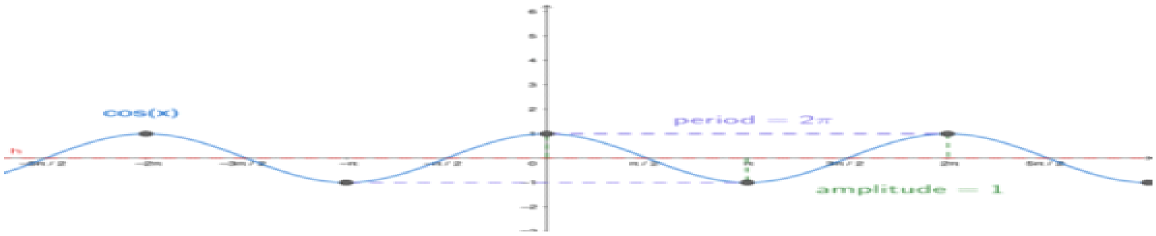
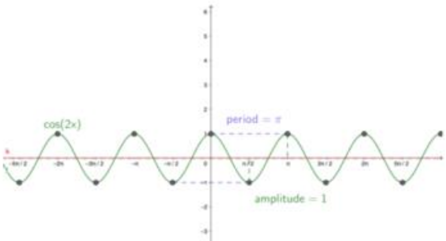
26 January 2023 03:34

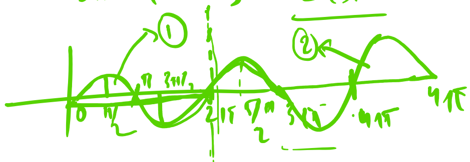
Angles & Arc length



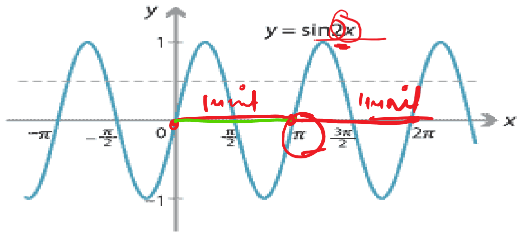


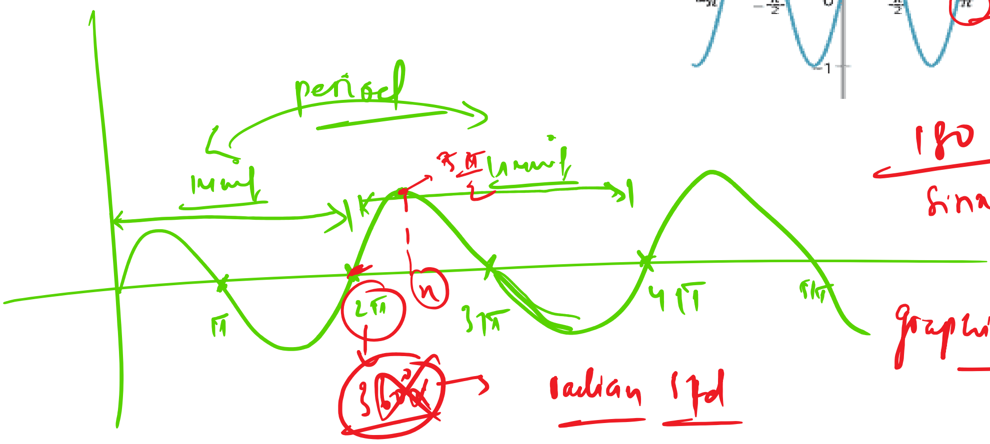
The number 2 in the function cos(2x) horizontally compresses the graph, causing the period to be smaller than the period of cos(x).

sin x → 2π



0 → 2π, 2π → 4π





180°

sin x → 2π / 2 = π

graphical

mathematical $f(x) = f(x + T)$ → eq^n is valid (period)

sin(x)

sin x = sin(x + 2π)

2π + x

cos(x + 2π

sin(5π/2) = sin(2π + x) → sin(x)

sin(x) −

2π → nπ

sin(5π/2) = sin(x)

sin(2π + π/2) = sin(2π + π/2

$\sin(2\pi + \pi/2) = \sin(2\pi + \pi/2)$

$\sin(\pi/2) = \sin(\pi/2)$ this periodic

$\cos(x) \rightarrow$

Q $\cos^2 x$ — periodic

$(2\pi), (2\pi)$

$\boxed{f(x) = f(x+T)}$ $\cos x, \sin x \rightarrow 2\pi$

$\cos^2(x)$ has T period $\rightarrow$

$\cos(x+T)^2 = \cos^2(x)$ $\cos^{-1}$ (inverse of trigonometric function

$\cancel{\cos^{-1}}(\cancel{\cos}(x+T))^2 = \cancel{\cos^{-1}}(\cancel{\cos}^2(x))$

