Physics I
Class 13

General
Rotational Motion

Definitions

Angular Position: $\vec{\theta}$ (normally in radians)

Angular Displacement: $\Delta\vec{\theta} = \vec{\theta} - \vec{\theta}_0$

Average or mean angular velocity is defined as follows:

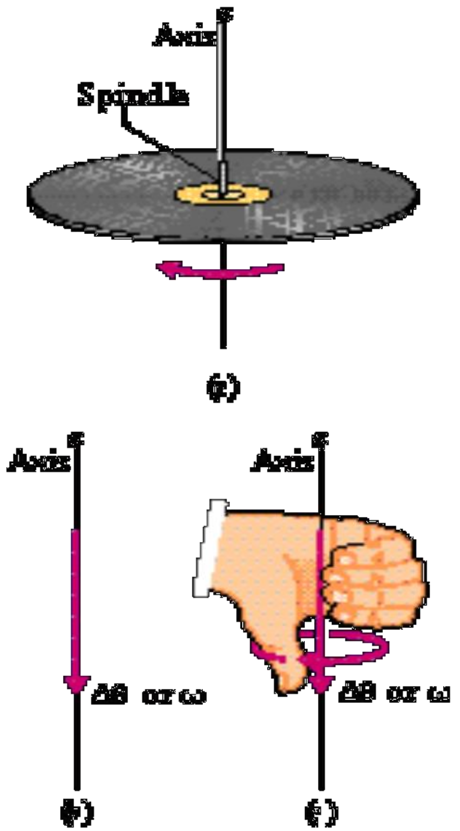
$$\vec{\omega}_{avg} \equiv \frac{\vec{\theta} - \vec{\theta}_0}{t - t_0} \equiv \frac{\Delta\vec{\theta}}{\Delta t}$$

Instantaneous angular velocity or just " angular velocity":

$$\vec{\omega} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} \equiv \frac{d\,\vec{\theta}}{d\,t}$$

Wait a minute! How can an angle have a vector direction?

Direction of Angular Displacement and Angular Velocity



- Use your right hand.
- Curl your fingers in the direction of the rotation.
- Out-stretched thumb points in the direction of the angular velocity.

Angular Acceleration

Average angular acceleration is defined as follows:

$$\vec{\alpha}_{avg} \equiv \frac{\vec{\omega} - \vec{\omega}_0}{t - t_0} \equiv \frac{\Delta\vec{\omega}}{\Delta t}$$

Instantaneous angular acceleration or just “angular acceleration”:

$$\vec{\alpha} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} \equiv \frac{d\,\vec{\omega}}{d\,t} \equiv \frac{d^2\,\vec{\theta}}{d\,t^2}$$

The easiest way to get the direction of the angular acceleration is to determine the direction of the angular velocity and then…

- If the object is speeding up, angular velocity and acceleration are in the same direction.
- If the object is slowing down, angular velocity and acceleration are in opposite directions.

Equations for Constant α

1. $\omega = \omega_0 + \alpha(t - t_0)$
2. $\theta = \theta_0 + \omega_0(t - t_0) + \frac{1}{2}\alpha(t - t_0)^2$
3. $\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)(t - t_0)$
4. $\theta = \theta_0 + \omega(t - t_0) - \frac{1}{2}\alpha(t - t_0)^2$
5. $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$x \rightarrow \theta$
$v \rightarrow \omega$
$a \rightarrow \alpha$

$\boxed{\text{We recommend always using radians for angles.}}$

Relationships Among Linear and Angular Variables

$$s = \theta r$$

$$v = \omega r$$

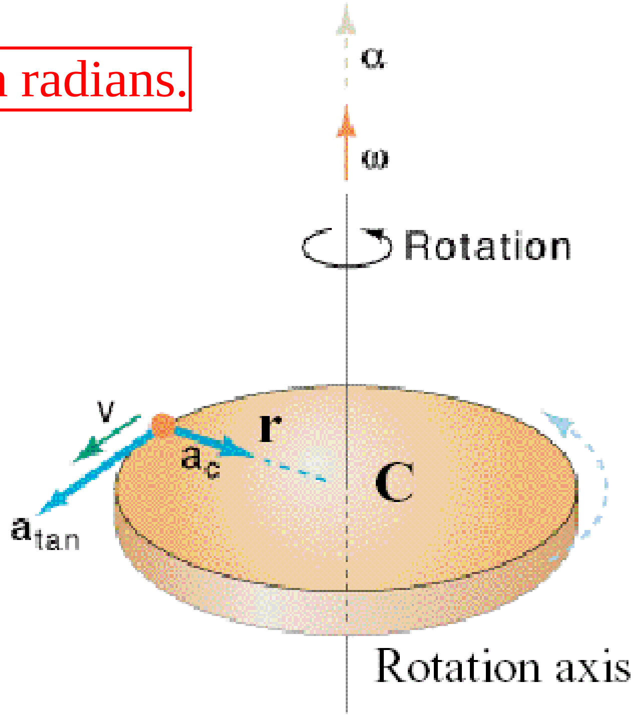
MUST express angles in radians.

$$a_{tangential} = \alpha r$$

$$a_{centripetal} = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r$$

The radial direction is defined to be + outward from the center.

$$a_{radial} = -a_{centripetal}$$



Energy in Rotation

Consider the kinetic energy in a rotating object. The center of mass of the object is not moving, but each particle (atom) in the object is moving at the same angular velocity (ω).

$$K = \sum \tfrac{1}{2} m_i v_i^{\,2} = \sum \tfrac{1}{2} m_i \omega^2 r_i^{\,2} = \tfrac{1}{2} \omega^2 \sum m_i r_i^{\,2}$$

The summation in the final expression occurs often when analyzing rotational motion. It is called **rotational inertia** or the **moment of inertia**.

