Permutations and Combinations

Fundamental principle of counting:

There are two fundamental counting principles i.e. *Multiplication principle* and *Addition principle*.

Multiplication principle: If an operation can be performed independently in 'm' different ways, and another operation can be performed in 'n' different ways, then both operations can be performed by m x n ways.

In other words, if a job has n parts and the job will be completed only when each part is completed, and the first part can be completed in a_1 ways, the second part can be completed in a_2 ways and so on… the n^{th} part can be completed in a_n ways then the total number of ways of doing the jobs is $a_1.a_2.a_3\cdots\cdots\cdots a_n$.

Ex: – A person can travel from Sambalpur to Bargarh in four routes and Bargarh to Bolangir in five routes then the number of routes that the person can travel is from Sambalpur to Bolangir via Bargarh is 4 x 5 = 20 routes.

Addition principle: If one operation can be performed independently in 'm' different ways, a second operation can be performed in 'n' different ways, then there are(m + n) possible ways when one of these operations be performed.

Ex: – A person has 4 shirts and 5 pants. The number of ways he wears a pant or shirt is 4 + 5 = 9 ways

Problems:

1. There are three letters and three envelopes. Find the total number of ways in which letters can be put in the envelopes so that each envelope has only one letter. [Ans:6]
2. Find the number of possible outcomes of tossing a coin twice.[Ans:4]
3. In a class there are 20 boys and 15 girls. In how many ways can the teacher select one boy and one girl from amongst the students of the class to represent the school in a quiz competition?[Ans:300]
4. A teacher has to select either a boy or a girl from the class of 12 boys and 15 girls for conducting a school function. In how many ways can she do it?[Ans:27]
5. There are 5 routes from A to B and 3 routes from place B to C. Find how many different routes are there from A to C?[Ans:15]
6. How many three lettered codes is possible using the first ten letters of the English alphabets if no letter can be repeated?[Ans:720]
7. If there are 20 buses plying between places A and B, in how many ways can a round trip from A be made if the return journey is made on
   i) same bus[Ans:20]   ii) a different bus[Ans:380]
8. A lady wants to choose one cotton saree and one polyester saree from 10 cotton and 12 polyester sarees in a textile shop. In how many ways she can choose?[Ans:120]
9. How many three digit numbers with distinct digits can be formed with out using the digits 0, 2, 3, 4, 5, 6.[Ans:24]
10. How many three digit numbers are there between 100 and 1000 such that every digit is either 2 or 9?[Ans:8]
11. In how many ways can three letters be posted in four letter boxes?[Ans:64]
12. How many different signals can be generated by arranging three flags of different colors vertically out of five flags?[Ans:60]

13. In how many ways can three people be seated in a row containing seven seats?[Ans:210]
14. There are five colleges in a city. In how many ways can a man send three of his children to a college if no two of the children are to read in the same college?[Ans:60]
15. How many even numbers consisting of 4 digits can be formed by using the digits 1, 2, 3, 5, 7?[Ans:24]
16. How many four digit numbers can be formed with the digits 4,3,2,0 digits not being repeated?[Ans:18]
17. How many different words with two letters can be formed by using the letters of the word JUNGLE, each containing one vowel and one consonant?[Ans:16]
18. How many numbers between 99 and 1000 can be formed with the digits 0, 1, 2, 3, 4 and 5?[Ans:180]
19. There are three multiple choice questions in an examination. How many sequences of answers are possible, if each question has two choices?[Ans:8]
20. There are four doors leading to the inside of a cinema hall. In how many ways can a person enter into it and come out?[Ans:16]
21. Find the number of possible outcomes if a die is thrown 3 times.[Ans:216]
22. How many three digit numbers can be formed from the digits 1,2,3,4, and 5, if the repetition of the digits is not allowed.[Ans:60]
23. How many numbers can be formed from the digits 1,2,3, and 9 , if the repetition of the digits is not allowed.[Ans:24]
24. How many four digit numbers greater than 2300 can be formed with the digits 0,1,2,3,4,5 and 6, no digit being repeated in any number.[Ans:560]
25. How many two digit even numbers can be formed from the digits 1,2,3,4,5 if the digits can be repeated?[Ans:10]
26. How many three digits numbers have exactly one of the digits as 5 if repetition is not allowed?[Ans:200]
27. How many 5 digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 59 and no digit appears more than once.[Ans:210]
28. In how many ways can four different balls be distributed among 5 boxes, when
    i) no box has more than one ball[Ans:120]
    ii) a box can have any number of balls[Ans:625]
29. Rajeev has 3 pants and 2 shirts. How many different pairs of a pant and a shirt, can he dress up with?[Ans:6]
30. Ali has 2 school bags, 3 tiffin boxes and 2 water bottles. In how many ways can he carry these items choosing one each?[Ans:12]
31. How many three digit numbers with distinct digits are there whose all the digits are odd?[Ans:60]
32. A team consists of 7 boys and 3 girls plays singles matches against another team consisting of 5 boys and 5 girls. How many matches can be scheduled between the two teams if a boy plays against a boy and a girl plays against a girl.[Ans:50]
33. How many non- zero numbers can be formed using the digits 0, 1, 2, 3, 4, 5 if repetition of the digits is not allowed? [Ans:600]
34. In how many ways can five people be seated in a car with two people in the front seat including driver and three in the rear, if two particular persons out of the five can not drive?[Ans:72]
35. How many A.P's with 10 terms are there whose first term belongs to the set{1,2,3} and common difference belongs to the set {1,2,3,4,5}[Ans:15]

Factorial: The product of first n natural numbers is generally written as n! or $\angle n$ and is read factorial n.

Thus, n! = 1. 2. 3.………..n.

Ex: $6! = 6\times5\times4\times3\times2\times1 = 720$

Note:

1) 0! =1
2) (−r)! = ∞

Problems:

1. Evaluate the following:

   i) 7! ii) 5! iii) 8! iv) 8!−5! v) 4!−3! vii) 7!−5! viii) $\frac{6!}{5!}$

   ix) $\frac{7!}{5!}$ x) $\frac{8!}{6!2!}$ xi) $\frac{12!}{10!\,2!}$ xii) (3!)(5!) xiii) $\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}$ xiv) $2!^{3!}$

2. Evaluate $\frac{n!}{r!(n-r)!}$, when

   i) n=7, r=3 ii) n=15, r=12 iii) n=5, r=2

3. Evaluate $\frac{n!}{(n-r)!}$, when

   i) n=9, r=5 ii) n=6, r=2

4. Convert the following into factorials:

   i) 1.3.5.7.9.11 ii) 2.4.6.8.10 iii) 5.6.7.8.9 iv) (n+ 1)(n+ 2)(n+ 3)…………2n

5. Find x if

   i) $\frac{1}{5!}+\frac{1}{6!}=\frac{x}{7!}$ ii) $\frac{1}{8!}+\frac{1}{9!}=\frac{x}{10!}$

6. Find the value of n if

   i) (n+ 1)!=12(n−1)! ii) (2n)!n!=(n+ 1)(n−1)!(2n−1)!

7. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio 2:1 find the value of n.

8. Find the value of x if $\frac{(x+2)!}{(2x-1)!}.\frac{(2x+1)!}{(x+3)!}=\frac{72}{7}$ where $x\in N$

9. Show that n!(n+ 2)=n!+ (n+ 1)!

10. Show that 27! Is divisible by 2^{12}. What is the largest natural number n such that 27! is divisible by 2^{n}.

11. Show that 24! + 1 is not divisible by any number between 2 to 24.

12. Prove that $(n!)^2 \le n^n\, n! < (2n)!$

13. Find the value of x if $\frac{(2x+3)!}{(x+1)!}.\frac{(x-1)!}{(2x+1)!}=7$

14. Prove that the product of k consecutive positive integers is divisible by k! for $k\ge2$

15. Show that 2.6.10……..to n factors $=\frac{(2n)!}{n!}$.

Permutation:- The different arrangements which can be made by taking some or all at a time from a number of objects are called permutations. In forming permutations we are concerned with the order of the things. For example the arrangements which can be made by taking the letters a, b, c two at a time are six numbers, namely,

ab , bc, ca, ba, cb, ac

Thus the permutations of 3 things taken two at a time are 6.

a) Without repetition:

i) If there are n distinct objects then the number of permutations of n objects taking r at a time with out repetition is denoted by ^np_r or p (n ,r) and is defined as

$$^np_r = \frac{n!}{(n-r)!} \quad , \ 0 \le r \le n$$

Proof: Arrangements of n objects, taken r at a time, is same to filling r places with n things

1^{st} place can be filled up in n ways

2^{nd} place can be filled up in n-1 ways

3^{rd} place can be filled up in n-2 ways

……………………………………….

