

Practice Series Problems Solutions – Math 112 – Fall 2001

1. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2}}$ Compare to $\frac{1}{n^{\frac{3}{2}}}$ to show convergence.
2. $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^3}$ Since \tan^{-1} is always less than $\frac{\pi}{2}$, compare to $\frac{\pi}{2n^3}$ to show convergence.
3. $\sum_{n=1}^{\infty} \left(\frac{n^2-1}{n^3+3}\right)^{\frac{1}{3}}$ Compare to $\frac{1}{n^{\frac{1}{3}}}$ to show divergence.
4. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$ Use ratio test to show convergence.
5. $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$ Use limit comparison to $\frac{10}{n^2}$ to show convergence.
6. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{5^n}$ This is a geometric series that converges to $\frac{4}{3}$.
7. $\sum_{n=1}^{\infty} \frac{n^2+1}{n}$ Use term test for divergence (to show divergence).
8. $\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{1}{n}\right)$ Use term test for divergence (to show divergence).
9. $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$ Use term test for divergence (to show divergence).
10. $\sum_{n=1}^{\infty} \frac{n}{e^n}$ Use ratio test to show convergence.
11. $\sum_{n=2}^{\infty} \left(\frac{1}{\sqrt{n-1}} - \frac{1}{\sqrt{n+1}}\right)$ Use algebra to combine the fractions to get $\frac{2}{n-1}$. This is a multiple of a harmonic series, so it diverges.
12. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ This is a p-series that converges, $p = 3/2 > 1$.
13. $\sum_{n=1}^{\infty} (n+3)!n!3^n$ Use term test for divergence (to show divergence).
14. $\sum_{n=1}^{\infty} \cos(n\pi)$ Use term test for divergence (to show divergence).
15. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+10}}$ Use alternating series to show convergence.
16. $\sum_{n=1}^{\infty} \frac{1}{4^n}$ This is a geometric series that converges to $\frac{1}{3}$.
17. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin^2 n}{n^{\frac{3}{2}}}$ Show converges absolutely by comparison to $\frac{1}{n^{\frac{3}{2}}}$.
18. $\sum_{n=1}^{\infty} \frac{2}{n^2+n}$ Compare to $\frac{2}{n^2}$ to show convergence.
19. $\sum_{n=1}^{\infty} \frac{1}{(n+3)^2}$ Compare to $\frac{1}{n^2}$ to show convergence.
20. $\sum_{n=1}^{\infty} (-1)^n \frac{n^4 3^n}{n!}$ Show converges absolutely by the ratio test.
21. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5^n}$ Compare to $\frac{1}{5^n}$ to show convergence.
22. $\sum_{n=1}^{\infty} \int_n^{n+1} \frac{1}{x^3} dx$ Rewrite as an improper integral and then evaluate to show convergence to $\frac{1}{2}$. Note: This is not the integral test!