OPTICS

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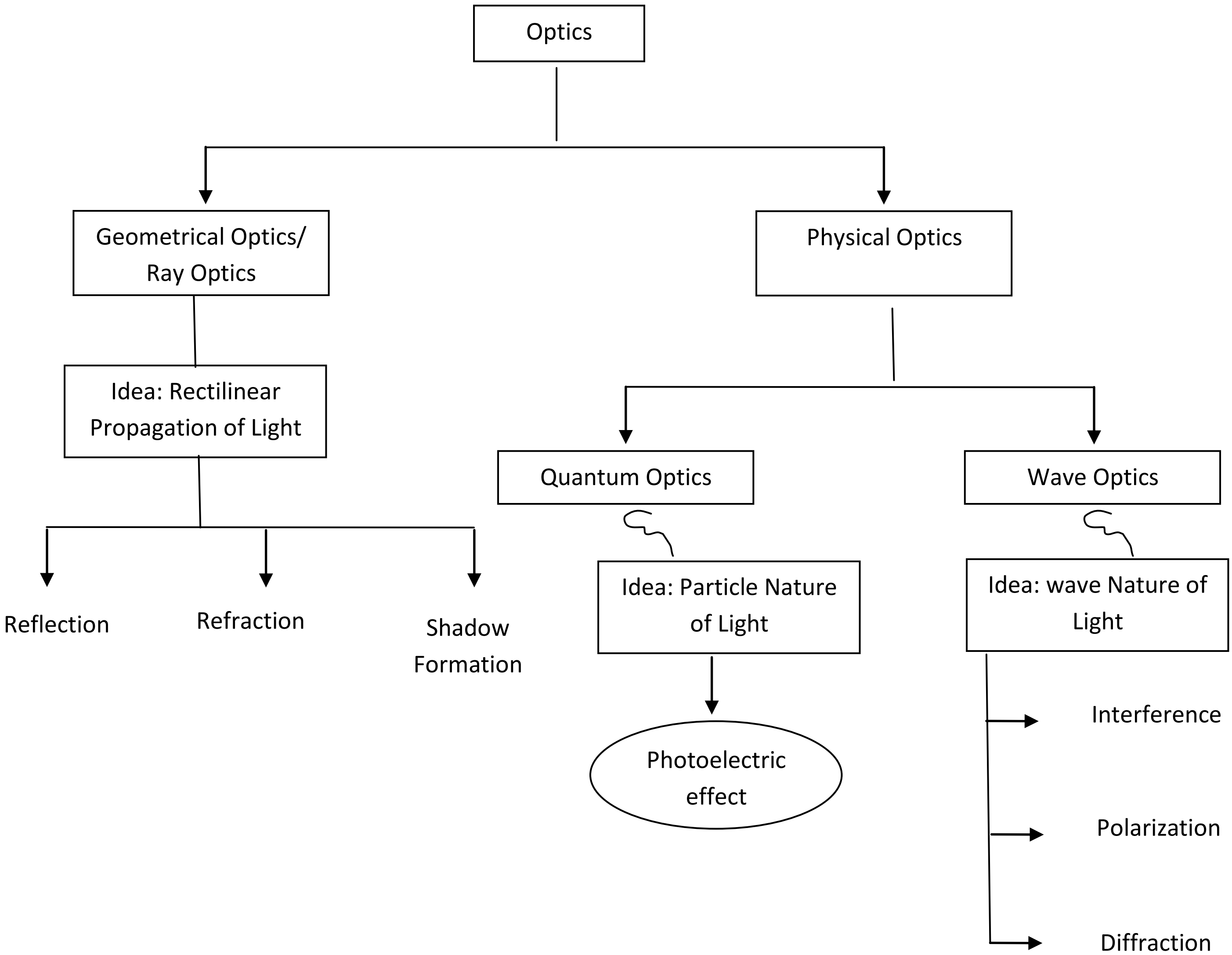
Optics

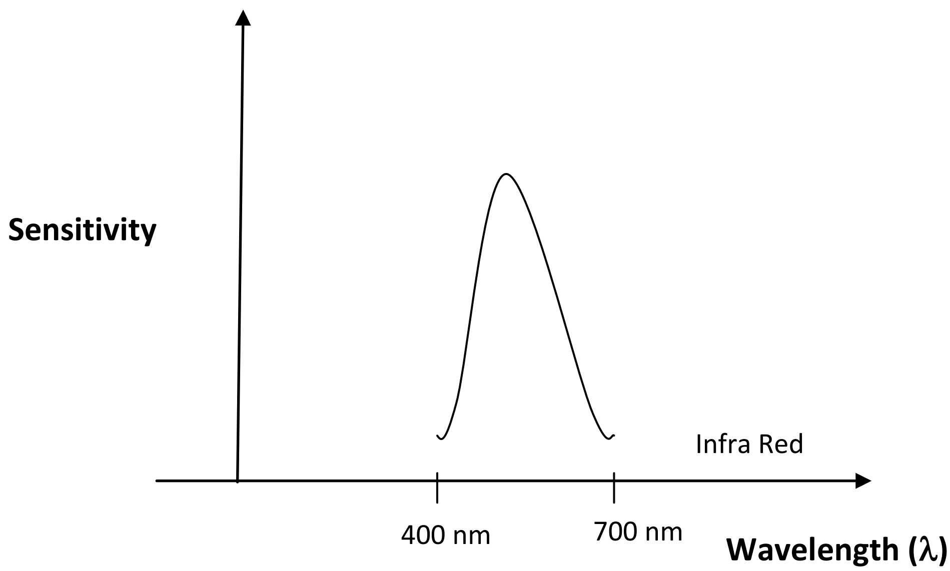
The branch of Physics which deals with the study of **Light** and **Vision** is called Optics

“The sense of vision doesn’t lie in the eye rather it lies in the mind”



Light

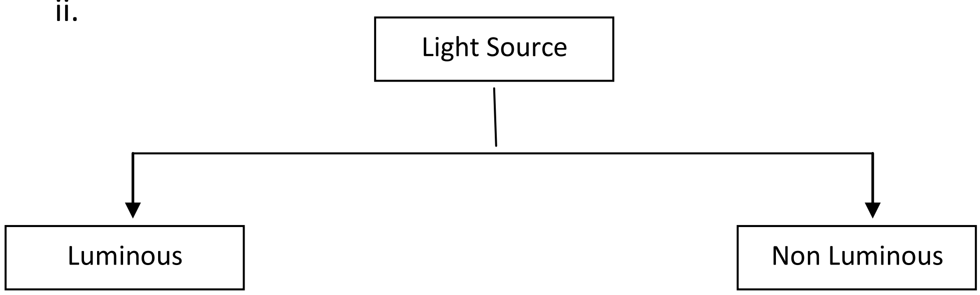
- This is the form of energy which produces sensation of **Vision** upon hitting your eyes
- One of the interesting facts about light is that **light itself is invisible** although it makes all objects visible
- Light is **an electromagnetic wave** having wavelength ranging from **400nm to 700nm**
- Human eyes are not equally sensitive for all wavelengths. The variation of sensitivity is shown below:



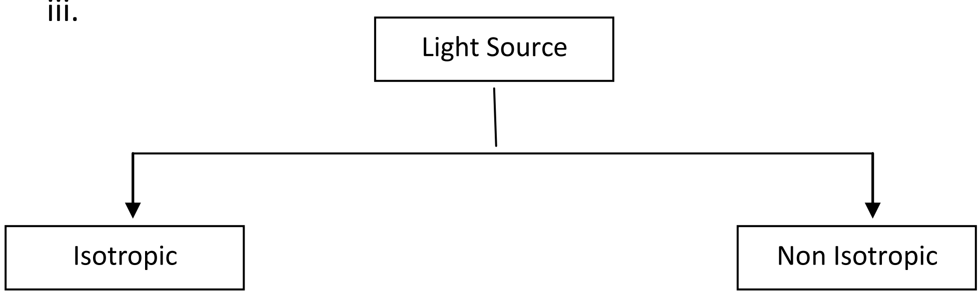
- Geometrical Optics can be looked upon as the limiting case of wave optics when
    - Size of obstacle or opening is large compared to the wavelength of light
- In situation of conflict between geometrical and wave optics, conclusion from and wave optics will dominate