……………………………………….

r^{th} place can be filled up in n-(r-1) ways

$\therefore$ the number of arrangements

$$^np_r = n(n-1)(n-2)\ldots\ldots\ldots\ldots(n-(r-1))$$

$$= \frac{n(n-1)(n-2)\ldots\ldots\ldots\ldots(n-r+1)(n-r)!}{(n-r)!}$$

$$^np_r = \frac{n!}{(n-r)!}.$$

ii) Number of arrangements of n different things taken all at a time without repetition $= {}^np_n = \frac{n!}{(n-n)!} = n!$

b) With repetition:

i) If there are n distinct objects then the number of permutations of n objects taking r at a time with repetition is n^r.

ii) Number of arrangements of n different things taken all at a time with repetition is n^n.

c) If p objects of one kind, q objects of second kind are there then the total number of permutations of all the p + q objects are given by $\frac{(p+q)!}{p!q!}$.

In general If a_i objects of i^{th} kind, i= 1, 2, 3…..,r are there then the number of permutations of all the $a_1 + a_2 + \cdots\cdots\cdots + a_r$ objects is given by $\frac{(a_1+a_2+a_3+\ldots\ldots\ldots+a_r)!}{a_1!a_2!\ldots\ldots\ldots a_r!}$.

d) Circular arrangements:

i) The number of circular arrangements of n distinct objects taking all at a time is (n−1)!

ii) The number of circular arrangements of n distinct objects when clockwise and anti-clockwise circular permutations are considered as same is $\frac{(n-1)!}{2}$.

iii) The number of circular permutations of n different things taken r at a time is $\frac{{}^{n}p_{r}}{r}$ (if clockwise and anti-clockwise circular permutations are considered as different)

Ex: The number of which 29 persons be seated in a round table if there are 9 chairs is $\frac{{}^{29}p_{9}}{9}$

iv) The number of circular permutations of n different things taken r at a time is $\frac{{}^{n}p_{r}}{2r}$ (if clockwise and anti-clockwise circular permutations are considered as same).

Restricted permutations:

1) The number of permutations of n dissimilar things taken r at a time when one particular thing always occurs is r $^{n-1}P_{r-1}$

2) The number of permutations of n dissimilar things taken r at a time when one particular thing taken is $^{n-1}P_{r}$.

3) The number of permutations of n dissimilar things taken r at a time when p particular things always occurs $= {}^{n-p}C_{r-p}.r!$

4) The number of permutations of n dissimilar things taken r at a time when p particular things never occurs ${}^{n-p}C_{r}.r!$

Zero Factorial:

The value of Zero factorial is 1 i.e. 0! =1

Proof:

By the fundamental principle of counting we know that the number of permutations of n different objects taken all at a time with out repetition is $n(n-1)(n-2)..........3.2.1 = n!$ ⋯⋯⋯(1)

And we have seen $^{n}p_{r} = \frac{n!}{(n-r)!}$ ⋯⋯⋯⋯⋯ (2)

From (2) the number of permutations of n different objects taken all at a time with out repetition is

$$^{n}p_{n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

from (1) and (3) $n! = \frac{n!}{0!}$

and this can be hold true if 0! is 1.

$$\therefore 0! = 1$$

Problems:

1. Find r if P(20,r) = 13. P(20,r−1)
2. Find n if P(n,4) = 12. P(n,2)
3. If P(n−1,3): P(n+1,3) = 5 : 12, find n
4. Find m and n if P(m+n,2)=56, P(m−n,2)=12
5. Show that P(n, n) = P(n, n−1) for all positive integers.
6. Show that P(m, 1)+ P(n ,1) = P(m+n, 1) for all positive integers
7. Prove that P(n,n) = 2 P(n, n−2)
8. Find n if ${}^{n-1}P_3 : {}^{n}P_4 = 1:9$
9. Find r if $5\ {}^{4}P_r = 6\ {}^{5}P_{r-1}$
10. If ${}^{n}P_5 = 42\ {}^{n}P_3$, for n>4, then find the value of n.
11. If ${}^{n}P_4 = 360$, find n.
12. If ${}^{n}P_3 = 9240$, find n.
13. If ${}^{10}P_r = 720$, find r.
14. Find n if ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$
15. Prove that ${}^{1}P_1 + 2\ {}^{2}P_2 + 3\ {}^{3}P_3 + 4\ {}^{4}P_4 + \cdots\cdots\cdots + n\ {}^{n}P_n = {}^{n+1}P_{n+1} - 1$
16. In how many ways can five people be arranged in a row? [Ans: 5!]
17. In how many ways can three guests be seated if there are six chairs in your home?[Ans: ${}^{6}p_3$]
18. How many four digit numbers are there, with no digit repeated?[Ans: 9. ${}^{9}p_3$]
19. How many numbers of four digits can be formed with the digits 1, 2,4,5,7 if no digit being repeated?[Ans: ${}^{5}p_4$]
20. How many even numbers of three digits can be formed with the digits 1, 2, 3,4,5,7 if no digit being repeated?[Ans: $2.{}^{5}p_2$
21. How many numbers between 100 and 1000 can be formed with the digits 1,2,3,4,5,6,7 if no digit being repeated? [Ans: ${}^{7}p_3$]
22. How many different numbers greater than 5000 can be formed with the digits 0,1,5,9 if no digit being repeated? [Ans:12]
23. In how many ways can four persons sit in a row?[Ans:4!]
24. In how many ways can three men and four women be arranged in a row such that all the men sit together?[[Ans:5!3!]
25. In how many ways can three men and four women be arranged in a row such that all the men and all the women will sit together?[Ans:2!3!4!]
26. In how many ways can 8 Indians, 4 English men and 4 Americans be seated in a row so that all the persons of the same nationality sit together? [Ans:3!8!4!4!]
27. In how many ways can 10 question papers be arranged so that the best and the worst papers never come together?[Ans:10!−2!9!]
28. In how many ways can 5 boys and 3 girls be seated in a row so that all the three girls do not sit together?[Ans:8!−3!6!]
29. In how many ways can 5 boys and 4 girls be seated in a row so that no two girls sit together?[Ans: ${}^{7}p_4 5!$]
30. In how many ways the word MISSISSIPPI can be arranged?[Ans: $\frac{11!}{4!4!2!}$]
31. In how many ways the word MISSISSIPPI can be rearranged? [Ans: $\frac{8!}{4!4!2!} - 1$]
32. In how many ways the word GANESH can be arranged?[Ans:6!]

33.In how many ways can the word CIVILIZATION be arranged so that four I's come together?[Ans:9!]

34.In how many ways can 4 boys and 4 girls be seated in a row so that boys and girls occupy alternate seats?[2.4!.4!]

35.In a class there are 10 boys and 3 girls. In how many ways can they be arranged in a row so that no two girls come consecutive?[$^{11}p_3 10!$]

36.How many different words can be formed with the letters of the word UNIVERSITY so that all the vowels are together?[Ans:7! $\frac{4!}{2!}$]

37.In how many ways can the letters of the word DIRECTOR be arranged so that the three vowels are never together?[Ans: $\frac{8!}{2!}-\frac{6!}{2!}3!$]

38.Find the number of rearrangements of the letters of the word BENEVOLENT. How many of them end with L.[Ans: $\frac{10!}{3!2!}, \frac{9!}{3!2!}$]

39.In how many ways the letters of the word ALZEBRA can be arranged in a row if

i) the two A's are together[Ans: $\frac{6!2!}{2!}$ ii) the two A's are not together[Ans: $\frac{7!}{2!}-\frac{6!2!}{2!}$]

40.How many words can be formed with the letters of the word PATALIPUTRA with out changing the relative order of the vowels and consonants?[$\frac{6!}{2!2!}.\frac{5!}{3!}$]

41.How many different can be formed if with the letters of the word PENCIL when vowels occupy even places.[$^{3}p_2 4!$]

42.In how many ways can the letters of the word ARRANGE be arranged so that

i) the two R's are never together

ii) the two A's are together but not the two R's

iii) neither the two R's nor two A's are together

41.The letters of the word OUGHT are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word TOUGH in this dictionary.[Ans:89]

42.Find the number of words which can be made using all the letters of the word AGAIN. If these words are written in a dictionary, what will be the fiftieth word?[Ans:NAAIG]

43.In how many ways can 8 people sit in a round table?[Ans:7!]

44.In how many ways three men and three women sit in a round table so that no two men can occupy adjacent positions?[Ans:2!3!]

45.In how many ways a garland can be prepared if there are ten flowers of different colors?[Ans: $\frac{9!}{2}$]

46.In how many ways can four people be seated in a round table if six places are available?

[Ans: $\frac{^{6}p_4}{4}$]

Combination: - The different groups or selections which can be made by taking some or all at a time from a number of things are called combinations. Thus in combinations we are only concerned with the number of things each group contains irrespective of the order.