Algebraic

x^2

$(x+T)^2 = x^2$

$\cos^2(x)$

$2n\pi + \alpha^2$

$2n\pi$

$(x+T)^2 = x^2$

$\cancel{x^2} + 2Tx + T^2 = \cancel{x^2}$ $T($

$T = -1$ $2Tx = -T^2$

$2Tx + T^2 = 1$

$2\cancel{T}x = \cancel{T}\left(\frac{1}{T} - T^2\right)$

number

$2Tx + T^2 = 1$ $x(2T + T^2) = 1$

$2\pi n + r^2 = 1$

$n\left(2\pi + \frac{r^2}{n}\right) = 1$

π depend on (α)

$2\pi + \frac{r^2}{n} = \frac{1}{n}$

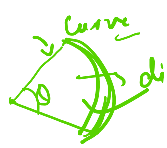
$n \to 0 - 360^\circ$, 2π

Change in

Change in r → not fixed

4–5

Angles & Arc length



→ direction (length), mass

↓

Complex length

length

length

physical quantity

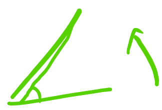
① Mathematics / com

- length ↔ m
- mass → kg
- Temp — K
- Time → sec/min.

Ew – or size → Electricity → Ampere

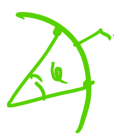
→ Amount of sub → (mole

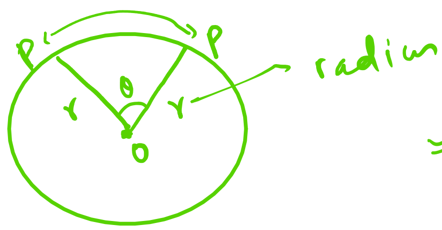
MW → luminous intensity → Candela

⇒ 

Arc length → dist (α) along the part of circumference of any circle or any curve

(12th, 11) ⇒ interspace b/w two points along a section of a curve





Arc → curve of a part of cir

Arc length → length of curve of a part —

↓

(s)

Arc →

Arc length formula

(1) $\Rightarrow \boxed{s = r\theta}$ $\longrightarrow$ $\dfrac{180}{\pi}$ ✗

$\theta \rightarrow$ radian

$\rightarrow \boxed{\theta \times \dfrac{\pi}{180} \text{ or } \rightarrow}$ θ – (degree)

π rad = 180° degree

$\frac{2\pi}{3}$ rad →

$\frac{2}{3}\pi$ radian   180 –   60°

π ÷ $\frac{\pi}{3}$ → Changing

⇒

$\boxed{\pi \text{ rad} = 180^\circ}$

$\boxed{\theta \text{ rad} \rightarrow \left(\dfrac{180}{\pi}\right) d}$

$\boxed{\pi \text{ rad} = 180^\circ \text{ degree}}$

1 rad → $\dfrac{180}{\pi}$ degree

Qus 9 ₹ price 9 apple

price of 12 apple

$\frac{2\pi}{3}$ rad → $\dfrac{\cancel{180}^{60}}{\cancel{\pi}} \times \dfrac{2\cancel{\pi}}{\cancel{3}}$ → 120° degree

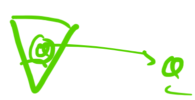
2 apple → ₹ 4

1 → $\frac{₹4}{2}$

12 → $\frac{4}{2} \times 12$

(1) –

$\boxed{\pi \rightarrow 180^\circ}$

Arc measure (θ°) →  θ

The degree measure of an arc is equal

to measure of

to measure of central angle that intercepts the arc.

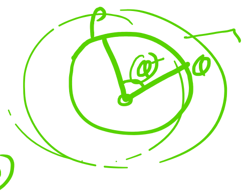
approximation

$$\frac{\text{Arc measure } (\theta)}{360^\circ} = \frac{\text{arc length}}{\text{circumference}}$$

deg^n

$$\frac{\theta}{360^\circ} = \frac{\text{Arc length}}{\text{circumference}}$$

$\angle s$ $\quad$ $2\pi r$



$\theta \rightarrow 360$

$\theta \rightarrow 60$

$$\frac{30^\circ}{360} = \frac{2\,\text{cm}}{2\pi \times 7\,\text{cm}}$$

θ in circle O $r = 8$ inches and minor arc $\overset{\frown}{AB}$ is intercepted by central $\angle s$ of 110° (degree). Find the length of $\overset{\frown}{AB}$

$\angle s$

$$\frac{44\pi}{9}$$

$$s = r\theta$$

$$8 \times 110^\circ \times \frac{\pi}{180^\circ}$$

π rad $\rightarrow 180^\circ$

$\frac{\pi}{180} \leftarrow 1^\circ$

Radian measurement

one radian $\rightarrow$ central angle that subtends an arc length of one radius ($s = r$)

