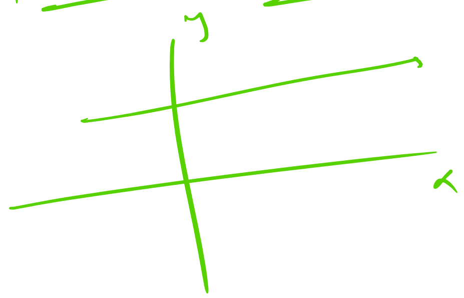
various form of Equation of a Line

Horizontal



Q find eqⁿ ||r to axes passes through (-2, 3)

↓ x, y

$$y = mx + c \quad \text{—} ①$$

$$② \quad y - y_0 = m(x - x_0) + c \qquad c = 0, \; c = ?$$

$$y = 3, \quad \boxed{x = -2}$$

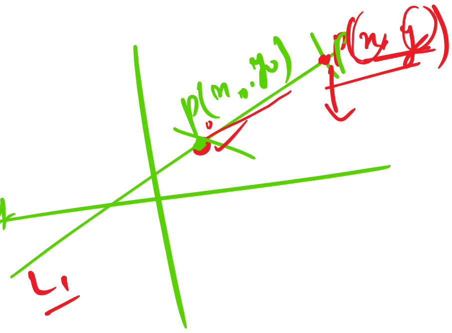
1 point

1 point

point - slope form

general formula two point

$$\tan\theta = m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{—} ①$$



point slope

$P_1(x_0, y_0)$, $P_2(x, y)$

$$m = \frac{y - y_0}{x - x_0} \quad (\text{slope})$$

$$m = \frac{y - y_0}{x - x_0}$$

$$\cancel{y = mx + c}$$

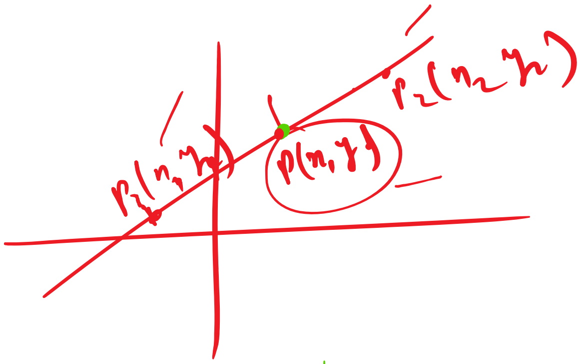
$$\boxed{y - y_0 = m(x - x_0)} \longrightarrow \text{point slope}$$

$p(x_0, y_0)$ ①

let $p(x, y)$ point on the line

$$m = \frac{y - y_0}{x - x_0} \Rightarrow \boxed{y - y_0 = m(x - x_0)}$$

Two - point



Two point

Colinear

Colin

slope of line

① $\dfrac{y - y_1}{x - x_1} = m_1$, $m_2 = \dfrac{y_2 - y}{x_2 - x}$

⑪ $m_3 \Rightarrow \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = m_3$ ① (colinear)

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

m

m

$$\boxed{y - y_1 = m(x - x_1)}$$

Two point

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

Q write eqn $P_1(1, -1)$ & $P_2(3, 5)$

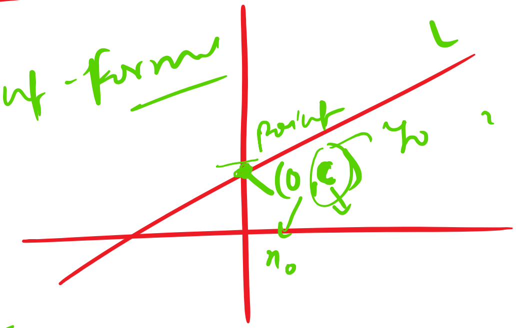
$y - 3x + 4 = 0$

- slope - intercept form

Case ①

$$\boxed{y = mx + c}$$

point - form



$y - c = m(x - 0)$

y-intercept

$$\boxed{y = mx + c}$$

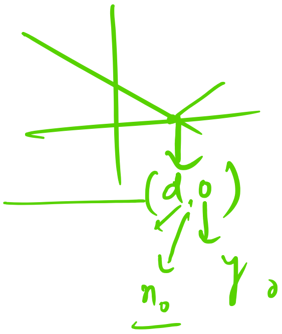
② $\boxed{y - y_0 = m(x - x_0)}$

Two point

m Two m

① $\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$

Case (ii) x-axis intercept 'd'