For examples the combinations which can be made by taking the letters a, b, c two at a time are 3 in number namely, ab, bc, ca

The number of combinations of n dissimilar things taken r at a time denoted by ^nC_r or C(n,r) and is given by $^nc_r = \frac{n!}{r!(n-r)!}$

Proof:

Let there are n objects and let us denote the number of combinations of n objects taking r at a time as ^nc_r. Therefore every combination contains r objects and these r objects can be arranged in r! ways, which gives us the total number of permutations of n objects taking r at a time.

$$\text{Hence } ^np_r = r!\,^nc_r$$

$$\Rightarrow {}^nc_r = \frac{^np_r}{r!}$$

$$\Rightarrow {}^nc_r = \frac{n!}{r!(n-r)!}$$

Note: Relation between ^np_r and ^nc_r is $^np_r = r!\,^nc_r$

Restricted combinations

1) The number of combinations of n dissimilar thing taken r at a time when p particular things always occur = $^{n-p}C_{r-p}$

2) The number of combinations of n dissimilar things taken r at a time when p particular things never occur = $^{n-p}C_r$

Properties of ^nc_r **:**

1) $^nc_r = {}^nc_{n-r} = \frac{n}{r}\,^{n-1}c_{r-1}$

Proof:

$$^nc_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)!}{r(r-1)![(n-1)-(r-1)]!} = \frac{n}{r}\,^{n-1}c_{r-1}$$

3) If $^nc_x = {}^nc_y$ then either $x = y \text{ or } x + y = n$

Proof:

Case (i) given $^nc_x = {}^nc_y$

$$\Rightarrow x = y$$

Case (ii) given $^nc_x = {}^nc_y$

$$\Rightarrow {}^nc_x = {}^nc_{n-y} \Rightarrow x = n - y \Rightarrow x + y = n$$

4) $^{n}c_{r} + {}^{n}c_{r-1} = {}^{n+1}c_{r}$

Proof: we have

$$^{n}C_{r} + {}^{n}C_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r.(r-1)!(n-1)!} + \frac{n!}{(r-1)!(n-r+1).(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!} \cdot \frac{n+1}{r(n-r+1)}$$

$$= \frac{(n+1)n!}{r!(n-r+1)!}$$

$$= {}^{n+1}C_{r}$$

$$Hence\, {}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

5) $^{n}C_{r} = \frac{^{n}p_{r}}{r!} = \frac{n(n-1)(n-2)........(n-r+1)}{r!}$

6) $^{n}C_{n} = \frac{n!}{n!(n-n!)} = \frac{n!}{n!0!} = 1$

7) $^{n}C_{0} = \frac{n!}{0!(n-0!)} = \frac{n!}{n!0!} = 1$

8) $\sum_{r=1}^{n} c(n,r) = \sum_{r=1}^{n} \frac{p(n,r)}{r!} = 2^{n} - 1$

9) Number of divisors or factors of a given number n>1, which can be expressed as $p_1^{k_1} . p_2^{k_2} .... p_r^{k_r}$ where $p_1, p_2, ..., p_r$ are distinct primes and $k_1, k_2, ..., k_r$ are positive integers, are $(k_1+1)(k_2+1).........(k_r+1)$ (including 1and n).

10) Number of selections from n objects, taking at least one is $^{n}c_{1} + {}^{n}c_{2} + {}^{n}c_{3} + \cdots\cdots\cdots. + {}^{n}c_{n} = 2^{n} - 1$

Ex: There are 15 bulbs in a room. Each one of them can operated independently. The number of ways in which the room can be lightened is $^{15}c_{1} + {}^{15}c_{2} + {}^{15}c_{3} + \cdots\cdots\cdots. + {}^{15}c_{15} = 2^{15} - 1$

11) The number selection of r objects out of n identical objects is 1.

12) The number of selection of none or more objects from n identical objects is equal to n+ 1.

13) Number of ways of dividing m different things into 3 sets consisting a, b, c things such that a, b, c are distinct and a + b + c=m is $^{m}c_{a}\, {}^{m-a}c_{b}\, {}^{m-a-b}c_{c} = \frac{m!}{a!b!c!}$

14) Number of ways of distributing m different things among three persons such that each person gets a, b, c things is $\frac{m!}{a!b!c!}3!$

15) Number of ways dividing 3m different things into three groups having m things in each group is $\frac{m!}{(m!)^{3}3!}$

16) Number of ways distributing 3m different things to three persons having m things is $\frac{m!}{(m!)^3}$

17) If there are n points in the plane then the number of line segments can be drawn is $^{n}c_2$

18) If there are n points out of which m are collinear then the number of line segments can be drawn is $^{n}c_2 - ^{m}c_2 + 1 = \frac{1}{2}(n-m)(n+m-1)$

19) If there are n points in the plane then the number of triangles can be drawn is $^{n}c_3$

20) If there are n points out of which m are collinear then the number of triangles can be drawn is $^{n}c_3 - ^{m}c_3$

21) Number of diagonals in a regular polygon having n sides is $^{n}c_2 - n$.

Ex: Number of diagonals in a regular decagon is $^{10}c_2 - 10$.

Problems:

1. Compute the following

   i) $^{12}c_3$ ii) $^{15}c_{12}$ iii) $^{9}c_4 + ^{9}c_5$ iv) $^{7}c_3 + ^{6}c_4 + ^{6}c_3$
2. Prove that $\sum_{r=1}^{5} {}^{5}c_r = 31$
3. Evaluate $^{25}c_{22} - ^{24}c_{21}$
4. If $^{5}c_{3r} = ^{15}c_{r+3}$, find r
5. If $^{18}c_r = ^{18}c_{r+2}$, find $^{r}c_5$
6. Determine n, if $^{2n}c_3 : ^{n}c_3$=11:1.
7. If $^{n}c_8 = ^{n}c_6$, determine n and hence find $^{n}c_2$
8. Determine n, if $^{n}c_6 : ^{n-3}c_3$=33 : 4.
9. Prove that $^{n}c_r \times ^{r}c_s = ^{n}c_s \times ^{n-s}c_{r-s}$
10. If $^{n-1}c_r : ^{n}c_r : ^{n+1}c_r = 6:9:13$, find n and r
11. Find the value of the expression $^{47}c_4 + \sum_{j=1}^{5} {}^{52-j}c_3$
12. How many diagonals does a polygon have?[$^{n}c_2 - n$]
13. Find the number of sides of a polygon having 44 diagonals.[Ans:11]
14. In how many ways three balls can be selected from a bag containing 10 balls?[$^{10}c_3$]
15. In how many ways two black and three white balls are selected from a bag containing 10black and 7 white balls? [$^{10}c_2$ $^{7}c_3$]
16. A delegation of 6 members is to be sent abroad out of 12 members. In how many ways can the selection be made so that i) a particular person always included [$^{11}c_5$] ii) a particular person never included[$^{11}c_6$]
17. A man has six friends. In how many ways can he invite two or more friends to a dinner party?[Ans:57]
18. In how many ways can a student choose 5 courses out of the courses $c_1, c_2, \ldots\ldots\ldots, c_9$ if c_1, c_2 are compulsory and c_6, c_8 can not be taken together?
19. In a class there are 20 students. How many Shake hands are available if they shake hand each other?[$^{20}c_2$]

20.Find the number of triangles which can be formed with 20 points in which no two points are collinear?[${}^{20}c_3$]

21.There are 15 points in a plane, no three points are collinear. Find the number of triangles formed by joining them. [${}^{15}c_3$]

22.How many lines can be drawn through 21 points on a circle?[${}^{21}c_2$]

23.There are ten points on a plane, from which four are collinear. No three of remaining six points are collinear. How many different straight lines and triangles can be formed by joining these points?[Ans: ${}^{10}c_2 - {}^{4}c_2 + 1,\ {}^{10}c_3 - {}^{4}c_3$]

24.To fill 12 vacancies there are 25 candidates of which 5 are from S.C. If three of the vacancies are reserved for scheduled caste, find the number of ways in which the selections can be made. [Ans: ${}^{20}c_9\,{}^{5}c_3$]

25.On a New Year day every student of a class sends a card to every other student. If the post man delivers 600 cards. How many students are there in the class?[Ans:25]

26.There are n stations on a railway line. The number of kinds of tickets printed (no return tickets) is 105. Find the number of stations.[Ans:15]

27.In how many ways a cricket team containing 6 batsmen and 5 bowlers can be selected from 10 batsmen and 12 bowlers?[${}^{10}c_6\,{}^{12}c_5$]

28.How many words can be formed out of ten consonants and 4 vowels, such that each contains three consonants and two vowels?[${}^{10}c_3\,{}^{4}c_2\,5!$]

29.How many words each of three vowels and two consonants can be formed from the letters of the word INVOLUE? [${}^{4}c_3\,{}^{3}c_2\,5!$]

30.A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of i) exactly 3 girls[Ans: ${}^{9}c_4\,{}^{4}c_3$]

ii) at least three girls.[${}^{9}c_4\,{}^{4}c_3 + {}^{9}c_3\,{}^{4}c_4$]

31.A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has i) no girls ii) at least one boy iii) at least one boy and one girl iv) at least three girls.

