Inequality: (definition)

Two real numbers or two algebraic expression related by the symbol '$<$', '$>$' '$\leq$' or '$\geq$' form an inequality.

e.g.

$ax+b < 0$

$ax+b > 0$

} (strict inequality)

$ax+b \leq 0$

$ax+b \geq 0$

$$\frac{ax+by > 0}{ax+b < 0}$$

Solution of linear inequalities:

$30x < 200$

$x \geq 0$ $(x \in W)$

$x=0$, $30 \times 0 < 200$

$0 < 200$ true

$x=1$, $30 \times 1 < 200$

$x=2$, $30 \times 2 < 200$

$x=3$, $30 \times 3 < 200$

$x=4$, $30 \times 4 < 200$

$x=5$, $30 \times 5 < 200$

$x=6$, $30 \times 6 < 200$

$x=7$ $30 \times 7 < 200$ (false)

Solution of inequality $= \{0, 1, 2, 3, 4, 5, 6, 7\}$

Rules for solving inequalities:

Rule 1:

1) $30x < 200$

$30x + 5 < 200 + 5$

$30x - 5 < 200 - 5$

Rule 2: (a) $30x < 200$

(i) $10 \times 30x < 10 \times 200$

↑ (+ve) ↑ (+ve)

$\frac{30x}{-10} > \frac{200}{-10}$

Rule 1:

(ii) $\frac{30x}{10} < \frac{200}{10}$

(b) when multiply by (-ve) numbers.

$30x < 200$

$(-10) \times 30x > 200 \times (-10)$

Ex 6.1:

1: $24x < 100$ (i) $x \in N$ (ii) $x \in Z$

divide both sides by 24. we get -

$$\frac{24x}{24} < \frac{100}{24}$$

$$x < \frac{100}{24} \quad \frac{50}{12} \quad \frac{25}{6} \quad 4.. \quad x < 4.5$$

(i) $x = \{1, 2, 3, 4\}$

(ii) Solution set $= \{-\infty, - \cdots 0, 1, 2, 3, 4\}$

(2)

$-12x > 30$

(i) $x \in N$

$x \in Z$

(ii)

$$\frac{-12x}{-12} < \frac{30}{-12}$$

$$x < -\frac{5}{2} \quad = -2.5$$

Solution set

$= \{-\infty, \ldots -3\}$

$\therefore$ The inequality has no solution.

$\because x \in N.$

(3) $5x - 3 < 7$ (i) $x \in Z$

$5x - 3 + 3 < 7 + 3$

$5x < 10$

$\frac{5x}{5} < \frac{10}{5}$

$x < 2$

Solution set $= \{-\infty, \ldots, 0, 1,\}$

$x \in R$

(ii) The solution set

$= (-\infty, 2)$

$(-\infty, \infty)$ → open interval

$(2, 4]$ → Closed interval

→ 4 is part of Solution

→ 2 - excluded.

4: $3x+8 > 2$

(i) $x \in \mathbb{Z}$ (ii) $x \in \mathbb{R}$

(i) $3x+8-8 > 2-8$

$3x > -6$

$\frac{3x}{3} > \frac{-6}{3}$

$x > -2$

Solution set $= \{-2, -1, \ldots 0 \ldots \infty\}$

(ii) Solution set $= (-2, \infty)$

5: $4x+3 < 5x+7$

$4x - 5x < 7 - 3$

$-x < 4$

multiply both sides by (-1)

$x > (-4)$

$x > -4$

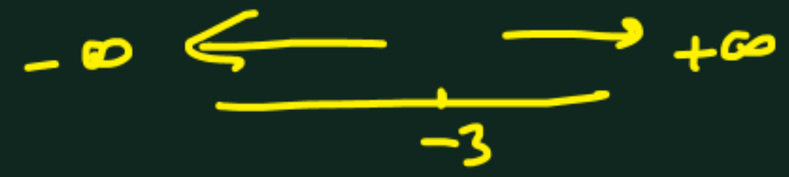
$\therefore$ Solution set $= (-4, \infty)$

(6) $3x - 7 > 5x - 1$

$3x - 5x > 7 - 1$

$-2x > 6$

$\frac{-2x}{-2} < \frac{6}{-2}$



$x < -3$

$\therefore$ Solution set $= (-\infty, -3)$

(8) $3(2-x) \geq 2(1-x)$

$6 - 3x \geq 2 - 2x$

$-3x + 2x \geq 2 - 6$

$-x \geq -4$

multiplying (-1)

$\boxed{x \leq 4}$ → less than or equal.