using y

$y - y_0 = m(x - x_0)$

$y - 0 = m(x - d)$

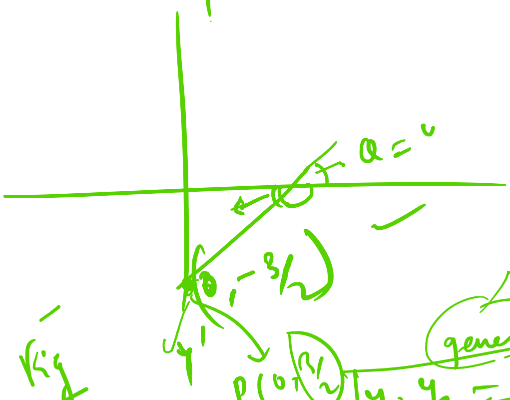
$$\boxed{y = m(x - d)}$$

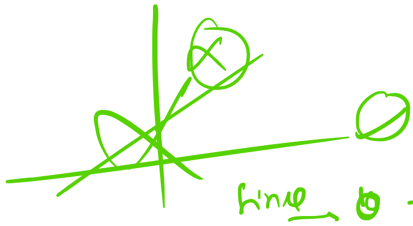
x-intercept

$\tan\theta = \frac{1}{2}$ (i)

θ is inclination

(i) y-intercept $= \left(-\frac{3}{2}\right)$ (ii) $x \to y$





fig

$P_1\left(0, -\frac{3}{2}\right)$ point

general

$$\boxed{y - y_1 = m(x - x_1)}$$

$y + \frac{3}{2} = m(x - 0)$

$y = \frac{1}{2}x - \frac{3}{2}$

$\tan\theta = \frac{1}{2}$

$\sin\theta \to$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta \to$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

1.107 rad

$1.107 \times \frac{180}{3.14}$

63.435

$y = \frac{x-3}{2}$

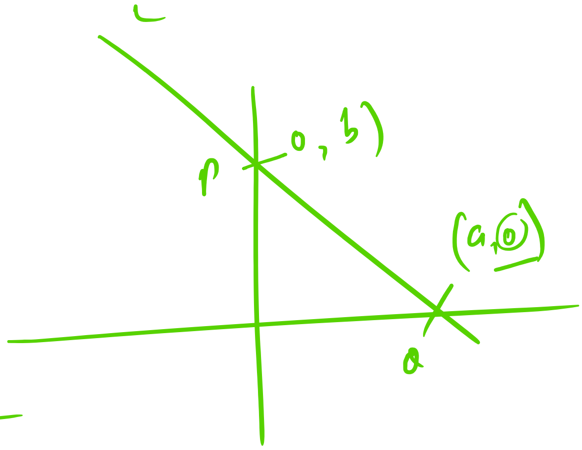
$$\boxed{2y = x - 3}$$

(ii) $2y = x - 4$

(9)

Carry 1, (8, 9)

intercept form



this general for

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 0 = \frac{b - 0}{0 - a}(x - a)$$

$$y = \frac{b}{-a}(x - a)$$

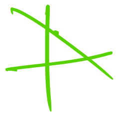
$$\frac{bx}{-a}$$

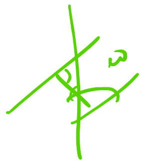
$$\frac{y}{b} = \frac{-x}{a} + 1$$

$$\boxed{\frac{y}{b} + \frac{x}{a} = 1}$$

$$\boxed{\frac{y}{b} + \frac{x}{a} = 1}$$

$$\boxed{\frac{y}{b} + \frac{x}{a} - 1 = 0}$$





Normal Form

(i) length of ⊥ (normal) from origin to the line

(ii) Angle which normal makes with the positive direction of x-axis.