32.In how many ways four cards selected from the pack of 52 cards? [${}^{52}c_4$]

33.How many factors do 210 have?[16(including 1) and 15(excluding 1)]

34.How many factors does 1155 have that are divisible by 3?[Ans:8]

35.Find the number of divisors of 21600.[71(excluding 1)]

36.In an examination minimum is to be scored in each of the five subjects for a pass. In how many ways can a student fail?[Ans:31]

37.In how many number of ways 4 things are distributed equally among two persons. [$\frac{4!}{(2!)^2}$]

38.In how many ways 12 different things can be divided in three sets each having four things? [Ans: $\frac{12!}{(4!)^3 3!}$]

39.In how many ways 12 different things can be distributed equally among three persons?[Ans: $\frac{12!}{(4!)^3}$]

40.How many different words of 4 letters can be made by using the letters of the word EXAMINATION?[Ans:2454]

41.How many different words of 4 letters can be made by using the letters of the word BOOKLET?[

42.How many different 5 lettered words can be made by using the letters of the word INDEPENDENT?[Ans:72]

43.From 5 apples, 4 oranges and 3 mangos how many selections of fruits can be made?[Ans:119]

44.Find the number of different sums that can be formed with one rupee, one half rupee and one quarter rupee coin.[Ans:7]

45.There are 5 questions in a question paper. In how many ways can boy solve one or more questions?[Ans:31]

Important formulas:

1. The number of arrangements taking not more than q objects from n objects, provided every object can be used any number of times is given by $\sum_{r=1}^{q} n^r$.
2. Number of integers from 1 to n which are divisible by k is $\left[\frac{n}{k}\right]$, where [ ] denotes the greatest integral function.
3. The total number of selections of taking at least one out of $p_1 + p_2 + \dots\dots\dots + p_n$ objects where p_1 are alike of one kind, p_2 are alike of another kind and so on ……..p_n are alike of another kind is equal to $[(p_1+1)(p_2+1)\dots\dots\dots(p_n+1)]-1$
4. The total number of selections taking of at least one out of $p_1 + p_2 + \dots\dots\dots + p_n + s$ objects where p_1 are alike of one kind, p_2 are alike of another kind and so on ……..p_n are alike of another kind and s are distinct are equal to $\{[(p_1+1)(p_2+1)\dots\dots\dots(p_n+1)]2^s\}-1$
5. The greatest value of ${}^nc_r$ is ${}^nc_k$ where

$$k = \frac{n}{2} \, if \; n \in 2m, m \in N$$
$$= \frac{n-1}{2} \; or \; \frac{n+1}{2} \, if \; n \in 2m+1 \, \forall m \in N$$

6. Number of rectangles of any size in a square of size $n \times n = \sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$
7. Number of squares of any size in a square of size $n \times n = \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
8. Number of squares of any size in a rectangle of size $m \times n = \sum_{r=1}^{n} (m-r+1)(n-r+1)$
9. If m points of one straight line are joined to n points on the another straight line, then the number of points of intersections of the line segment thus obtained $= {}^mc_2 \, {}^nc_2 = \frac{mn(m-1)(n-1)}{4}$.
10. Number of rectangles formed on a chess board is ${}^9c_2 \, {}^9c_2$.
11. Number of rectangles of any size in a rectangle of size $m \times n = (n \le m) = {}^{m+1}c_2 \, {}^{n+1}c_2 = \frac{mn}{4}(m+1)(n+1)$
12. The total number of ways of dividing n identical objects into r groups if blank groups are allowed is ${}^{n+r-1}c_{r-1}$.

13. The total number of ways of dividing n identical objects into r groups if blank groups are not allowed is ${}^{n-1}c_{r-1}$.
14. The exponent of k in n! is $E_k(n!) = \left[\frac{n}{k}\right] + \left[\frac{n}{k^2}\right] + \left[\frac{n}{k^3}\right] + \left[\frac{n}{k^4}\right] + \ldots\ldots \left[\frac{n}{k^p}\right]$, where $k^p < n$
15. The sum of the digits in unit's place of the numbers formed by n nonzero distinct digits is
(sum of the digits) (n−1)!
16. The sum of the numbers formed by n nonzero distinct digits is (sum of the digits) (n−1)!$\left(\frac{10^n - 1}{9}\right)$
17. **Derangements:** If n items are arranged in a row, then the number of ways in which they can be rearranged so that no one of them occupies the place assigned to it is $n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots\ldots\ldots + (-1)^n \frac{1}{n!}\right]$

<u>Exercise:</u>

1. In how many ways can 5 beads out 7 different beads be strung into a string?
2. A person has 12 friends, out of them 8 are his relatives. In how many ways can he invite his 7 friends so as to include his 5 relatives?
(a) ${}^{8}C_3 \times {}^{4}C_2$ (b) ${}^{12}C_7$ (c) ${}^{12}C_5 \times {}^{4}C_3$ (d) none of these
3. It is essential for a student to pass in 5 different subjects of an examination then the no. of method so that he may failure
(a) 31 (b) 32 (c) 10 (d) 15
4. The number of ways of dividing 20 persons into 10 couples is
(a) $\frac{20!}{2^{10}}$ (b) ${}^{20}C_{10}$ (c) $\frac{20!}{(2!)^9}$ (d) none of these
5. The number of words by taking 4 letters out of the letters of the word 'COURTESY', when T and S are always included are
(a) 120 (b) 720 (c) 360 (d) none of these
6. The number of ways to put five letters in five envelopes when one letter is kept in right envelope and four letters in wrong envelopes are−
(a) 40 (b) 45 (c) 30 (d) 70
7. ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$ is equal to
(a) ${}^{51}C_4$ (b) ${}^{52}C_4$ (c) ${}^{53}C_4$ (d) none of these
8. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from each group. The number of ways in which he can make up his choice is
(a) 100 (b) 200 (c) 300 (d) 400

9.Out of 10 white, 9 black and 7 red balls, the number of ways in which selection of one or more balls can be made, is

(a) 881 (b) 891 (c) 879 (d) 892

10.The number of diagonals in an octagon are

(a) 28 (b) 48 (c) 20 (d) none of these

Q26.Out of 10 given points 6 are in a straight line. The number of the triangles formed by joining any three of them is

(a) 100 (b) 150 (c) 120 (d) none of these

Q27.In how many ways the letters AAAAA, BBB, CCC, D, EE, F can be arranged in a row when the letter C occur at different places?

(a) $\frac{12!}{5!3!2!}$ x ${}^{13}C_3$ (b) $\frac{12!}{5!3!2!}$ x ${}^{13}P_3$ (c) $\frac{13!}{5!3!2!3!}$ (d) none of these

Q28.A is a set containing n elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of chosen P and Q so that P Ç Q = f is

(a) $2^{2n} - {}^{2n}C_n$ (b) 2^n (c) $2^n - 1$ (d) 3^n

Q29.A parallelogram is cut by two sets of m lines parallel to the sides, the number of parallelograms thus formed is

(a) $\frac{m^2}{4}$ (b) $\frac{(m+1)^2}{4}$ (c) $\frac{(m+2)^2}{4}$ (d) $\frac{(m+2)^2(m+1)^2}{4}$

Q30.Along a railway line there are 20 stations. The number of different tickets required in order so that it may be possible to travel from every station to every station is

(a) 380 (b) 225 (c) 196 (d) 105

Q31.The number of ordered triplets of positive integers which are solutions of the equation x + y + z = 100 is

(a) 5081 (b) 6005 (c) 4851 (d) none of these

Q32.The number of numbers less than 1000 that can be formed out of the digits 0, 1, 2, 3, 4 and 5, no digit being repeated, is

(a) 130 (b) 131 (c) 156 (d) none of these

Q33.A variable name in certain computer language must be either a alphabet or alphabet followed by a decimal digit. Total number of different variable names that can exist in that language is equal to

(a) 280 (b) 290 (c) 286 (d) 296

Q34.The total number of ways of selecting 10 balls out of an unlimited number of identical white, red and blue balls is equal to

(a) ${}^{12}C_2$ (b) ${}^{12}C_3$ (c) ${}^{10}C_2$ (d) ${}^{10}C_3$

Q35.Total number of ways in which 15 identical blankets can be distributed among 4 persons so that each of them get atleast two blankets equal to

(a) ${}^{10}C_3$ (b) ${}^{9}C_3$ (c) ${}^{11}C_3$ (d) none of these

Q36.The number of ways in which three distinct numbers in AP can be selected from the set {1, 2, 3, ···, 24}, is equal to

(a) 66 (b) 132 (c) 198 (d) none of these

Q37.The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is:

(a) 5 (b) 21 (c) 3^8 (d) ${}^{8}C_3$

Q38.The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by:

(a) 6! x 5! (b) 30 (c) 5! x 4! (d) 7! x 5!