Basic Definitions

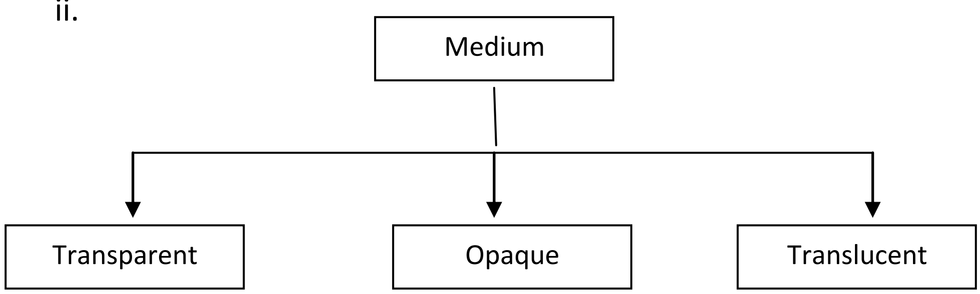
a. Light Source:
   i. The substance which **emits light** is called light source
   ii.



   iii.

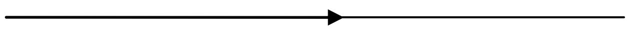


b. Medium:
   i. The substance which through which light propagates or tends to propagate is called a medium
   ii.



c. Light Ray:

   i. The straight line path along which light energy travels in a homogeneous medium is called a ray

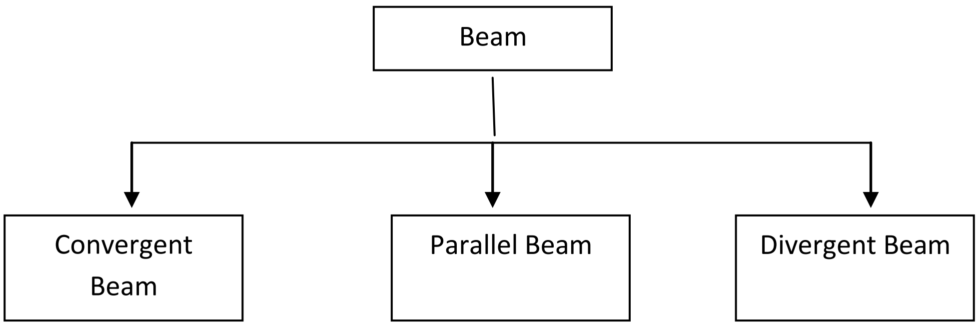


   ii. A single ray cannot be isolated from a source of light

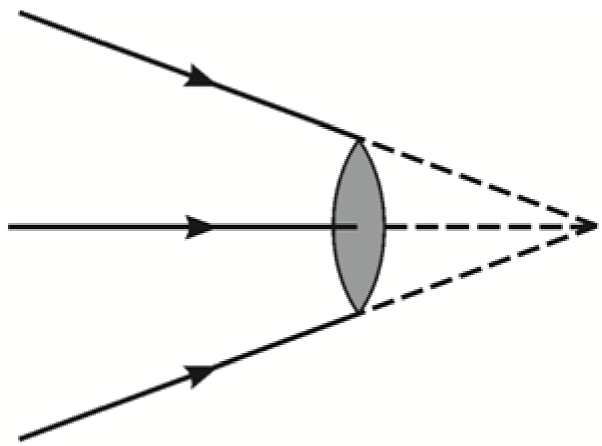
d. Light Beam:

   i. A bundle or bunch of rays in called a beam

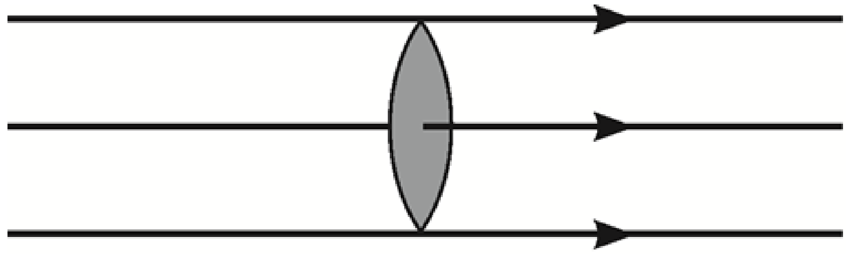
   ii.

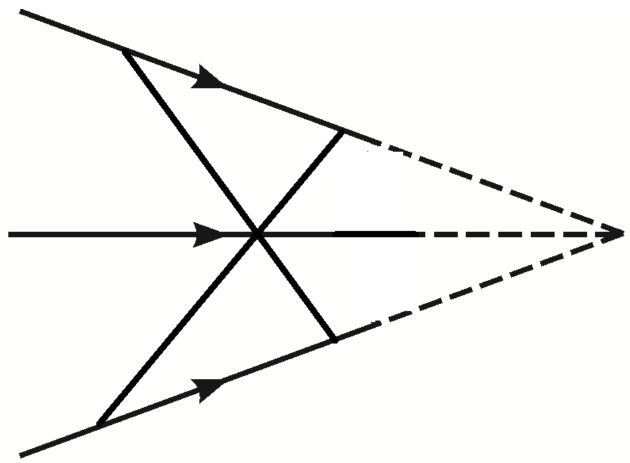


Convergent beam of light

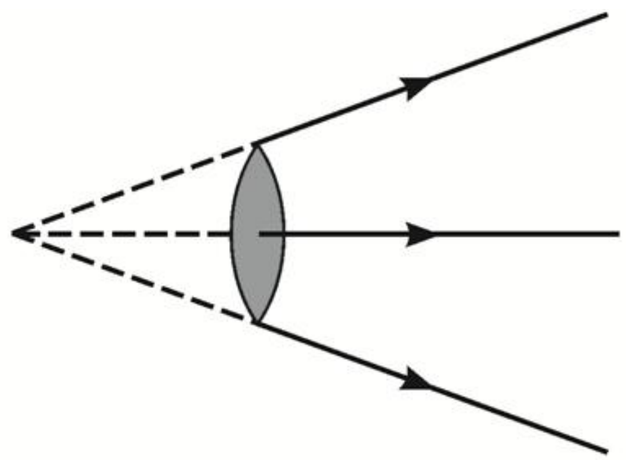


Parallel beam of light





Divergent beam of light



e. Optical Object:

   i. An optical object is defined by incident rays on mirror, prism or lens or any optical instrument

   ii.