$s = r\theta$ (1)

$\boxed{s = r}$

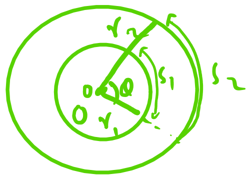
$360^\circ =$

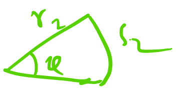
Relationship b/w degree & radian

Justify the length of the arc intercepted by a central angle is proportional to the radius

Concentric (common) centre

$$\frac{S_1}{S_2} = \frac{r_1}{r_2}$$





$S_2 = r_2\theta$ — (1)

$S_1 = r_1\theta$

$$\frac{S_1}{S_2} = \frac{r_1}{r_2}$$

$$\boxed{S = \frac{\theta}{360} \times 2\pi r}$$

arc

Q Calculate the arc length of a curve with sector area 25 sq

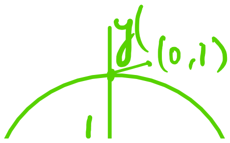
Area of sector $= \frac{1}{2} \times r^2\theta$

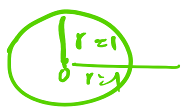
Definition of trigonometric function with the help of unit circle

$\sin^2\theta + \cos^2\theta = 1$

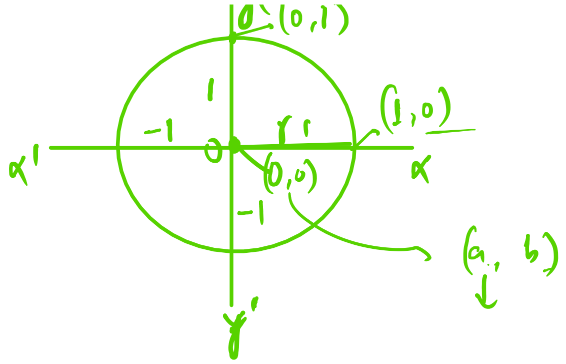
have radius = 1

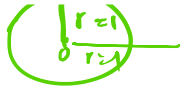
$1 - \sec^2\theta =$





Step (i)





Step (ii)

circle → $\boxed{x^2+y^2=r^2}$ → result

center, radius

$x^2+y^2+6x+9y+9=0$

gen $\boxed{(x-a)^2+(y-b)^2=r^2}$

$a, b \to$ center $2\times(3)$ $2\times(4.5)$

$\downarrow g$ $\downarrow f$

0, 0

$(x-0)^2+(y-0)^2=r^2$

$\boxed{x^2+y^2+2gx+2fy+c=0}$

$x^2+y^2=r^2$ (unit circle)

$\downarrow$

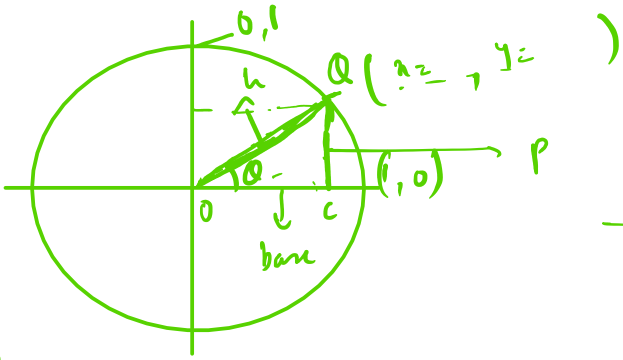
1

center → $(-g, -f)$

$\boxed{x^2+y^2=1}$

$(-3, -4.5)$

$\boxed{\text{radius} = \sqrt{g^2+f^2-c}}$



⊥ = y axis

in △OCQ

$\sin\theta = \dfrac{CQ}{OQ} = \dfrac{P}{h}$ ← → 1

$\sin\theta = x_y$ — (i)

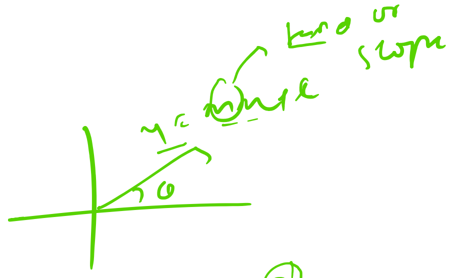
Trigonom

$\sin\theta = \frac{y}{1}$ — (i)

$\cos\theta = \frac{OC}{OQ} \Rightarrow \frac{x}{1}$

$\cos\theta = x$ — (ii)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \boxed{\frac{y}{x}}$ = m or slope