Q39.If ${}^{n}C_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^{n}C_{r+1} + {}^{n}C_{r-1} + 2 \times {}^{n}C_r$ equals:

(a) ${}^{n+2}C_r$ (b) ${}^{n+2}C_{r+1}$ (c) ${}^{n+1}C_r$ (d) ${}^{n+1}C_{r+1}$

Q40.If the letters of the word SACHIN are arranged in all possible ways and these are written out as in dictionary, then the word SACHIN appears at serial number

(a) 600 (b) 601 (c) 602 (d) 603

Q26.The number of numbers is there between 100 and 1000 in which all the digits are distinct is

(a)648 (b) 548 (c)448 (d) none of these

Q27.The number of arrangements of the letters of the word 'CALCUTTA' is

(a)5040 (b) 2550 (c)40320 (d) 10080

Q28. How many different words can be formed with the letters of the word "PATLIPUTRA" without changing the position of the vowels and consonants?
(a)2160 (b) 180 (c)720 (d) none of these

Q29. How many different words ending and beginning with a consonant can be formed with the letters of the word 'EQUATION'?
(a)720 (b) 4320 (c)1440 (d) none of these

Q30.The number of 4 digit numbers divisible by 5 which can be formed by using the digits 0, 2, 3, 4, 5 is
(a)36 (b) 42 (c) 48 (d) none of these

Q31.The number of ways in which 5 biscuits can be distributed among two children is
(a)32 (b) 31 (c) 30 (d) none of these

Q32.How many five-letter words containing 3 vowels and 2 consonants can be formed using the letters of the word "EQUATION" so that the two consonants occur together?
(a)1380 (b) 1420 (c)1440 (d) none

Q33.If the letters of the word 'RACHIT' are arranged in all possible ways and these words are written out as in a dictionary, then the rank of this word is
(a)365 (b) 702 (c)481 (d) none of these

Q34.On the occasion of Dipawali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
(a)$^{20}C_2$ (b) $2 \cdot {}^{20}C_2$ (c)$2 \cdot {}^{20}P_2$ (d) none of these

Q35.The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a)18 (b) 108 (c)432 (d) 144

Q36.How many six digits numbers can be formed in decimal system in which every succeeding digit is greater than its preceding digit
(a) $^{9}P_6$ (b) $^{10}P_6$ (c)$^{9}P_3$ (d) none of these

Q37.How many ways are there to arrange the letters in the work GARDEN with the vowels in alphabetical order?

(a) 120 (b) 240 (c)360 (d) 480

Q38.A five-digit numbers divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is

(a)216 (b) 240 (c)600 (d) 3125

Q39.How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(a)16 (b) 36 (c) 60 (d) 180

Q40.The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

(a)40 (b) 60 (c) 80 (d) 100

THE BINOMIAL THEOREM

Binomial expression:

An algebraic expression consisting of only two terms is called a binomial expression.

Ex: i) x+ y ii) 4x −3y iii) x^2+y^2 iv) x^2-1/a^2

Binomial theorem:

The formula by which any power of a binomial expression can be expanded in the form of a series is known as binomial theorem. This theorem is given by Sir Issac Newton.

Binomial theorem for positive integral index:

If n is a positive integer

$$(x+y)^n = {}^nc_0x^ny^0 + {}^nc_1x^{n-1}y^1 + {}^nc_2x^{n-2}y^2 + {}^nc_3x^{n-3}y^3 + \cdots\cdots\cdots\cdots\cdots .. + {}^nc_nx^{n-n}y^n$$

Note:

1) Number of terms in the expansion of $(x+y)^n$ is n+ 1.
2) In the expansion of $(x+y)^n$, the sum of the powers of x and y is equal to n.
3) ${}^nc_0, {}^nc_1, {}^nc_2, \cdots\cdots\cdots, {}^nc_n$ are called coefficients of 1^{st}, 2^{nd} ,……..,$(n+1)^{th}$ terms respectively. These are called binomial coefficients.

Pascal's triangle:

The coefficients of the binomial expansion for different values of n are written in the form of triangle as shown below.

1 ———— n=0
1 1 ———— n=1
1 2 1 ———— n=2
1 3 3 1 ———— n=3
1 4 6 4 1 ———— n=4
1 5 10 10 5 1 ———— n=5
1 6 15 20 15 6 1 ———— n=6

This triangular array is called *Pascal's Triangle*.

Each row gives the binomial coefficients. That is, the row 1 2 1 are the coefficients of $(a+b)^2$. The next row, 1 3 3 1, are the coefficients of $(a+b)^3$; and so on.

To construct the triangle, write 1, and below it write 1 1. Begin and end each successive row with 1. To construct the intervening numbers, add the two numbers immediately above.

Thus to construct the third row, begin it with 1, and then add the two numbers immediately above: 1 + 1. Write 2. Finish the row with 1.

To construct the next row, begin it with 1, and add the two numbers immediately above: 1 + 2. Write 3. Again, add the two numbers immediately above: 2 + 1 = 3. Finish the row with 1.

Some special forms of Binomial expansion:

$$(x+y)^n = {}^nc_0x^ny^0 + {}^nc_1x^{n-1}y^1 + {}^nc_2x^{n-2}y^2 + {}^nc_3x^{n-3}y^3 + \cdots\cdots\cdots\cdots + {}^nc_nx^{n-n}y^n \cdots (1)$$

$$= \sum_{r=0}^{n} {}^nc_r x^{n-r} y^r$$

Put −x in place of x, we get

$$(x-y)^n = {}^nc_0x^ny^0 - {}^nc_1x^{n-1}y^1 + {}^nc_2x^{n-2}y^2 - {}^nc_3x^{n-3}y^3 + \cdots\cdots + (-1)^n\,{}^nc_nx^{n-n}y^n \cdots (2)$$

$$= \sum_{r=0}^{n} (-1)^r\ {}^nc_r x^{n-r} y^r$$

Put x = 1 in (1)

$$(1+y)^n = {}^nc_01^ny^0 + {}^nc_11^{n-1}y^1 + {}^nc_21^{n-2}y^2 + {}^nc_31^{n-3}y^3 + \cdots\cdots\cdots\cdots + {}^nc_n1^{n-n}y^n$$

$$= 1 + {}^nc_1y + {}^nc_2y^2 + {}^nc_3y^3 + \cdots\cdots\cdots\cdots + y^n$$

$$= \sum_{r=0}^{n} {}^nc_r y^r$$

Put x = 1 in (2)

$$(1-y)^n = {}^nc_01^ny^0 - {}^nc_11^{n-1}y^1 + {}^nc_21^{n-2}y^2 - {}^nc_31^{n-3}y^3 + \cdots\cdots\cdots\cdots + (-1)^n\,{}^nc_n1^{n-n}y^n$$

$$= 1 - {}^nc_1y + {}^nc_2y^2 - {}^nc_3y^3 + \cdots\cdots\cdots\cdots + (-1)^ny^n$$

$$= \sum_{r=0}^{n} (-1)^r\ {}^nc_r y^r$$

Problems:

1) Expand $(x - 1)^6$.

Solution: According to Pascal's triangle, the coefficients are

1 6 15 20 15 6 1.

In the binomial, x is "x", and −1 is "y". The signs will alternate:

$$(x - 1)^6 = x^6 - 6x^5\cdot 1 + 15x^4\cdot 1^2 - 20x^3\cdot 1^3 + 15x^2\cdot 1^4 - 6x\cdot 1^5 + 1^6$$

$$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$$

2) The term a^8b^4 occurs in the expansion of what binomial?

Answer. $(a + b)^{12}$. The sum of 8 + 4 is 12.

3). Use Pascal's triangle to expand the following.

a) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
b) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
c) $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
d) $(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
e) $(x - 1)^5 = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$
f) $(x + 2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
g) $(2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1$

Exercise:

1) Expand i) $\left(x+\frac{1}{x}\right)^6$ ii) $\left(x-\frac{1}{y}\right)^4, y\neq 0$ iii) $(2x-3y)^4$ iv) $(x^2+2a)^5$ v) $(1+x+x^2)^3$

vi) $(1-x+x^2)^4$

2) Expand $(a+b)^6-(a-b)^6$ .hence find the value of $(\sqrt{2}+1)^6-(\sqrt{2}-1)^6$

3) Simplify $(x+\sqrt{x-1})^6-(x-\sqrt{x-1})^6$

4) If A be the sum of odd terms and B be the sum of even terms in the expansion of $(x+a)^n$, then prove that

i) $A^2-B^2=(x^2-a^2)^n$ ii) $2(A^2+B^2)=(x+a)^{2n}+(x-a)^{2n}$

5) The first three terms in the expansion $(1+y)^n$ are 1, 10 and 40, find the expansion.

6) Using binomial theorem compute $(99)^5$

7) Find the exact value of $(1.01)^5$

8) Which is larger $(1.2)^{4000}$ or 800?

9) Which is greater $(1.1)^{10000}$ or 1000?

10) Show that $(101)^{50} > (100)^{50} + (99)^{50}$.

11) Prove that $\sum_{r=0}^{n} {}^nc_r 3^r = 4^n$.