Optical Object

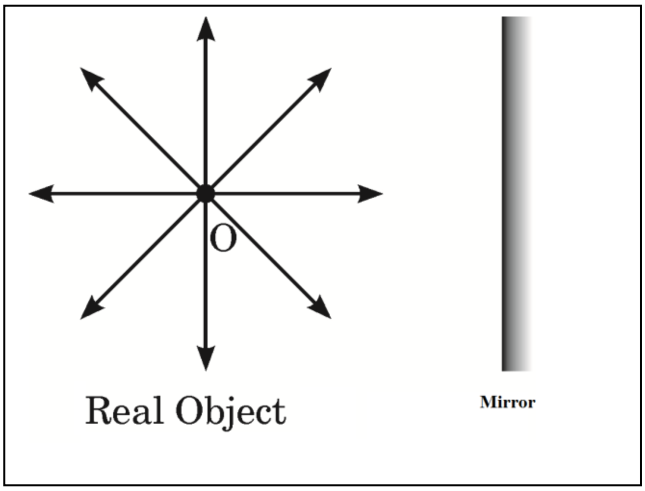
Real Object

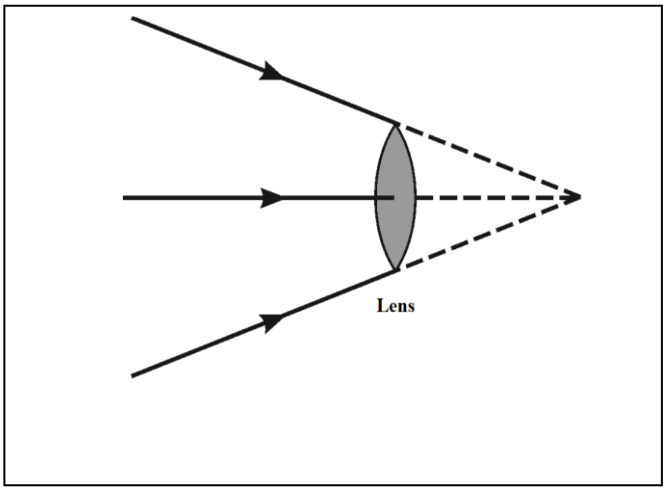
Virtual Object

Diverging incident rays

Diverging incident

rays





f. Optical Image:

i. An optical image is defined by **reflected** or **refracted** rays

ii.

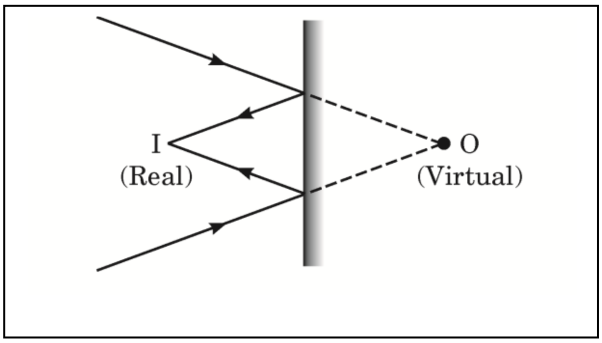
Optical Image

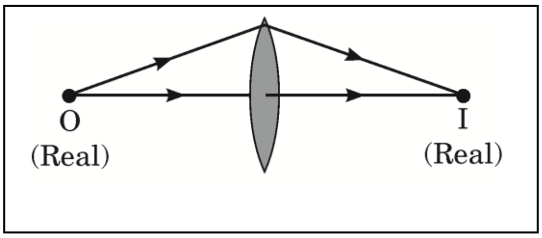
Real Image

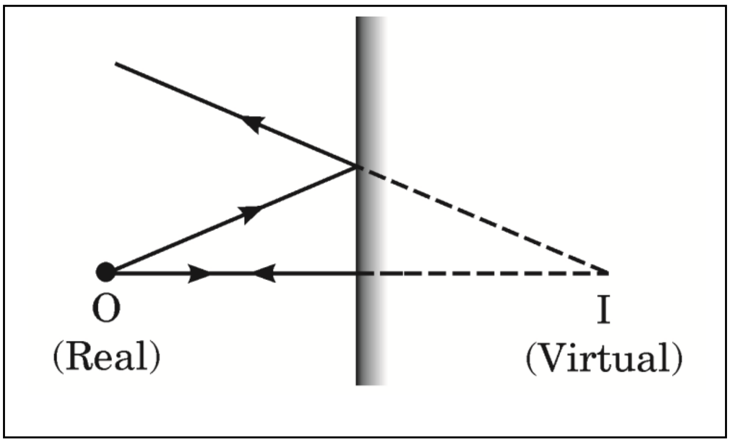
Intersection is real

Virtual Image

Intersection is apparent







Special Points:

- Real image can be obtained on screen whereas virtual image can't be obtained on screen
- Human eyes can't distinguish between real image and virtual image
- Human eyes can't see virtual object
- Virtual images can be photographed

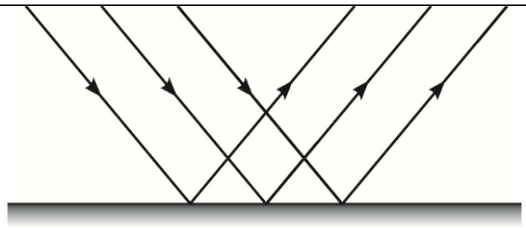
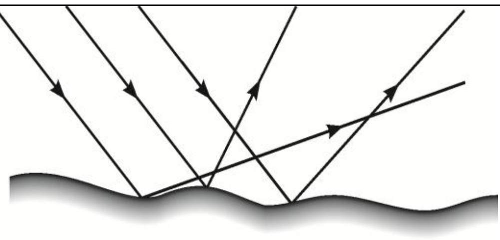
Reflection

i. The phenomenon of bouncing back light energy into the same medium when it is obstructed by same surface is called Reflection

ii.

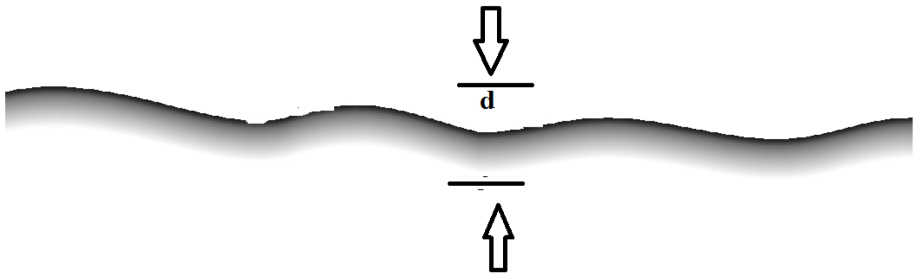
Reflection

Regular | Irregular

a) Light is reflection in well defined reflection	a) Reflected light doesn't go in well defined direction
b) Surface is smooth	b) Surface is rough
c)  Regular reflection	c)  Irregular reflection

Remarks

I. It is irregular reflection that makes an object visible from different directions
II. For image formation, reflection must be regular
III. The **smoothness** or **roughness** is decided by comparing the wavelength of incident light and size of irregularities of surface



d = Size of surface irregularity

d > λ, surface is rough

d ≤ λ, surface is smooth

Laws of reflection

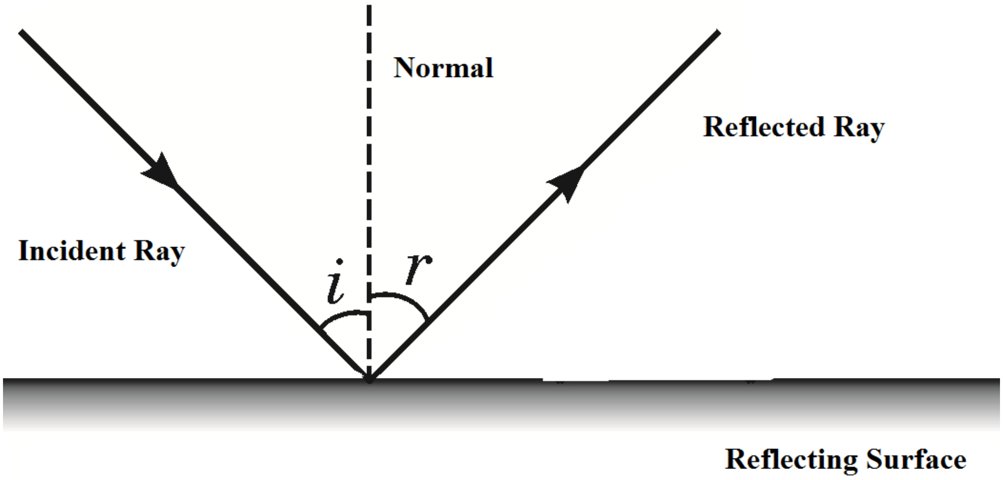
First Law:

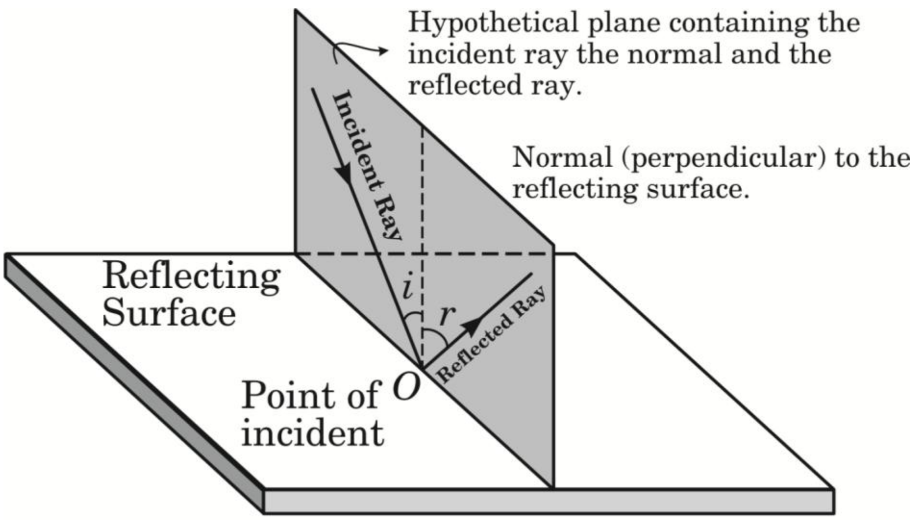
Incident ray, normal at the point of incidence and reflected ray are **coplanar** and this plane is **perpendicular** to the reflecting surface

Second Law:

Angle of incidence = Angle of reflection

$$\angle i = \angle r$$

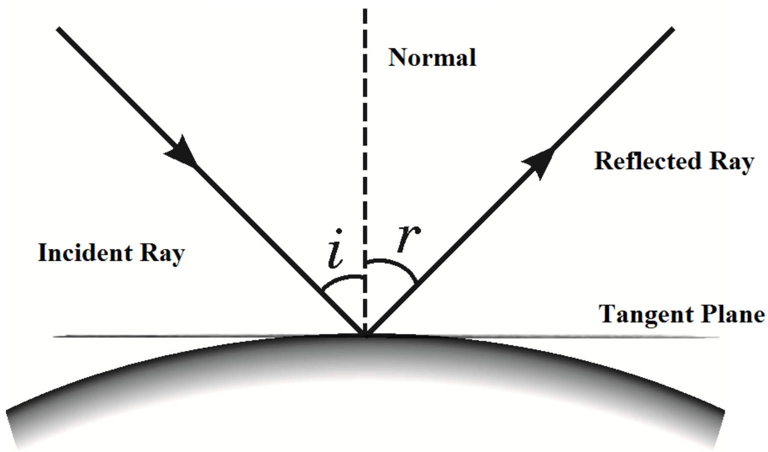




Reflection from a plane surface

Remarks

1. The laws of reflection are valid for any smooth surface irrespective of geometry

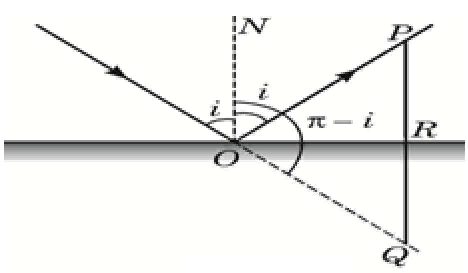


2. Vector Form:

Vector form of laws of reflection:

$$\hat{\boldsymbol{e}}_2 = \hat{\boldsymbol{e}}_1 - 2(\hat{\boldsymbol{e}}_1.\hat{\boldsymbol{n}})\hat{\boldsymbol{n}} \qquad ...(1)$$

where $\hat{e}_1$ = unit vector along incident ray



$\hat{n}$ = unit vector along normal

$\hat{e}_2$ = unit vector along reflected ray

From triangle law of addition of vectors

$$\overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{QP} \qquad \text{... (2)}$$

$$\overrightarrow{OP} = OP\,\hat{e}_2$$

$$\overrightarrow{OQ} = OQ\,\hat{e}_1$$

$$\overrightarrow{QP} = QP\,\hat{n} = 2\,QR\,\hat{n} \qquad (\because \Delta ORP \cong \Delta OQR)$$

$\therefore$ From eq. (2), we get ;

$$OP\,\hat{e}_2 = OQ\,\hat{e}_1 + 2QR\,\hat{n}$$

or, $$\hat{e}_2 = \hat{e}_1 + \frac{2QR}{OP}\hat{n} \qquad (\because OP = OQ) \qquad ..(3)$$

Also, $$\hat{e}_1.\hat{n} = 1.1\cos(\pi - i) = -\cos i$$

Further, $$\cos i = \frac{QR}{OQ} = \frac{QR}{OP}$$

Substituting in eq. (3), we get;

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}$$

Alternatively:

$$\hat{e}_1 = \vec{e}_{\parallel} + \vec{e}_{\perp}$$

where $\vec{e}_{\parallel}$ = component of $\hat{e}$ parallel to mirror

$$= \hat{e}_1 - (\hat{e}_1.\hat{n})\hat{n}$$

$\vec{e}_{\perp}$ = component of $\hat{e}_1$ perpendicular to mirror

$$= (\hat{e}_1.\hat{n})\hat{n}$$

Using remark (ii), we get ;

$$\hat{e}_2 = \vec{e}_{\parallel} - \vec{e}_{\perp} = \hat{e}_1 - 2\hat{n}(\hat{e}_1.\hat{n})$$

(iv) Following vector relation is also valid in reflection

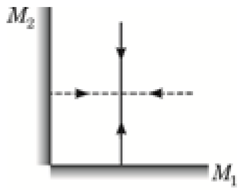
$$\boldsymbol{\hat{n} \times \hat{e}_2 = \hat{n} \times \hat{e}_1}$$

The above result can be extended to three plane mirrors arranged mutually perpendicular to each other. This arrangement of three mutually perpendicular mirrors is also known as **CORNER REFLECTOR**. If the incident ray is represented by $x\hat{i} + y\hat{j} + z\hat{k}$ then after three reflections final reflected ray is given by $-x\hat{i} - y\hat{j} - z\hat{k}$. We always see our reflected image (but only upside down), independent of our position.

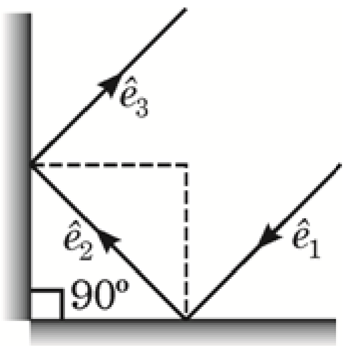
Example

Two plane mirrors are arranged mutually perpendicular to each other. Show that the emergent ray is always antiparallel to incident ray.

Solution: Case I : When incident ray suffers one reflection.



In this case, incident ray will strike either of the mirror normally and ray will be reflected back.

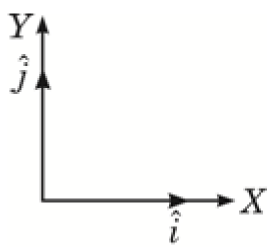


Case II : When incident ray suffers two successive reflection.