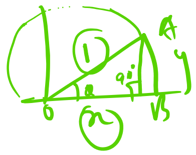


$y = 2x + 2$

in face $\frac{dy}{dx} = 2$

$\boxed{x^2 + y^2 = 1}$ — (i)

$\boxed{\cos^2\theta + \sin^2\theta = 1}$



$(OA)^2 = OB^2 + AB^2$

$\boxed{1 = x^2 + y^2}$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\frac{1}{\sin\theta}$

% $\cos^2\theta$

$(\cos^2\theta + \sin^2\theta = 1)$ % $\sin^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$1 + \tan^2\theta = \sec^2\theta$ (% $\cos^2\theta$)

$\frac{1}{\cos^2\theta}$

$\cos^2\theta + \sin^2\theta = (1)$ %

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\sec^2\theta - \tan^2\theta = 1$$

(1) $\sin^2\theta + \cos^2\theta = 1$

$$\frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$1 = \sec^2\theta - \tan^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$\cot^2\theta$

$$\rightarrow \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$1 + \tan^2\theta = \frac{1}{\cos^2\theta}$$

$$1 + \cot^2\theta = \frac{1}{\sin^2\theta}$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} \quad \sec\theta$$

EXERCISE 3

$\left(\text{Take } \pi = \frac{22}{7}\right)$

1. Express the following angles in degrees : $\frac{\pi}{6}, \frac{14}{15}\pi, \frac{11}{18}\pi, \frac{7}{90}\pi$.
2. Express the following angles in radians (*i*) 1′ (*ii*) 20° (*iii*) 135°.
3. Express in radians and also in degrees the angle of a regular polygon of (*i*) 40 sides, (*ii*) *n* sides.
4. The perimeter of a certain sector of a circle is equal to the length of the arc of the semi-circle having the same radius, express the angle of the sector in degrees, minutes and seconds.
5. The length of a pendulum is 8 m while the pendulum swings through 1/5 rad, find the length of the arc through which the tip of the pendulum passes.
6. The minute hand of a clock is 15 cm long. How far does the tip of the hand move during 40 minutes ? (Take $\pi = 3.14$)
7. A central angle of a circle of radius 50 cm intercepts an arc of 10 cm. Express the central angle θ in radians and in degrees.
8. The moon's distance from the earth is 360000 km and its diameter subtends an angle of 31′ at the eye of the observer. Find the diameter of the moon.
9. A railway train is travelling on a curve of 750 m radius at the rate of 30 km/h, through what angle has it turned in 10 seconds ?
10. A horse is tethered to a stake by a rope 810 cm long. If the horse moves along the circumference of a circle always keeping the rope tight, find how far it will have gone when the rope has traced out an angle of 70°?
11. The area of a sector is 5.024 cm² and its angle is 36°. Find the radius, ($\pi = 3.14$).
12. Find the area of sector of a circle, radius 5 m bounded by an arc of length 8 m.
13. The diagram shows a windscreen wiper cleaning a car windscreen.
    (*i*) What is the length of the arc swept out?
    (*ii*) What area of the windscreen is not cleaned?

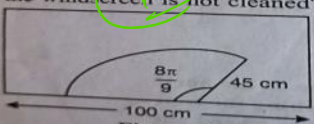


Fig. 3.16

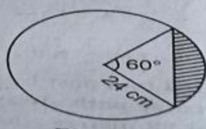
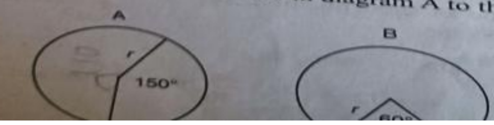


Fig. 3.17

14. Find the area of the shaded segment (Fig. 3.17).
15. What is the ratio of the areas of the major sector in diagram A to the minor sector in a diagram B?



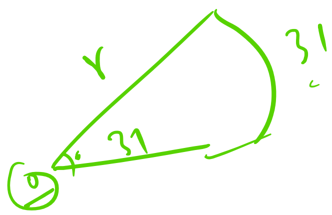
Angles
1. 30°
3. $\frac{19}{20}$
5. 12
8. 324
10. 99
13. (*i*)
4. (*i*)
(*ii*)
5. 2*r*
1. D
2. R
3. R

1° = 60 min
1 min = 60 sec
60° → 360°
30 km / hr
240° degree
$\frac{240 \times \pi}{180} = \frac{4\pi}{3}$ ⇒ rad
rad → 15 cm
exp
write
step

Steps in chap

30 km / hr
30 km → in hr
1 hr

speed = dist / t
velocity
vector
direction
disp
rate
$\frac{\Delta v}{\Delta t}$ → long
$\frac{dv}{dt}$ ∝ acc
instant
magnitude
0.021 sec
m ∝ acc
⇒ m × a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3!
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