12) Prove that ${}^nc_0 + {}^nc_1 + {}^nc_2 + {}^nc_3 + \cdots\cdots + {}^nc_n = 2^n$.

13) Prove that product of k consecutive numbers is divisible by k!.

General term in the expansion $(x+y)^n$:

$$(x+y)^n = {}^nc_0 x^n y^0 + {}^nc_1 x^{n-1} y^1 + {}^nc_2 x^{n-2} y^2 + {}^nc_3 x^{n-3} y^3 + \cdots\cdots + {}^nc_n x^{n-n} y^n$$

In the above expansion the (r+ 1)th term is given by

$$\boxed{T_{r+1} = {}^nc_r x^{n-r} y^r}$$

this is called the general term of the expansion.

Putting r=0,1,2,3,4…..,n we get 1^{st} ,2^{nd} ,……..,(n+ 1)th terms respectively.

Middle term in the expansion $(x+y)^n$:

Case- i) n is even

If n is even then the number of terms in the expansion is n+ 1 which is odd. Therefore the number of middle terms in the expansion is one and the term is $\frac{n}{2}+1$ th term.

Case- ii) n is odd

If n is odd then the number of terms in the expansion is n+1 which is even. Therefore the number middle terms in the expansion are two and the terms are $\frac{n+1}{2}$ th and $\frac{n+3}{2}$ th terms.

Greatest coefficient in the expansion (x + y)^n :

In any binomial expansion the middle term has the greatest coefficient. If there are two middle terms then their two coefficients are equal and greater.

Prob : If n be a positive integer, prove that the coefficients of the terms in the expansion of (x+y)^n equidistant from the beginning and from the end are equal.

In the expansion of (x+y)^n

Co efficient of 1st term from beginning = ${}^nc_0$

Co efficient of 2nd term from beginning = ${}^nc_1$

Co efficient of 3rd term from beginning = ${}^nc_2$

………………………………………..

………………………………………

Co efficient of r th term from beginning = ${}^nc_{r-1}$

Now

Co efficient of 1st term from end = ${}^nc_n$

Co efficient of 2nd term from end = ${}^nc_{n-1}$

Co efficient of 3rd term from end = ${}^nc_{n-2}$

………………………………………..

………………………………………

Co efficient of r th term from end = ${}^nc_{n-(r-1)}$

Since ${}^nc_{r-1}$ = ${}^nc_{n-(r-1)}$ are equal. We can say in the expansion of (x+y)^n , the co efficient of r th term from beginning and end are equal.

Note: ***In the binomial expansion, the r th term from the end is equal to (n-r+2)th term from the beginning.***

Problems:

1) Find the 4 th term in the expansion of (x-2y)^{12}

2) Find the 13 th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0.$

3) Find the 5 th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$.

4) Write the general term in the expansion of $(x^2-y)^6$.

5) If x >1 and the third term in the expansion of $\left(\frac{1}{x}+x^{\log_{10}x}\right)^5$ is1000, find the value of x.

6) If the 21^{st} and 22^{nd} terms in the expansion of $(1+x)^{44}$ are equal then find the value of x.

7) In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then find $\frac{a}{b}$.

8) Find the middle term in the expansion of $\left(\frac{x}{3}-9y\right)^{10}$.

9) Find the middle term in the expansion of $\left(x-\frac{1}{2x}\right)^{12}$.

10) Find the middle term in the expansion of $\left(2x^2-\frac{1}{x}\right)^7$.

11) Find the middle term in the expansion of $(1-2x+2x^2)^n$.

12) Prove that the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5.7......(2n-1)2^n}{n!}$

13) Show that the greatest coefficient in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5.7......(2n-1)2^n}{n!}$.

14) Show that the coefficient of the middle term in $(1+x)^{2n}$ is equal to the sum of the coefficients of two middle terms in $(1+x)^{2n-1}$.

15) Find the coefficient of $1/y^2$ in $\left(y-\frac{c^3}{y^2}\right)^{10}$.

16) Find the coefficient of x^9 in $(1+3x+3x^2+x^3)^{15}$.

17) Find the coefficient of x^{40} in $(1+2x+x^2)^{27}$.

18) Find the term independent of x in $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$.

19) Given that the fourth term in the expansion of $\left(px+\frac{1}{x}\right)^n$.is 5/2, find n and p.

20) Find the value of k so that the term independent of x in $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405.

21) In the expansion of $(1+a)^{m+n}$, prove that the coefficient of a^m and a^n are equal.

22) Find a if the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.

23) If the coefficients of a^{r-1}, a^r, a^{r+1} in the binomial expansion of $(1+a)^n$ are in A.P. prove that $n^2-n(4r+1)+4r^2-2=0$.

24) Find the coefficient of x^{-1} in $(1+3x^2+x^4)\left(1+\frac{1}{x}\right)^8$.

25) If n be a positive integer, then prove that $6^{2n}-35n-1$ is divisible by 1225.

26) Find the

a) 7^{th} term in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$

b) 9^{th} term in the expansion of $\left(\frac{x}{a}-\frac{3a}{x^2}\right)^{12}$

c) 5^{th} term in the expansion of $\left(\frac{a}{3}-3b\right)^7$ and $\left(2x^2-\frac{1}{3x^3}\right)^{10}$

27) Find a, if the 17^{th} and 18^{th} terms of the expansion $(2+a)^{50}$ are equal.

28) Find the r th term from the end in $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$

29) Write the general terms in the following expansions.

i) $(1-x^2)^{12}$. ii) $\left(x-\frac{3}{x^2}\right)^{10}$ iii) $\left(x^2-\frac{1}{x}\right)^{12}, x \neq 0$

30) Find the general term and middle term in the expansion of $\left(\frac{x}{y}+\frac{y}{x}\right)^{2n+1}$ n being positive integer.

31) If n is a positive integer, show that

i) 4^n-3n-1 is divisible by 9.

ii) $2^{5n}-31n-1$ is divisible by 961.

32) Using binomial theorem prove that 6^n-5n always leaves the remainder 1 when divided by 25 for all positive integers n.

33) Find the middle terms in the expansions

i) $\left(\frac{2x}{3}-\frac{3y}{2}\right)^{20}$ ii) $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$ iii) $\left(\frac{x}{y}-\frac{y}{x}\right)^7$ iv) $(1+x)^{2n}$ v) $(1-2x+x^2)^n$ vi) $\left(3-\frac{x^3}{6}\right)^7$

34) Find the coefficient of

i) x in the expansion of $\left(2x-\frac{3}{x}\right)^9$

ii) x^7 in the expansion of $\left(3x^2+\frac{1}{5x}\right)^{11}$

iii) x^9 in the expansion of $\left(2x^2-\frac{1}{x}\right)^{20}$

iv) x^{24} in the expansion of $\left(x^2-\frac{3a}{x}\right)^{15}$

v) x^9 in the expansion of $\left(x^2-\frac{1}{3x}\right)^9$

vi) x^{-7} in the expansion of $\left(2x-\frac{1}{3x^2}\right)^{11}$

vii) x^5 in the expansion of $(x+3)^8$

viii) x^5 in the expansion of $(x+3)^9$

ix) a^5b^7 in the expansion of $(a-2b)^{12}$

x) x^6y^3 in in the expansion of $(x+y)^9$

35) If the coefficients of x, x^2 and x^3 in the binomial expansion $(1+x)^{2n}$ are in A.P then prove that $2n^2-9n+7=0$.

36) Find the positive value of m for which the coefficient of x^2 in the expansion of $(1+x)^m$ is 6.

37) Find the term independent of x in the following binomial expansion($x\neq 0$).

i) $\left(x+\frac{1}{x}\right)^{2n}$ ii) $\left(x-\frac{1}{x}\right)^{14}$ iii) $\left(2x^2+\frac{1}{x}\right)^{13}$ iv) $\left(x^2+\frac{1}{x}\right)^{12}$ v) $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$

vi) $\left(2x^2-\frac{1}{x}\right)^{12}$ vii) $\left(2x^2-\frac{3}{x^3}\right)^{25}$ viii) $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^{6}$ ix) $\left(x^3-\frac{3}{x^2}\right)^{15}$ x) $\left(x^2-\frac{3}{x^3}\right)^{10}$

xi) $\left(\frac{x^{1/3}}{2}+x^{-1/3}\right)^{8}$ xii) $\left(x-\frac{1}{x}\right)^{12}$ xiii) $\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}$

38) If three consecutive coefficients in the expansion of $(1+x)^n$ be 56, 70 and 56., find n and the position of the coefficients.

39) If three successive coefficients in the expansion of $(1+x)^n$ be 220, 495 and 972., find n.

40) If coefficients of (r−1)th, rth and (r+1)th terms in the expansion of $(x+1)^n$ are in the ratio 1:3:5. Find n and r.

41) If the coefficients of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1+x)^n$ are in A.P, Find n.