From vector form of laws of reflection

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j} \qquad ...(1)$$

Similarly, $$\hat{e}_3 = \hat{e}_2 - 2(\hat{e}_2 \cdot \hat{i})\hat{i} \qquad ...(2)$$



From equation (1) and (2) we get;

$$\hat{e}_3 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j} - 2[\{\hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j}\} \cdot \hat{i}]\hat{i}$$

$$= \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{j})\hat{j} - 2(\hat{e}_1 \cdot \hat{i})\hat{i} + 4[(\hat{e}_1 \cdot \hat{j})\hat{j} \cdot \hat{i}]\hat{i}$$

$$= \hat{e}_1 - 2[(\hat{e}_1 \cdot \hat{i})\hat{i} + (\hat{e}_1 \cdot \hat{j})\hat{j}]$$

$(\because \hat{i} \cdot \hat{j} = 0)$

or, $$\hat{e}_3 = \hat{e}_1 - 2\hat{e}_1$$

or, $$\hat{e}_3 = -\hat{e}_1$$

Hence, $\hat{e}_3$ is antiparallel to $\hat{e}_1$.

Alternatively :

Let $\hat{e}_1 = x\hat{i} + y\hat{j}$

then, $\hat{e}_2 = x\hat{i} - y\hat{j}$

and $\hat{e}_3 = -x\hat{i} - y\hat{j}$

Clearly, $\hat{e}_3 = -\hat{e}_1$

Not that such a combination of mirrors is used in the construction of periscope.

Remark

The above result can be extended to three plane mirrors arranged mutually perpendicular to each other. This arrangement of three mutually perpendicular mirrors is also known as CORNER REFLECTOR. If the incident ray is represented by *xi+yj+zk* then after three reflections final reflected ray is given by *-xi-yj-zk*. We always see our reflected image (but only upside down), independent of our position

Image formed by a plane mirror

a. Point object:

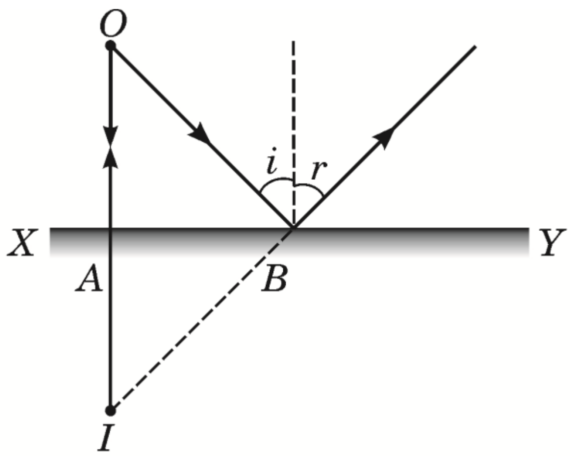


Image of a point source

i) MO = MI

ii) Line joining object and image is perpendicular to mirror

b. Extended object

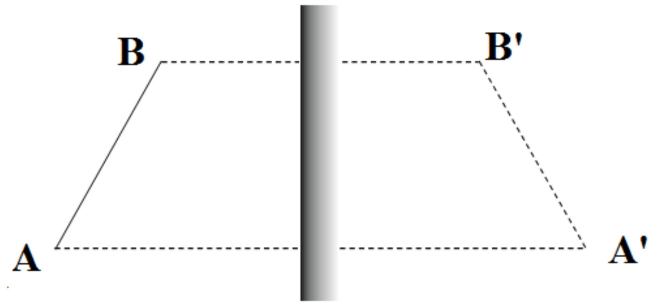
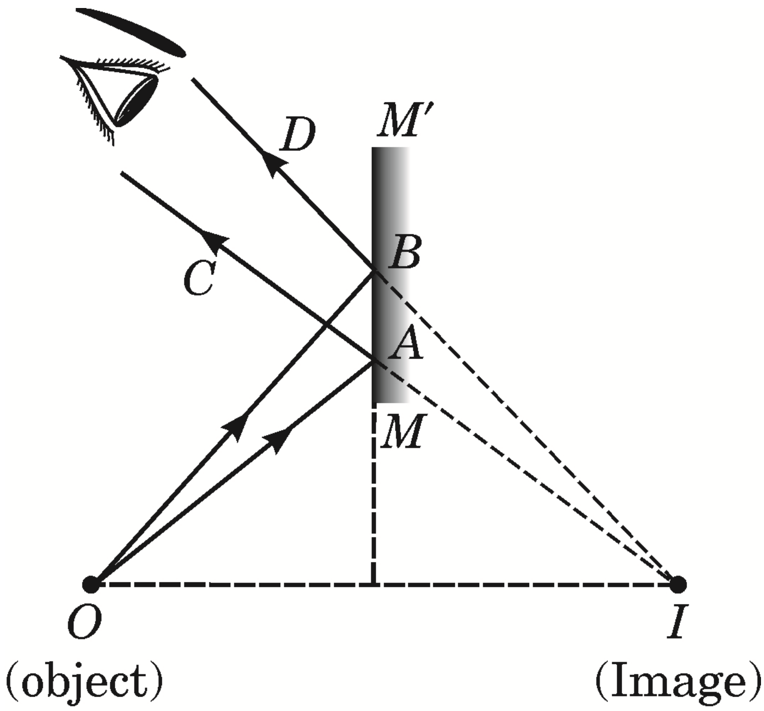


Image of an Extended Source



Remarks:

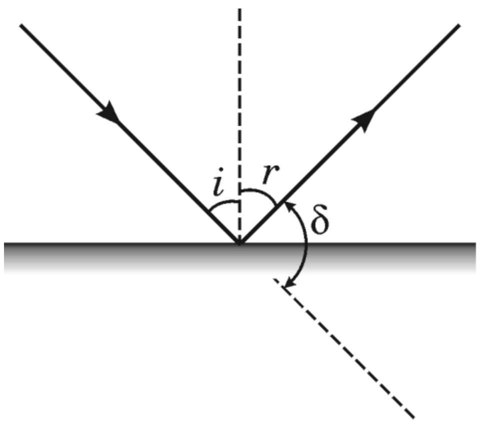
(i) For image formation, it is not necessary that object should be present in front of mirror.

(ii) It is not possible to locate an object by a single ray. It is for this reason that the surface of reflecting mirror is not visible to us. Any point like A on the surface sends only one ray AC into our eye corresponding to the incident ray OA. Rays incident at A in directions other than OA, are reflected in other directions and do not enter our eye. A neighbouring point B no doubt sends another ray BD, but these rays together locate the point I (image) and not the point A or B. Therefore, what is visible to us is I (Image) and not the surface MM′.

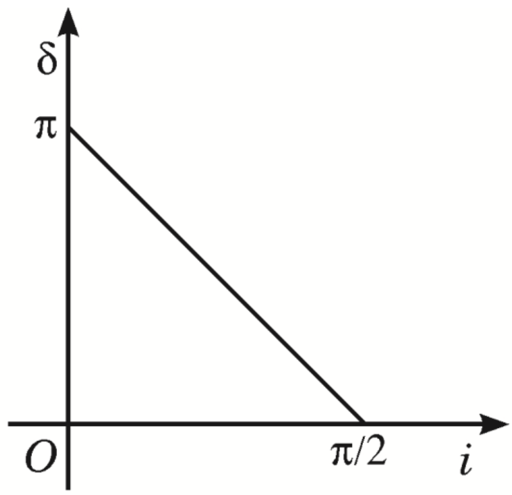
Deviation

a. Single reflection

$\delta = \pi - 2i$



Deviation



Graph between deviation versus angle of incidence

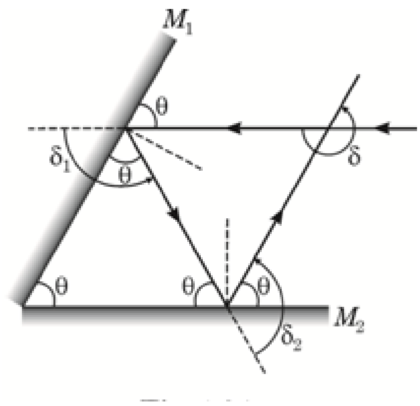
b. Multiple reflection:

   $\boldsymbol{\delta_{net} = \in \delta_i}$

Example:

Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror and parallel to the second is reflected from the second mirror parallel to the first mirror. Determine the angle between the two mirrors. Also determine the total deviation produced in the incident ray due to the two reflections.