polar

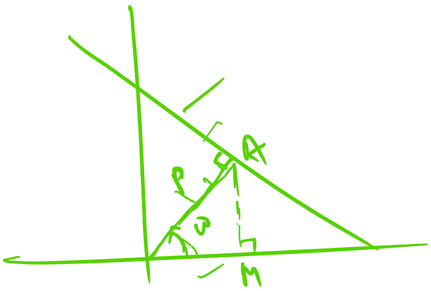
cartes

$$\boxed{y - p\sin\omega = -\frac{\cos\omega}{\sin\omega}\left(x - p\cos\omega\right)}$$

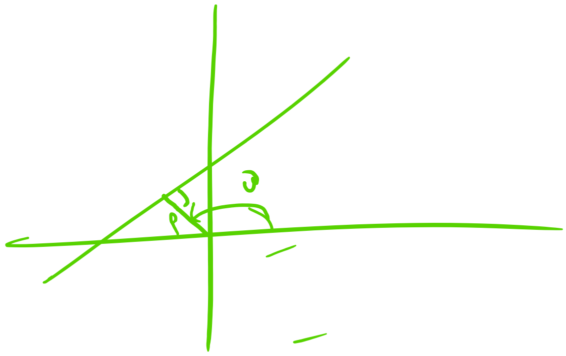
$$\downarrow$$

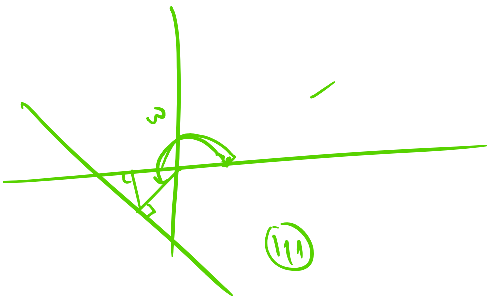
$$\boxed{x\cos\omega + y\sin\omega = p}$$

new

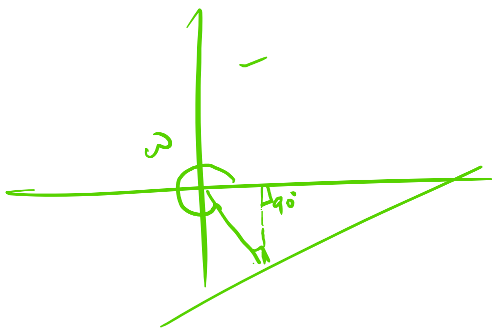


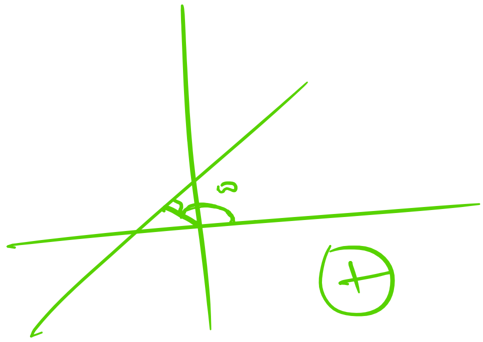
(i)





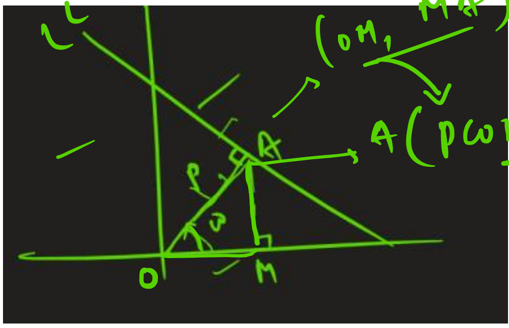
(iii)