42) If the coefficients of 2^{nd} , 3^{rd} and 4^{th} terms in the expansion of $(1+x)^n$ are in A.P, show that $2n^2-9n+7=0$.

43) In the expansion of $(1+a)^{m+n}$, prove that the coefficient of a^m and a^n are equal.

44) Find a if the coefficient of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.

45) If coefficients of a^{r-1}, a^r,a^{r+1} in the expansion of $(1+a)^n$ are in A.P. Prove that

$n^2-n(4r+1)+4r^2-2=0$.

46) Find the coefficient of x^4 in the expansion of $\left(1+3x+10x^2\right)\left(x+\frac{1}{x}\right)^{10}$

47) Find the coefficient of x^{-1} in the expansion of $\left(1+3x^2+x^4\right)\left(x+\frac{1}{x}\right)^{8}$

48) Find n if the if the coefficient of 4^{th} and 13^{th} terms in the expansion of $(a+b)^n$ are equal.

49) If in the expansion of $(1+x)^{43}$ the coefficient of (2r+1)th term is equal to the coefficient of (r+2) th term , find r.

50) If three consecutive coefficients in the expansion of $(1+x)^n$ be 165, 330 and 462., find n and the position of the coefficients.

51) If a_1,a_2,a_3 and a_4 be any four consecutive coefficients in the expansion of $(1+x)^n$, prove that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$.

52) If 2nd, 3^{rd} and 4^{th} terms in the expansion of $(x+y)^n$ be 240,720 and 1080 respectively find x, y and n.

53) If the coefficients of three consecutive terms in the expansion of $(1+a)^n$ are in the ratio 1:7:42. Find n .

54) if in the binomial expansion a,b,c and d be 6^{th} ,7^{th} ,8^{th} and 9^{th} terms respectively, prove that $\frac{b^2-ac}{c^2-bd}=\frac{4a}{3c}$.

Binomial expansion for fractional index:

$$(1+x)^{-n}=1-{}^{n}c_1x+{}^{n+1}c_2x^2-{}^{n+2}c_3x^3+\cdots.+(-1)^r\ {}^{n+r-1}c_rx^r+\cdots..\cdots, |x|<1, n\in Q$$

To determine numerically greatest term in the expansion of $(x+y)^n$ $(\forall n\in N)$ **:-**

It is always better to consider $(1+x)^n$ in place of $(x+y)^n$. For this take one of x and y common preferably the greater one. For example $(5+7)^{10}=7^{10}\left(1+\frac{5}{7}\right)^{10}$, now one should find the greatest term of $\left(1+\frac{5}{7}\right)^{10}$ and multiply it by 7^{10}. So it is sufficient to consider the expansion of $(1+x)^n$, $|x|<1$.

Method to determine numerically greatest term in the expansion of $(1+x)^n$ **:**

Steps:

1. Calculate $r=\left|\frac{x(n+1)}{x+1}\right|$
2. If r is an integer then T_r and T_{r+1} are equal and both are greatest terms.
3. If r is not an integer, there $T_{[r]+1}$ is the greatest term where[ ] denotes the greatest integer part.

Some important conclusions from the binomial theorem:

1) If n is odd then $(x+a)^n-(x-a)^n$ and $(x+a)^n+(x-a)^n$ both have equal no of terms and the number of terms are $\frac{n+1}{2}$.

2) If n is even then $(x+a)^n-(x-a)^n$ has $\frac{n}{2}$ terms and $(x+a)^n+(x-a)^n$ has $\frac{n}{2}+1$ terms.

Some important products:

1) $r^2=r(r-1)+r$
2) $r^3=r(r-1)(r-2)+3r(r-1)+r$
3) $r^4=r(r-1)(r-2)(r-3)+6r(r-1)(r-2)+7r(r-1)+r$
4) $(x-a)(x-b)(x-c)=x^3-(a+b+c)x^2+(ab+bc+ca)x-abc$
5) $(x-a_1)(x-a_2)(x-a_3)(x-a_4)=x^4-\sum_{i=1}^{4}a_i\,x^3+\sum_{i\neq j=1}^{4}a_ia_j\,x^2-\sum_{i\neq j\neq k=1}^{4}a_ia_ja_k\,x-\prod_{i=1}^{4}a_i$

Some important short cuts:

1) If a, b, c are three consecutive coefficients in the expansion of $(1+x)^n$ then the values of n and r are given by

$$n = \frac{2ac + b(a+c)}{b^2 - ac} \quad \text{and} \quad r = \frac{a(b+c)}{b^2 - ac}$$

2) If the coefficient of x^r, x^{r+1} in the expansion $\left(a + \frac{x}{b}\right)^n$ are given then the value of n is

$$n = ab(r+1) + r$$

3) If the coefficients of T_r, T_{r+1}, T_{r+2} in the expansion of $(1+x)^n$ are in A.P then the value of r is given by

$$r = \frac{n \pm \sqrt{n+2}}{2}, \forall n \in N$$

4) If the coefficients of T_r, T_{r+1}, T_{r+2} in the expansion of $(1+x)^n, \forall n \in N$ are in the ratio a : b : c then the value of r is given by

$$r = \frac{a(b+c)}{b^2 - ac} \quad \text{and} \quad n = \frac{2ac + b(a+c)}{b^2 - ac}$$

5) If in the expansion of $(1+x)^n$, p^{th} term = q^{th} term then p + q = n + 2

Identities involving Binomial coefficients:

We know the binomial coefficients are $^nc_0$, $^nc_1$, $^nc_2$, $^nc_3$, …………, ^nc_n. Through out this chapter we write these coefficients as $c_0, c_1, c_2 \ldots\ldots\ldots, c_n$ for convenience.

1. Prove that $c_0 + c_1 + c_2 + \ldots\ldots\ldots + c_n = 2^n$

Proof:

we have

$$(1+y)^n = {}^nc_0 1^n y^0 + {}^nc_1 1^{n-1} y^1 + {}^nc_2 1^{n-2} y^2 + {}^nc_3 1^{n-3} y^3 + \cdots\cdots\cdots + {}^nc_n 1^{n-n} y^n$$

Put y = 1 we get

$$c_0 + c_1 + c_2 + \ldots\ldots\ldots + c_n = 2^n \quad \ldots\ldots\ldots\ldots. (1)$$

2. Prove that $c_0 - c_1 + c_2 - \ldots\ldots\ldots + (-1)^n c_n = 0$

Proof:

we have

$$(1+y)^n = {}^nc_0 1^n y^0 + {}^nc_1 1^{n-1} y^1 + {}^nc_2 1^{n-2} y^2 + {}^nc_3 1^{n-3} y^3 + \cdots\cdots\cdots + {}^nc_n 1^{n-n} y^n$$

Put y = −1 we get

$$c_0 - c_1 + c_2 - \ldots\ldots\ldots + (-1)^n c_n = 0 \quad \ldots\ldots\ldots\ldots. (2)$$

3. Prove that $c_0 + c_2 + c_4 + \ldots\ldots\ldots = 2^{n-1}$ and $c_1 + c_3 + c_5 + \ldots\ldots\ldots = 2^{n-1}$

Proof:

Adding (1) and (2) we get $c_0 + c_2 + c_4 + \ldots\ldots\ldots = 2^{n-1}$

Subtracting (1) and (2) we get $c_1 + c_3 + c_5 + \ldots\ldots\ldots = 2^{n-1}$

4. Prove that $\left(1+\frac{c_1}{c_0}\right)\left(1+\frac{c_2}{c_1}\right)\left(1+\frac{c_3}{c_2}\right)\text{............}\left(1+\frac{c_n}{c_{n-1}}\right)=\frac{(n+1)^n}{n!}$

Proof:

Let us take $\frac{c_r}{c_{r-1}}=\frac{n!}{r!(n-r)!}\times\frac{(r-1)!(n-r+1)!}{n!}=\frac{n-r+1}{r}$

Now putting r=1,2,3……,n we get

$$\frac{c_1}{c_0}=n,\ \frac{c_2}{c_1}=\frac{n-1}{2},\ \frac{c_3}{c_2}=\frac{n-2}{3}\ \cdots\cdots\cdots.,\ \frac{c_n}{c_{n-1}}=\frac{1}{n}$$

now

$$\left(1+\frac{c_1}{c_0}\right)\left(1+\frac{c_2}{c_1}\right)\left(1+\frac{c_3}{c_2}\right)\text{............}\left(1+\frac{c_n}{c_{n-1}}\right)$$

$$=(1+n)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)\text{............}\left(1+\frac{1}{n}\right)$$

$$=\frac{(1+n)(1+n)\text{.............}(1+n)(n\,times)}{1.2.3\text{.........}n}=\frac{(n+1)^n}{n!}$$

5. If P be the sum of the odd terms and Q be the sum of the even terms in the expansion of $(a+x)^n$, then prove that $(a^2-x^2)^n=P^2-Q^2$