Solution: From figure



$$3\theta = 180º$$

or, $$\theta = 60º$$

$$\delta_1 = 180º - 2 \times 30º$$

$$= 120º ↺$$

$$\delta_2 = 180º - 2 \times 30º = 120º ↺$$

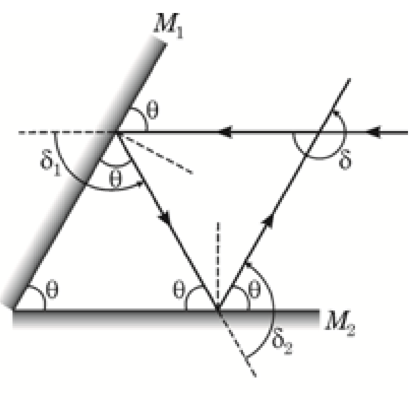
∴ Total deviation $= \delta_1 + \delta_2$

$$= \mathbf{240º ↺ \text{ or } 120º ↻} \textbf{ Ans.}$$

Alternatively :

From fig. 1.24,

$$\delta = 180º + \theta = 240º \text{ anticlockwise or } 120º \text{ clockwise.}$$



$$3\theta = 180º$$

or, $$\theta = 60º$$

$$\delta_1 = 180º - 2 \times 30º$$

$$= 120º ↺$$

$$\delta_2 = 180º – 2 \times 30º = 120º \circlearrowleft$$

$\therefore$ Total deviation $= \delta_1 + \delta_2$

$= \mathbf{240º \circlearrowleft \text{ or } 120º \circlearrowright}$ **Ans.**

Alternatively :

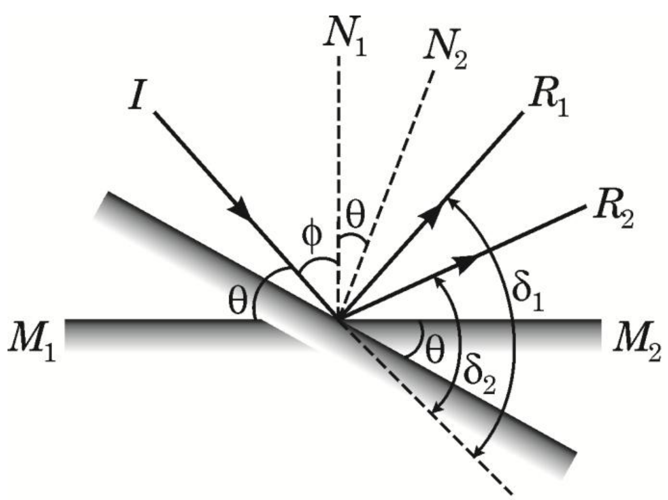
From figure,

$$\delta = 180º + \theta = 240º \text{ anticlockwise or } 120º \text{ clockwise.}$$

Rotation of Mirror

$\delta_1 = \pi – 2\ \Phi$

$\delta_2 = \pi – 2\ (\theta + \Phi)$



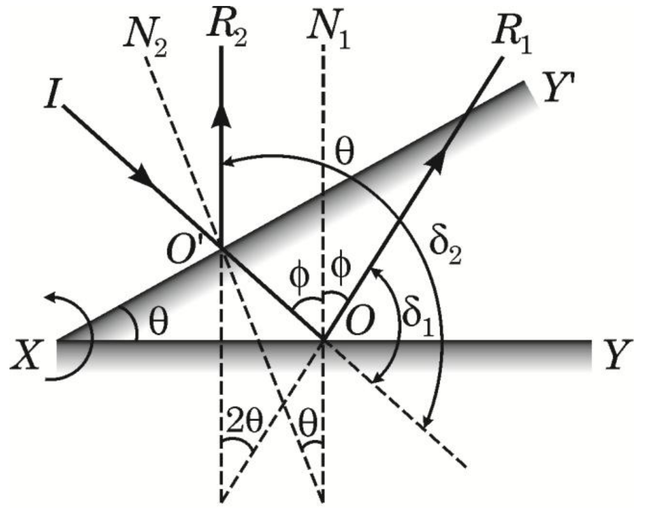
Rotation of plane mirror

Remarks:

(i) The above statement is valid for rotation of mirror about any point.

(ii) If mirror is kept fixed and incident ray is rotated, then reflected ray will rotate in opposite sense by same angle.

(iii) If mirror and incident ray both are rotated then net rotation suffered by reflected ray will be algebraic sum of rotation suffered by reflected ray due to mirror rotation and incident ray rotation separately keeping sense of rotation in mind.



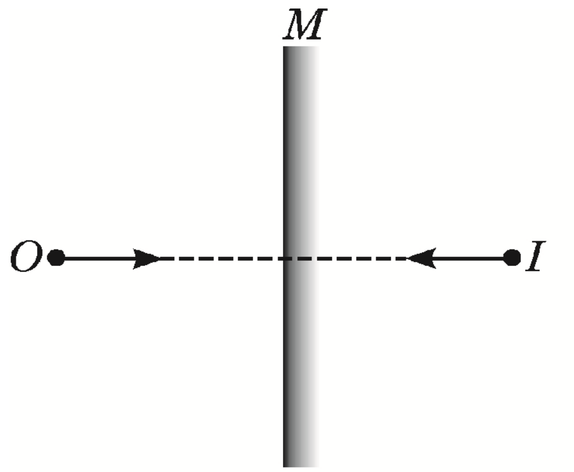
$\delta_1 = \pi - 2\,\Phi$

$\delta_2 = \pi - 2\,(\Phi - \theta)$

$\therefore\ \delta_1 - \delta_2 = 2\,\theta$

Note: Explain that parallel component remains unchanged and perpendicular component reverses in direction

Velocity of Image



i) MO = MI

$$\left(\overrightarrow{V_{I/M}}\right)_{\perp} = -\left(\overrightarrow{V_{I/M}}\right)_{\perp}$$

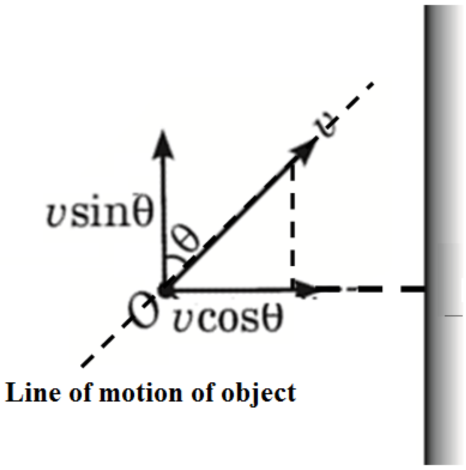
ii) Line joining object and image is perpendicular to mirror