$\oplus$

in each case



(OM, MA) → A(p cos ω, p sin ω)

$$\boxed{OM = p\cos\omega}$$

$$MA = p\sin\omega$$

$L \perp OA$ perpendicular

$(\because m_1 m_2 = -1)$

slope of the line $L = -\dfrac{1}{\text{slope } OA}$

inverse

$$\boxed{\text{slope of } L = -\frac{1}{\tan\omega} = -\frac{\cos\omega}{\sin\omega}}$$

$m_L \cdot m_{OA} = -1$

$$y - p\sin\omega = -\frac{\cos\omega}{\sin\omega}\left(x - p\cos\omega\right)$$

$$y\sin\omega - p\sin^2\omega = -x\cos\omega + p\cos^2\omega$$

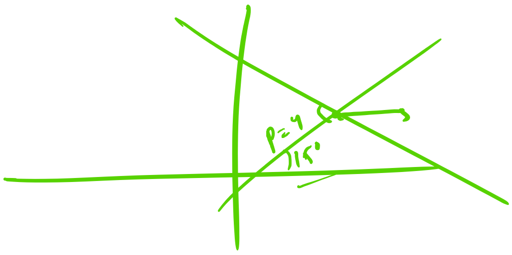
$$y\sin\omega + x\cos\omega = p\left(\cos^2\omega + \sin^2\omega\right)$$

$$1$$

$$\boxed{y\sin\omega + x\cos\omega = p}$$

$$\boxed{x = my + c}$$

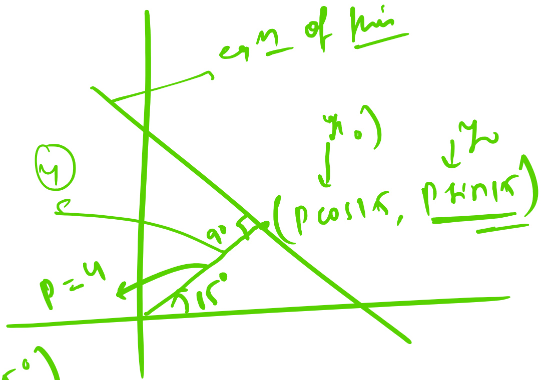
eqn of line whose normal dist (p) from origin ~~and~~ ~~$\angle \to \omega$~~ ~~cos~~ is 4 unit and $\angle$ → 15° +ve axis axis



Q eqn of line whose $\perp^r$ dist from (0,0) is 4 unit & $\angle$ which it makes normal to line is 15° with +ve x-axis

$$y - y_0 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_0)$$

tan15°



$$y - p\sin 15 = \tan 15°\,(x - p\cos 15°)$$

$$\frac{\sin 15}{\omega}$$

$$y\sin 15 + x\cos 15 = 4$$

$y \frac{}{2\sqrt{2}} + x \frac{\sqrt{3}+1}{2\sqrt{2}} = 4 \rightarrow$

Normal form

General equation of line

$\boxed{y = mx + c}$

$Ax + By + C = 0$ Que $\quad y = mx + c$

$By = -Ax - C$

$y = -\frac{A}{B}x -$

slope form

$y = mx + c$

$By = -C - Ax \qquad y = mx + c$

$y = -\frac{A}{B}x - \frac{C}{B}$ — (1)

$-\frac{A}{B} = m, \quad -\frac{C}{B} \rightarrow$ const

intercept form

either axes

$Ax + By + C = 0$

$y-i$, $-2, +ve$, $+ve$

make

slope

$y = -\frac{A}{B}x - \frac{C}{B}$

$C \neq 0$

$y = -\frac{C}{B}$

$0 =$

$x \rightarrow -\frac{A}{B}$

$y = -\frac{A}{B}x - \frac{C}{B}$

$$\boxed{\frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1}$$

$$\frac{-C}{A} \to a \qquad \frac{-C}{B} \to b$$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