6. Find the sum of $1+\frac{1}{2}c_1+\frac{1}{3}c_2+\text{..........}+\frac{1}{n+1}c_n$

Proof: 1st method

$$1+\frac{1}{2}c_1+\frac{1}{3}c_2+\text{..........}+\frac{1}{n+1}c_n$$

$$=1+\frac{1}{2}n+\frac{1}{3}\frac{n(n-1)}{2!}+\text{..........}+\frac{1}{n+1}$$

$$=\frac{1}{n+1}\left((n+1)+\frac{1}{2}n(n+1)+\frac{1}{3}\frac{(n+1)n(n-1)}{2!}+\text{..........}+1\right)$$

$$=\frac{1}{n+1}\left({}^{n+1}c_1+{}^{n+1}c_2+{}^{n+1}c_3+\text{.........}+{}^{n+1}c_{n+1}\right)$$

$$=\frac{1}{n+1}\left(2^{n+1}-1\right)$$

2nd method

we have

$$(1+y)^n=1+c_1y+c_2y^2+c_3y^3+\cdots\cdots\cdots\cdots+c_ny^n$$

now integrating both sides w.r.to y under the limits 0 and 1 we get the answer

7. Find the sum of $\frac{c_1}{c_0}+2\frac{c_2}{c_1}+3\frac{c_3}{c_2}+\text{..............}+n\frac{c_n}{c_{n-1}}$

Proof:

Let us take $\frac{c_r}{c_{r-1}}=\frac{n!}{r!(n-r)!}\times\frac{(r-1)!(n-r+1)!}{n!}=\frac{n-r+1}{r}$

Now putting r=1,2,3……,n we get

$$\frac{c_1}{c_0}=n,\ \frac{c_2}{c_1}=\frac{n-1}{2},\ \frac{c_3}{c_2}=\frac{n-2}{3}\ \cdots\cdots\cdots.,\ \frac{c_n}{c_{n-1}}=\frac{1}{n}$$

now

$$\frac{c_1}{c_0}+2\frac{c_2}{c_1}+3\frac{c_3}{c_2}+.............+n\frac{c_n}{c_{n-1}}$$
$$=n+2\frac{n-1}{2}+3\frac{n-1}{3}+........+n\frac{1}{n}$$
$$=n+(n-1)+(n-2)+...............+1$$
$$=\frac{n(n+1)}{2}$$

8. Show that i) $c_0^{\,2}+c_1^{\,2}+c_2^{\,2}+..........+c_n^{\,2}=\frac{(2n)!}{(n!)^2}$

ii) $c_0c_1+c_1c_2+c_2c_3+..........+c_{n-1}c_n=\frac{(2n)!}{(n-1)!(n+1)!}$

iii) $c_0c_2+c_1c_3+c_2c_4+..........+c_{n-2}c_n=\frac{(2n)!}{(n-2)!(n+2)!}$

Proof:

We have

$$(1+y)^n=c_0+c_1y+c_2y^2+c_3y^3+\cdots\cdots\cdots\cdots+c_ny^n\cdots\cdots\cdots\cdots\cdots.\ (1)$$

and $(y+1)^n=c_0y^n+c_1y^{n-1}+c_2y^{n-2}+c_3y^{n-3}+\cdots\cdots\cdots\cdots+c_n\cdots\cdots\cdots\cdots\cdots.(2)$

now multiplying (1) and (2) we get

$(1+y)^{2n}=(c_0+c_1y+c_2y^2+c_3y^3+\cdots\cdots+c_ny^n)(c_0y^n+c_1y^{n-1}+c_2y^{n-2}+\cdots..+c_n)\cdots\cdots..(3)$

from l.h.s

$(1+y)^{2n}={}^{2n}c_0+{}^{2n}c_1y+{}^{2n}c_2y^2+\cdots\cdots..+{}^{2n}c_{n-1}y^{n-1}+{}^{2n}c_ny^n+{}^{2n}c_{n+1}y^{n+1}+\cdots\cdots+{}^{2n}c_{2n}y^{2n}\cdots..(4)$

i) Equating the coefficients of y^n in the right hand side of (3) and (4) we get

$$c_0^{\,2}+c_1^{\,2}+c_2^{\,2}+..........+c_n^{\,2}={}^{2n}c_n$$
$$\Rightarrow c_0^{\,2}+c_1^{\,2}+c_2^{\,2}+..........+c_n^{\,2}=\frac{(2n)!}{(n!)^2}$$

ii) Equating the coefficients of y^{n-1} in the right hand side of (3) and (4) we get

$$c_0c_1+c_1c_2+c_2c_3+..........+c_{n-1}c_n={}^{2n}c_{n-1}$$
$$\Rightarrow c_0c_1+c_1c_2+c_2c_3+..........+c_{n-1}c_n=\frac{(2n)!}{(n-1)!(n+1)!}$$

iii) Equating the coefficients of y^{n-2} in the right hand side of (3) and (4) we get

$$c_0c_2+c_1c_3+c_2c_4+..........+c_{n-2}c_n={}^{2n}c_{n-2}$$
$$\Rightarrow c_0c_2+c_1c_3+c_2c_4+..........+c_{n-2}c_n=\frac{(2n)!}{(n-2)!(n+2)!}$$

9. Prove that $c_0 - 2^2 c_1 + 3^2 c_2 - \dots\dots\dots + (-1)^n (n+1)^2 c_n = 0$, $n > 2$

10. $\frac{c_0}{2} + \frac{c_1}{3} + \frac{c_2}{4} + \frac{c_3}{5} + \dots\dots\dots\dots + \frac{c_n}{n+2} = \frac{n2^{n+1}+1}{(n+1)(n+2)}$

11. Prove that

i) ${}^{2n}c_0 + {}^{2n}c_1 + {}^{2n}c_2 + \dots\dots + {}^{2n}c_{2n-1} + {}^{2n}c_{2n} = 2^{2n}$ [Hints: $c_0 + c_1 + c_2 + \dots\dots\dots + c_n = 2^n$]

ii) ${}^{2n}c_1 + {}^{2n}c_3 + {}^{2n}c_5 + \dots\dots + {}^{2n}c_{2n-1} = 2^{2n-1}$ [Hints: $c_1 + c_3 + c_5 + \dots\dots\dots = 2^{n-1}$]

iii) $c_1 + 2c_2 + 3c_3 + \dots\dots\dots + nc_n = n2^{n-1}$

[Hints: take $(1+x)^n$ then differentiate w.r.to x both sides then put x=1both sides]

iv) $c_0 + 3c_1 + 5c_2 + \dots\dots\dots + (2n+1)c_n = (n+1)2^n$

[Hints: write it as $(c_0 + c_1 + c_2 + \dots\dots\dots + c_n) + 2(c_1 + 2c_2 + 3c_3 + \dots\dots\dots + nc_n)$]

12. Find the sum of

i) $c_1 - 2c_2 + 3c_3 - \dots\dots\dots + (-1)^{n-1} nc_n$

[Hints: take $(1-x)^n$ then differentiate w.r.to x both sides then put x=1both sides]

ii) $1.2c_2 + 2.3c_3 + \dots\dots\dots + (n-1)nc_n$

[Hints: take $(1+x)^n$ then differentiate w.r.to x both sides then again differentiate both sides w.r.to x and then put x=1both sides]

iii) $c_1 + 2^2 c_2 + 3^2 c_3 + \dots\dots\dots + n^2 c_n$

[Hints: take $(1+x)^n$ then differentiate w.r.to x both sides then multiply x both sides then again differentiate both sides w.r.to x and then put x=1both sides]

iv) $c_0 + 2c_1 + 3c_2 + \dots\dots\dots + (n+1)c_n$

[Hints: take $(1+x)^n$ then multiply x both sides then differentiate w.r.to x both sides and then put x=1both sides]

v) $c_0 - 2c_1 + 3c_2 - \dots\dots\dots + (-1)^n (n+1)c_n$

[Hints: take $(1-x)^n$ then multiply x both sides then differentiate w.r.to x both sides and then put x=1both sides]

vi) $c_0 - \frac{1}{2}c_1 + \frac{1}{3}c_2 - \dots\dots\dots + (-1)^n \frac{1}{n+1} c_n$

[Hints: take $(1-x)^n$ then integrate both sides w.r.to x under the limits 0 and 1]

13. Show that

i) $c_1^2 + 2c_2^2 + 3c_3^2 + \dots\dots\dots + nc_n^2 = \frac{(2n-1)!}{[(n-1)!]^2}$

[Hints: do like problem no.8]

ii) $c_2 + 2c_3 + 3c_4 + \dots\dots\dots + (n-1)c_n = 1 + (n-2)2^{n-1}$

[Hints: take $(1+x)^n$ then divide by x both sides then differentiate w.r.to x both sides and then put x=1both sides]

14. The sum $\frac{1}{1!9!}+\frac{1}{3!7!}+.......+\frac{1}{9!1!}$ can be written in the form $\frac{2^n}{b!}$. Find a and b.