but it should be noted that for rotating shafts, some form of commutation is needed to get the signals from the rotating gages. One approach involves use of mechanical slip rings to maintain electrical contact with rotating components. Radio telemetry is also a viable option given the availability of miniature electronics.

Sensing Strain in Rings

The thin ring is one of the most effective structural forms for converting loads and displacements into proportional strains. A structural analysis of the thin or thick ring subjected to various kinds of loadings is beyond the scope of the present notes. Instead, only a simple engineering analysis of the ring will be used to infer the behavior when instrumented with strain gages. Rings are very commonly employed as sensing elements in load cells and for this purpose, more detailed structural models may be required. The basic concept is to apply a diametric load to a thin ring and sense the flexural strains in the inner and outer surfaces of the ring. This is shown in Figure 9 below. A simple analysis of the deformation of the ring (not the initial shape but rather just the change in shape) indicates that there will be four inflection points (where the curvature reverses) at roughly $\pm 45^\circ$ to the load axis. This means that the bending strains will reach local maximum values between the inflection points or roughly on the load axis and at 90° to it. As a result, these are good locations for strain gages, either on the inner or the outer surfaces of the ring. Since these are flexural strains, they will be nearly opposite in sign when located on opposite surfaces of the ring. This is illustrated in Figure 9 where gages are shown only at the middle locations. Using only the inner surfaces is often a better choice because it affords a measure of protection for the gage but it is often more difficult to install the gage.

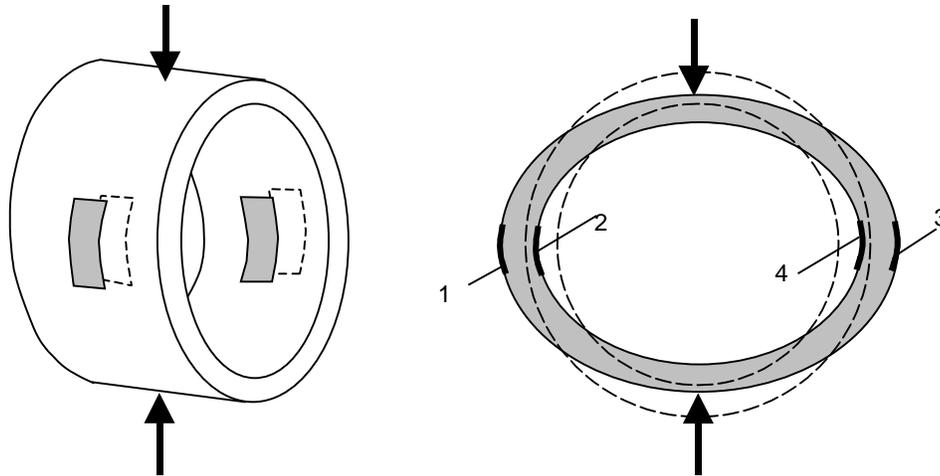


Figure 9. Simple Strain Gaged Ring

A variation of the ring gage involves use on only a half ring, sometimes called a “C” ring for obvious reasons, with the load applied across the opening of the C.. Since this is a statically determinate configuration, it is actually easier to analyze as a structure. In any event, the behavior is similar to that of the full ring but in this case the maximum strains are developed at the midpoint of the ring and this is where strain gages should be located.

It should also be noted that rings are most commonly employed as load transducers so the strain gages sense the strain induced in the ring due to an applied load. With proper design, such devices are very effective load sensors. At the same time, however, if the ring is made as thin as possible it eventually becomes so fragile that it is either a very sensitive load sensor with only a limited range. But at the same time, such a thin ring sensor can also be used as a displacement sensor with quite satisfactory performance. All that is necessary is to add two small spindles where the load is normally applied to transfer the deflection to a diametric change for the ring.

Multi-element Bridges

All of the configurations described above involve the use of up to 4 strain gages in a single Wheatstone Bridge. There is no reason not to add more than one gage in an arm of the bridge provided balancing can still be achieved. Balancing is achieved when the products of the opposite arm resistances are equal ($R_1R_3=R_2R_4$). For example a total of 8 strain gages could be wired with 2 in each arm. If the gages are installed on the structure in such a way so that each pair of gages in a given arm experience strains with the same sign, the bridge will respond accordingly. In general this approach does not yield an improved bridge sensitivity (K factor) because the fractional change in resistance in the arm is still roughly the same. However, the effect is to average the strains in the pair. On the other hand, pairing gages which respond with opposite signs to an external input into a single arm will render the bridge insensitive to that kind of input, which may be the desired result. (This can also be achieved by placement of the gages in a bridge with single elements.) One key advantage of multi-element bridge arms is that the increased bridge resistance will allow use of a higher excitation voltage without inducing excessive thermal heating and drift. The only real drawbacks to multi-element bridge arms are that the increased bridge resistance may increase the noise level in the instrumentation somewhat and they may be incompatible with commercial strain instrumentation that is designed for balancing and calibration of 120 or 350 Ohm bridges only.