$y \to$ $x \to 0$

$$y = -\frac{A}{B}x - \frac{C}{B}$$

$$y = -\frac{C}{B}$$

x - intercept $y \to 0$

$$0 = -\frac{A}{B}x - \frac{C}{B}$$

$$\frac{C}{\cancel{B}} = -\frac{A}{\cancel{B}}x$$

$$x = -\frac{C}{A}$$

$$Ax + By + C = 0$$

$$\frac{A}{\cos\omega} = \frac{B}{\sin\omega} = -\frac{C}{p}$$

—— α ———— α ————

(1) $3x - 4y + 10 = 0$

(i) slope (ii) x- and y intercepts

using slope

$$\Rightarrow y = \frac{3}{4}x + \frac{5}{2}$$

$$\frac{dy}{dx}\left(\;\right) = 3x - 4y + 10$$

$$3x + 10 = 4y$$

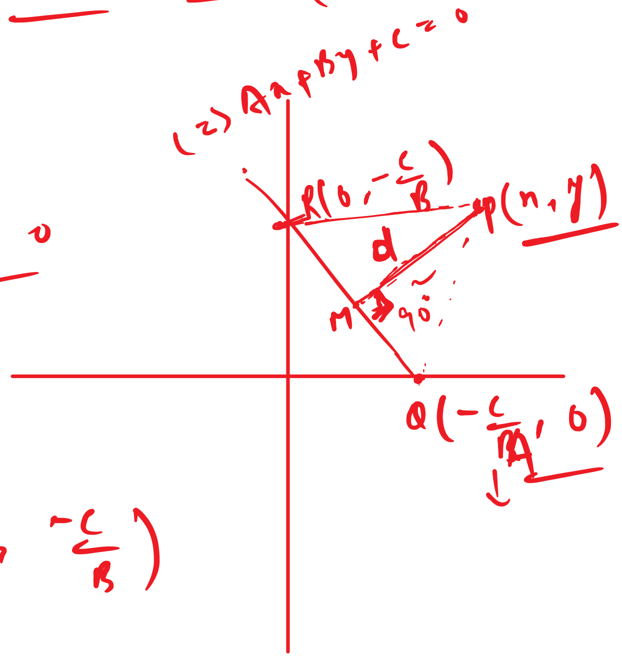
$$3 + 0 = 4\frac{dy}{dx} \quad m$$

$$\boxed{\frac{3}{4} = m}$$

Normal Dist of a point from a line ($\perp^r$

general equation

$l: Ax + By + C = 0$



$Q(-\frac{C}{A}, 0)$, $R \to (0, -\frac{C}{B})$

$P(x, y)$

area of $\triangle PQR = \frac{1}{2} \times PM \times QR$

$\Rightarrow PM = \frac{2 \text{ area of } \triangle PQR}{QR}$ —— (1)

area of PQR $\Rightarrow \frac{1}{2}\left[x_1\left(0+\frac{C}{B}\right) + \left(-\frac{C}{B}\right)\left(-\frac{C}{B} - y_1\right) + 0(y_1 - 0)\right]$

$\Rightarrow \frac{1}{2}\left| x_1\frac{C}{B} + y_1\frac{C}{B} + \frac{C^2}{AB} \right|$

PM = (2)