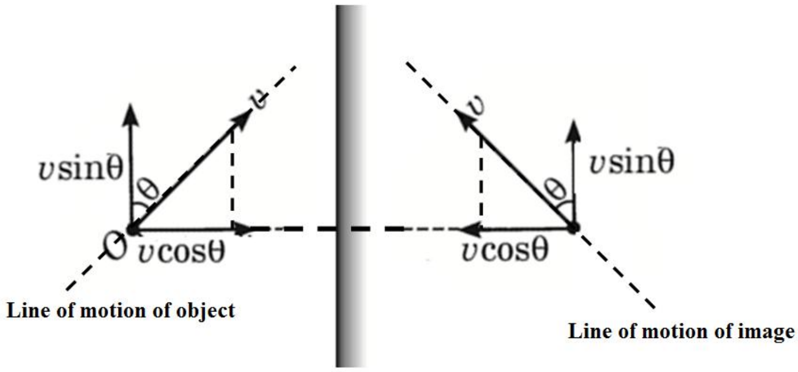
$$\left(\overrightarrow{V_{I/M}}\right)_{\parallel} = -\left(\overrightarrow{V_{I/M}}\right)_{\parallel}$$

iii)

$$\overrightarrow{V_{I/M}} = -\left(\overrightarrow{V_{I/M}}\right)_{\perp} + \left(\overrightarrow{V_{I/M}}\right)$$

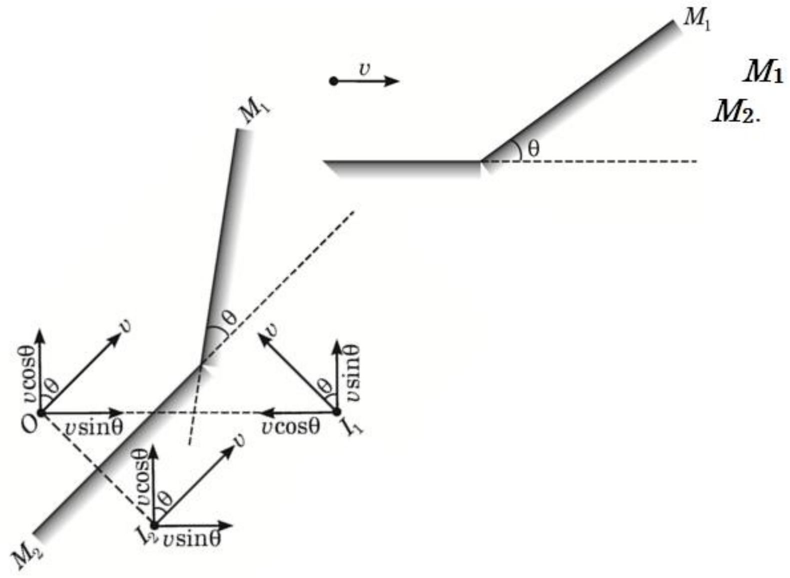
Show the line of motion of image in situation as shown in figure. Also, find the velocity of image w.r.t. object





$v_{I/O} = 2v\cos\theta$

A point object is moving with a speed *v* before an arrangement of two mirrors as shown in fig. Find the velocity of image in mirror M_1 with respect to image in mirror



Solution. Velocity of image in mirror M_1 and M_2 is as shown in fig. 1.33

$\vec{v}_{1/2}$ = Velocity of I_1 w.r.t. I_2

$= \vec{v}_1 - \vec{v}_2$

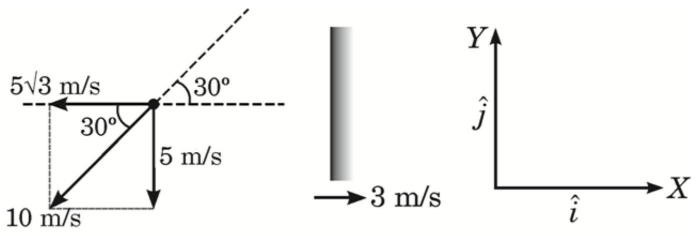
or $v_{1/2}$ = $\mathbf{2v\sin\theta}$ **Ans.**

Example:

A point object is moving with a speed of 10 m/s infront of a mirror moving with a speed of 3 m/s as shown in fig. 1.37. Find the velocity of image of the object with respect to mirror, object and ground.

Solution.

$\vec{V}_O$ = Velocity of object = $(-5\sqrt{3}\,\hat{i} - 5\,\hat{j})$ m/s



$\vec{V}_M$ = Velocity of mirror = $3\hat{i}$ m/s

For component of velocity perpendicular to mirror

$$(\vec{V}_{I/M})_\perp = -(\vec{V}_{O/M})_\perp = -(\vec{V}_O - \vec{V}_M)$$

$$= -(-5\sqrt{3}\hat{i} - 3\hat{i}) = (5\sqrt{3} + 3)\hat{i} \text{ m/s}$$

For component of velocity parallel to mirror

$$(\vec{V}_{I/M})_\parallel = (\vec{V}_{O/M})_\parallel$$

$$= \vec{V}_O - \vec{V}_M$$

$$= -5\,\hat{j} - 0 = -5\,\hat{j}$$

$$(\vec{V}_{I/M}) = (\vec{V}_{I/M})_\perp + (\vec{V}_{I/M})_\parallel$$

$$= \mathbf{(5\sqrt{3} + 3)\hat{i} - 5\,\hat{j}} \qquad \textbf{Ans.}$$

Also, $\vec{V}_{I/M} = \vec{V}_I - \vec{V}_M$

$$(5\sqrt{3} + 3)\hat{i} - 5\,\hat{j} = \vec{V}_I - (3\hat{i})$$

or, $(5\sqrt{3} + 3)\hat{i} + 3\hat{i} - 5\,\hat{j} = \vec{V}_I$

or, $\vec{V}_I = \mathbf{(5\sqrt{3} + 6)\hat{i} - 5\,\hat{j}}$ **Ans.**

Also, $\vec{V}_{I/O} = \vec{V}_I - \vec{V}_O$

$$= (5\sqrt{3}+6)\hat{i} - 5\hat{j} - (-5\sqrt{3}\,\hat{i} - 5\hat{j})$$

$$= (5\sqrt{3}+6+5\sqrt{3})\hat{i} + (5-5)\hat{j}$$

$$= \mathbf{(10\sqrt{3}+6)\,\hat{i}} \quad \textbf{Ans.}$$

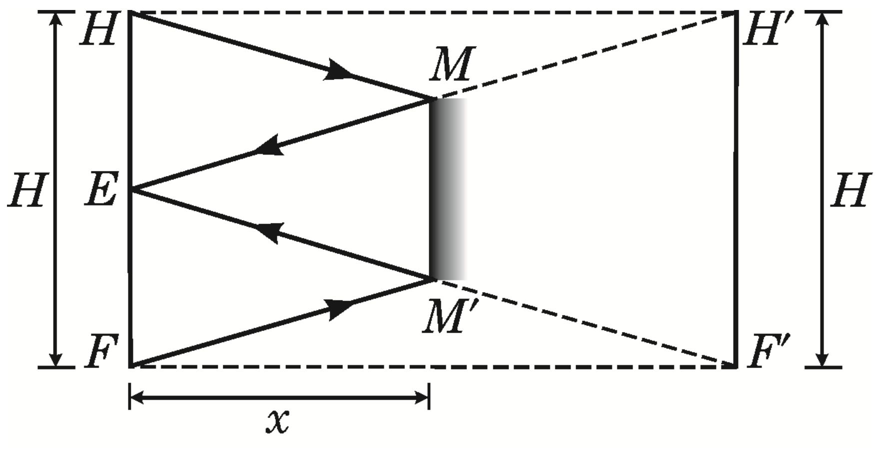
Minimum size of mirror

Thought process

The rays from extreme parts of object should reach the eyes of observer after hitting the mirror

Example

A man of height 'h' wishes to see his full image by using a plane mirror. Find the minimum size of mirror required.