Rotational Inertia or Moment of Inertia

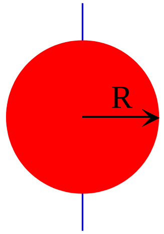
For a system of discrete “point” objects:

$$I = \sum m_i r_i^2$$

For a solid object, use an integral where ρ is the density:

$$I = \iiint \rho r^2 \, dx \, dy \, dz$$

We may ask you to calculate the rotational inertia for point objects, but we will give you a formula for a solid object or just give you its rotational inertia.



I for a solid sphere: $I = \frac{2}{5} M R^2$

I for a spherical shell: $I = \frac{2}{3} M R^2$

Characteristics of Rotational Inertia

The rotational inertia of an object depends on

- Its mass.
- Its shape.
- The axis of rotation.
- **NOT** the angular velocity or acceleration.

The rotational inertia is a measure of how difficult it is to get an object to start rotating or to slow down once started.

For two or more objects rotating around a common axis, the total rotational inertia is the sum of each individual rotational inertia.

$$I = \sum I_i$$

Introduction to Torque

For linear motion, we have "F = m a". For rotation, we have

$$\vec{\tau} = I\,\vec{\alpha}$$

The symbol "τ" is torque. We will define it more precisely next time.

Torque and angular acceleration are always in the same direction in Physics 1 because we consider rotations about a fixed axis.

Correspondence Between Linear and Rotational Motion

$$x \to \theta$$
$$v \to \omega$$
$$a \to \alpha$$
$$m \to I$$
$$F \to \tau$$
$$K = \tfrac{1}{2} I \omega^2$$
$$\vec{\tau} = I \vec{\alpha}$$

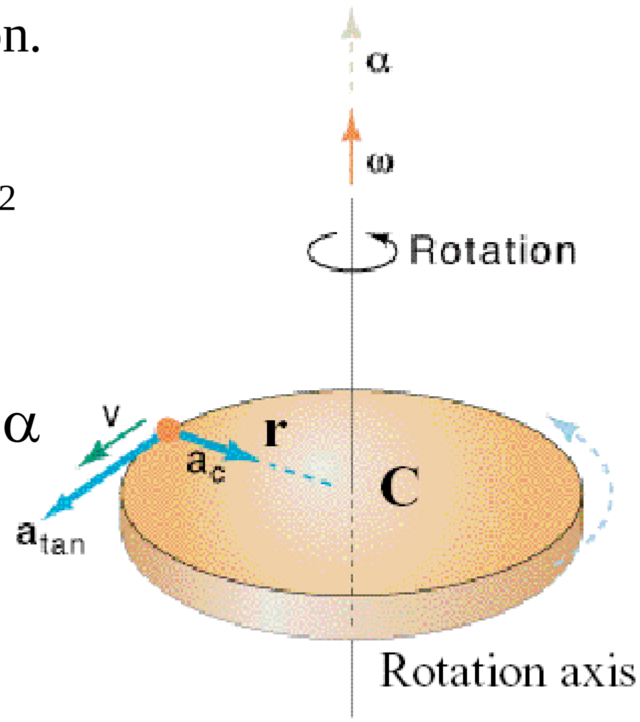
You will solve many rotation problems using exactly the same techniques you learned for linear motion problems.

Class #13
Take-Away Concepts

1. Definitions of rotational quantities: θ, ω, α.
2. Centripetal and tangential acceleration.
3. Rotational inertia: $I = \sum m_i r_i^2$
4. Rotational kinetic energy: $K = \frac{1}{2} I \omega^2$
5. Introduction to torque: $\vec{\tau} = I \vec{\alpha}$
6. Correspondence

$x \rightarrow \theta \qquad v \rightarrow \omega \qquad a \rightarrow \alpha$

$m \rightarrow I \qquad F \rightarrow \tau$



Class #13
Problems of the Day

___**1.** Lance Armstrong is riding his bicycle due east in the Tour de France bicycle race. What is the direction of the angular velocity of his bicycle wheels?

A) North.
B) South.
C) East.
D) West.
E) Up.
F) Down.

Class #13
Problems of the Day

2. A carousel takes 8 seconds to make one revolution when turning at full speed. It takes exactly one revolution to reach its full speed from rest. What is the <u>magnitude</u> of its angular acceleration while speeding up, assuming that its angular acceleration is constant?

Activity #13
Introduction to Rotation

Objective of the Activity:

1. Think about rotation concepts.
2. Try changing the rotational inertia of a simple object and see how that affects $\vec{\tau} = I\vec{\alpha}$.