Solution set = $(-\infty, 4]$

(9) $x + \frac{x}{2} + \frac{x}{3} < 11$

$\frac{6x + 3x + 2x}{6} < 11$

$\frac{11x}{6} < 11$

$\frac{11x}{6} \times 6 < 11 \times 6$

$\frac{11x}{11} < \frac{11 \times 6}{11}$

$\therefore \boxed{x < 6}$

Solution set = $(-\infty, 6)$

10) M1 $\frac{x}{3} > \frac{x}{2} + 1$

$\frac{x}{3} > \frac{x+2}{2}$ (3) (2)

$2x > 3x + 6$

$2x - 3x > 6$

$-x > 6$

$x < -6$

M2 $\frac{x}{3} - \frac{x}{2} > 1$

$\frac{-x}{6} > 1$

$\frac{-x}{6} \times 6 > 1 \times 6$

$-x > 6$

$x < -6$

denominator mein +ve no. hai multiply cross - possible.

multiply both sides by (-1)

Solution set = $(-\infty, -6)$

(11)

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

$$\frac{3x-6}{5} \leq \frac{10}{3} - \frac{5x}{3}$$

$$\frac{3x}{5} + \frac{5x}{3} \leq \frac{10}{3} + \frac{1}{5}$$

$$\frac{34x}{15} \leq \frac{68}{15}$$

$$\frac{34x}{15} \times \frac{15}{34} \leq \frac{\cancel{68}^{2}}{\cancel{15}} \times \frac{\cancel{15}}{\cancel{34}}$$

$x \leq 2$

Solution set $= (-\infty, 2]$

$9\frac{25}{25} = 9\frac{1}{16}$

(12)

$$\frac{1}{2}\left[\frac{3x}{5}+4\right] \geq \frac{1}{3}(x-6)$$

(-1) multiply both sides

$$\frac{1}{2}\left(\frac{3x+20}{5}\right) \geq \frac{1}{3}(x-6)$$

$x \leq 120$

Solution set $= (-\infty, 120]$

$$3(3x+20) \geq 10(x-6)$$

$$9x+60 \geq 10x-60$$

$$9x-10x \geq -60-60$$

$$-x \geq -120$$

14:

$$37-(3x+5) \geq 9x-8(x-3)$$

$$37-3x-5 \geq 9x-8x+24$$

$$32-3x \geq x+24$$

$$-3x-x \geq 24-32$$

$$-4x \geq -8$$

$$\frac{-4x}{-4} \leq \frac{-8}{-4}$$

$$x \leq 2$$

$\therefore$ Solution set $= (-\infty, 2]$

15:

$$\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$$

$$\frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$$

$$\frac{x}{4} < \frac{4x-1}{15}$$

$$15x < (4x-1)\,4$$

$$15x - 16x < -4$$

$$-x < -4$$

$$x > 4$$

Solution set = $(4, \infty)$

16:

$$\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$$

$$\frac{2x-1}{3} \geq \frac{15x-10-8+4x}{20}$$

$$\frac{2x-1}{3} \geq \frac{19x-18}{20}$$

$$(2x-1)\,20 \geq 3(19x-18)$$

$$40x - 57x \geq 20 - 54$$

$$-17x \geq -34$$

$$\frac{-17x}{-17} \leq \frac{-34}{-17}$$

$$x \leq 2$$

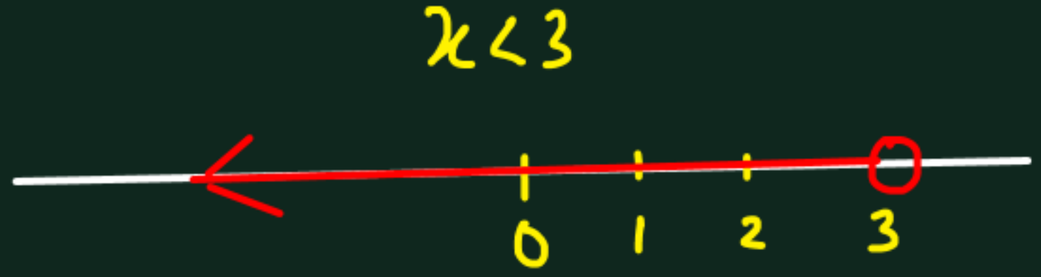
Set $= (-\infty, 2]$

17:

$3x - 2 < 2x + 1$

$3x - 2x < 2 + 1$

$\boxed{x < 3}$



Solution set $= (-\infty, 3)$

18

$5x - 3 \geq 3x - 5$

$5x - 3x \geq 3 - 5$

$2x \geq -2$

$\boxed{x \geq -1}$

Solution set $= [-1, \infty)$

(19) $3(1-x) < 2(x+4)$

$3-3x < 2x+8$

$-3x-2x < 8-3$

$-5x < 5$

$\boxed{x > -1}$

$(-1, \infty)$

$\left[\frac{-2}{7}, \infty\right)$

(20) $\frac{x}{2} \geq \left(\frac{5x-2}{3}\right) - \frac{7x-3}{5}$

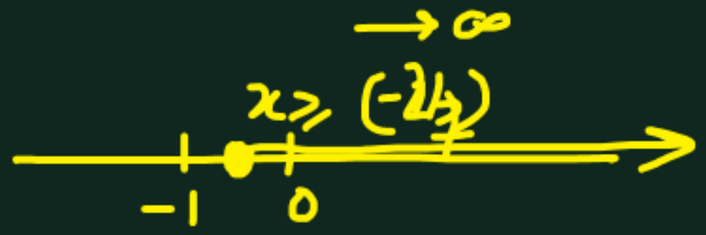
$\frac{x}{2} \geq \frac{25x-10-21x+9}{15}$

$\frac{x}{2} \geq \frac{4x-1}{15}$

$15x \geq 8x-2$

$7x \geq -2$

$x \geq \frac{-2}{7}$



21: Let x be the marks obtained by Rani in 3rd test.