From Similar $\triangle$s M_1M_2E & $F'H'E'$

$$\frac{M_1M_2}{x} = \frac{H}{2X}$$

Learning points

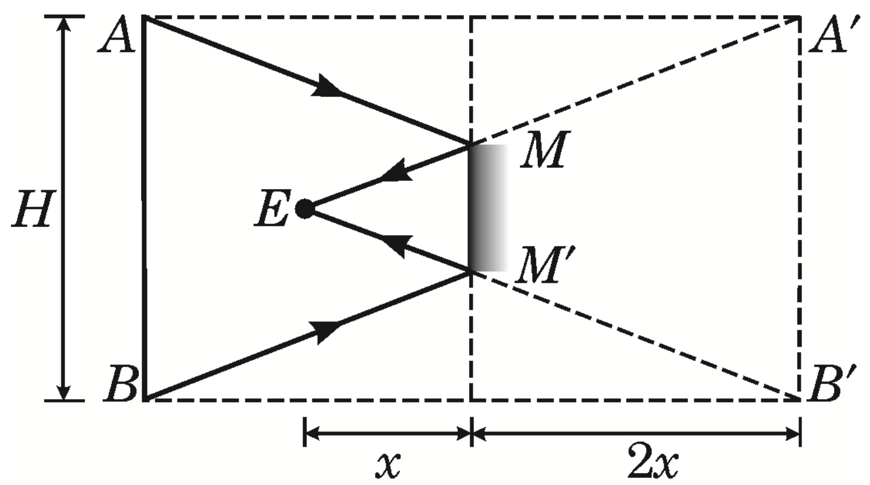
i) Minimum size is independent of distance between man and mirror.

ii) The minimum size mirror has to be positioned in a specific way

Height of lower edge of mirror = Half of height of eye level from foot

Example:

A man standing at the centre of room wishes to see full image of wall of height ‘h’ behind him by hanging a plane mirror on front wall. Find minimum size of mirror required.



From Similar $\triangle$s M_1M_2E & A’B’E

$$\frac{M_1M_2}{x} = \frac{H}{3X}$$

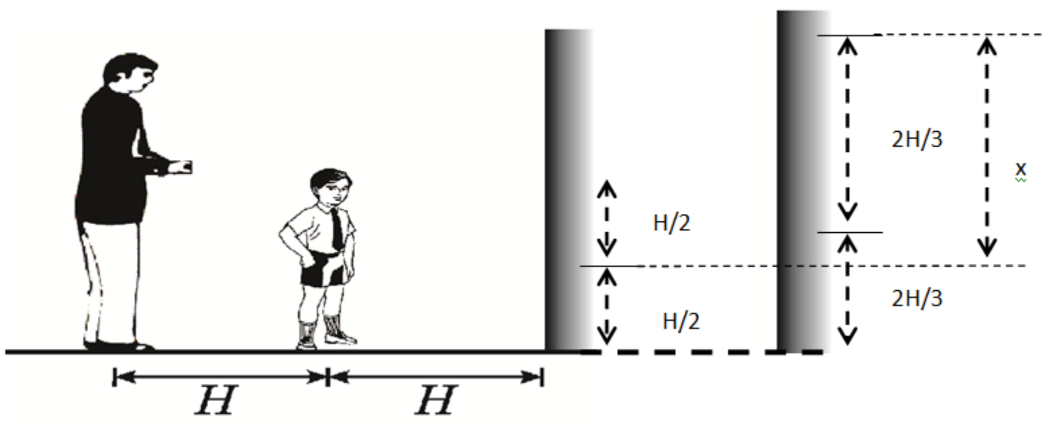
$$\Rightarrow M_1M_2 = \frac{H}{3}$$

Note that

Height of lower edge of mirror = 2/3rd of eye level

Example:

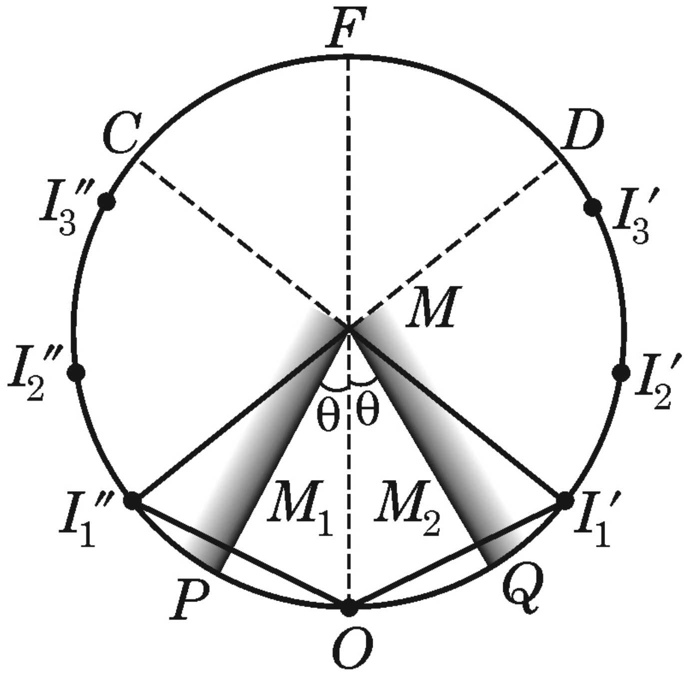
A child is standing in front of a straight plane mirror. His father is standing behind him, as shown in the fig. 1.113. The height of the father is double the height of the child. What is the minimum length of the mirror required so that the child can completely see his own image and his father's image in the mirror? Given that the height of father is 2H.



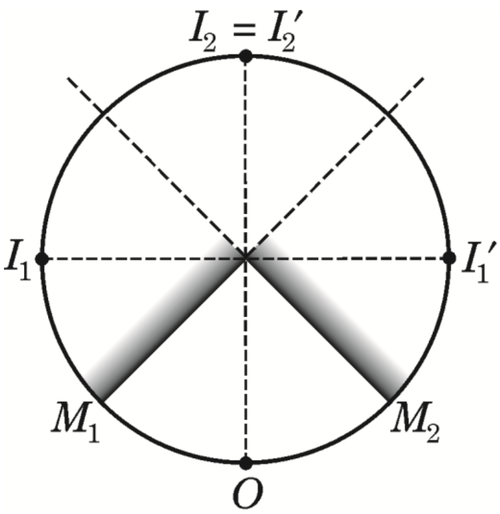
$$X = \frac{4H}{3} - \frac{H}{2}$$

$$X = \frac{4H}{6}$$

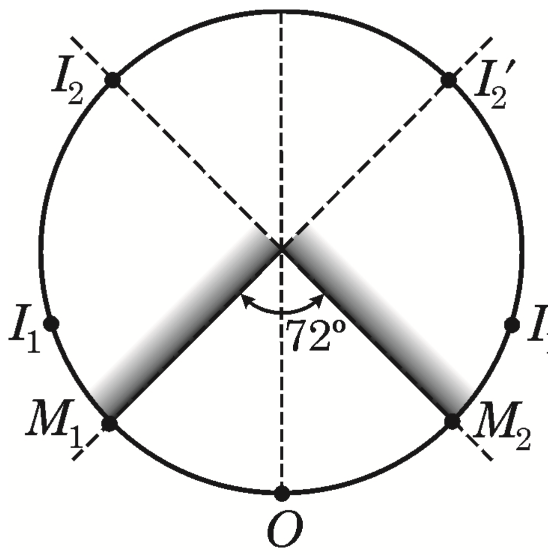
Number of images formed by two plane mirrors



$$K = \frac{2\pi}{\theta} \quad \text{is even } (2n)$$



$$K = \frac{2\pi}{\theta} \quad \text{is odd (2n)}$$



$$K = 2n \pm x\ ,\ 0 < x < 1$$