2 area of PQR $= \left|\frac{C}{AB}\right| \cdot |Ax_1 + By_1 + C|$

$QR = \sqrt{\left(0+\frac{C}{A}\right)^2 + \left(\frac{C}{B} - 0\right)^2} = \left|\frac{C}{AB}\right| \cdot \sqrt{A^2+B^2}$

$$\boxed{PM = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$

$\cdot p(x_1, y_1)$

$$3x + 4y + 7 = 0$$

$p(3, 4)$

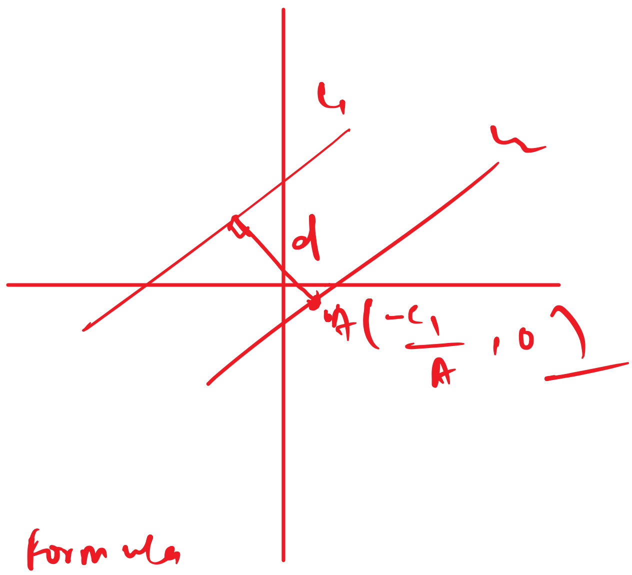
$$d = \frac{3 \times (3) + 4(4) + 7}{\sqrt{3^2 + 4^2}}$$

$$\Rightarrow \frac{9 + 16 + 7}{\sqrt{9+16}} \Rightarrow \frac{32}{5}$$

dist b/w two $\parallel$ lines

$y = mx + c_1$ —— (I)

$y = mx + c_2$ —— (II)



from $\perp^r$ dist formula

$$d = \left| \frac{(\cancel{m})\left(\frac{-c_1}{\cancel{m}}\right) + (-c_2)}{\sqrt{1+m^2}} \right|$$

$$d = \left| \frac{c_1 - c_2}{\sqrt{1+m^2}} \right|$$

Q1 Find dist of the point $(3, -5)$ from line $3x - 4y - 26 = 0$

$\frac{3}{5}$

Q2 find dist b/w $\parallel^r$ lines $3x - 4y + 7 = 0$

$3x - 4y + 5 = 0$

$\hookrightarrow \frac{2}{5}$

$$\frac{7-5}{\sqrt{3^2+4^2}} = \frac{2}{5}$$

✓ perpendicular intersection

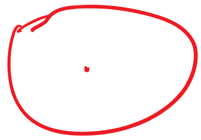
$$m_1 m_2 = -1$$

prove what is equation of the line that is ⊥' to $4x - 3y = 6$ through $(4, 6)$

$$m_1 m_2 = -1$$

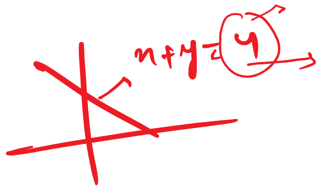
Circle

locus →



→ geometry condition the path trace out by a point in the plane is → locus

$$x + y = 4$$

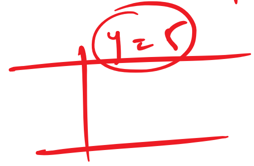


any random point lying x and y

If you take any other not on the line

add it → 4

Ex → find the locus of point moving on a plane which is at fixed dist r unit from x - axis.

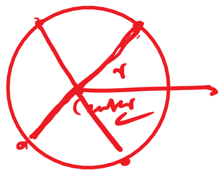


$y = r$

every point on which locus or curve has y - (r, or every point being at dist of r unit from x - axis

locus

↳ locus



either

random point

locus of the set of all points that are at fixed dist from a fixed point

circle → fixed point center

= fixed dist → radius

Q find the locus of a point that is at a distance of 4 unit from a point (-3, 2) in xy plane

$x^2 + y^2 + 6x - 4y - 3 = 0$

$x^2 + y^2 + 6x - 4y - 3 = 0$

find the locus of point which is at fixed dist 4 unit from origin