$\therefore \quad \frac{x+70+75}{3} \geq \frac{60}{1}$ $\quad [\because$ At least $\geq]$

$x + 145 \geq 180$

$x \geq 180 - 145$

$\boxed{x \geq 35} \longrightarrow$

Minimum obtain marks = 35 to get

22: Let x be the marks obtained by Sunita in the 5th exam.

$$\text{Average marks} = \frac{x + 87 + 92 + 94 + 95}{5} \geq 90$$

$$x + 368 \geq 450$$

$$x \geq 450 - 368$$

$$\boxed{x \geq 82}$$

Minimum marks $= 82$

23: Let x = Smaller of two consecutive +ve integers. x can take values 5 & 7

Other odd +ve integer = $x+2$

A.T.Q:

(i) $x < 10$ & $x+2 < 10$

$x < 10$ & $x < 8$ $\Rightarrow$ $x < 8$

pair odd no. = (5, 7)

(ii) $x + x+2 > 11$

$2x+2 > 11$

$x > \frac{9}{2}$ (4.5)

$4.5 < x < 8$

24: Let x : Smaller even positive no.

next number $= x+2$

$\therefore$ A.T.Q.

1) $x > 5$ & $x+2 > 5$

$x > 5$ & $x > 3$

(2) $x + x+2 < 23$

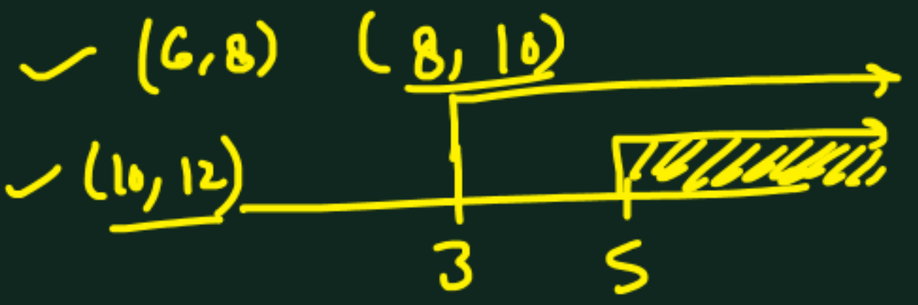
$2x < 21$

$x < \frac{21}{2}$ (10.5)

$\therefore x$ can take 6, 8, 10

(6,8) (8,10)

(10,12)



$x > 5$

$5 < x < 10.5$

25: Let shortest side of $\Delta = x$

longest side $= \underline{3x}$

third side $= 3x-2$

A. T. Q:

$$x + 3x + 3x - 2 \geq 61$$

$$7x - 2 \geq 61$$

$$7x \geq 63$$

$$\boxed{x \geq 9}$$

Minm value of $x = 9$

$\therefore$ Shortest side of $\Delta = 9$

26: Let the shortest piece length = x

second piece length = $x+3$

third piece length = $2x$

A.T.Q.

$$x + x+3 + 2x \leq 91$$

$$4x+3 \leq 91$$

$$4x \leq 88$$

$$x \leq 22 \quad \text{—(1)}$$

$8 \leq x \leq 22$

$\therefore$ from eq(1) & eq(2) -

2nd condition:

III piece − IInd piece ≥ 5

$2x - (x+3) \geq 5$

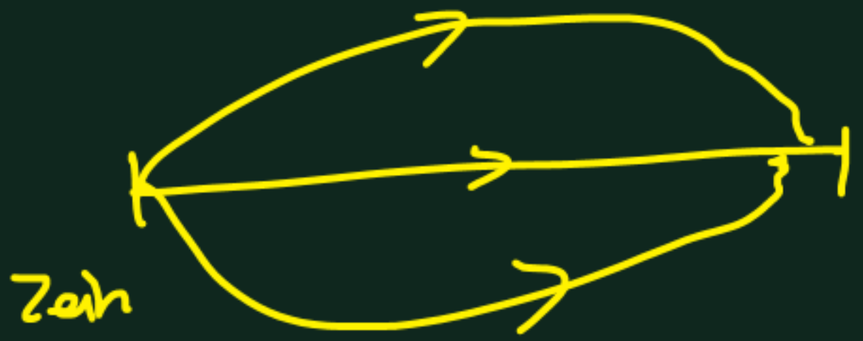
$2x - x - 3 \geq 5$

$x \geq 8$ —(2)

$\therefore$ The length of shortest board

$8 \leq x \leq 22$ ✓

12: Linear programming as tool



Linear

Optimization

$z =$

restrict

Shortest time

force mini

inequality

Calculation function

C

Ex 6.2

$6+3 < 5$

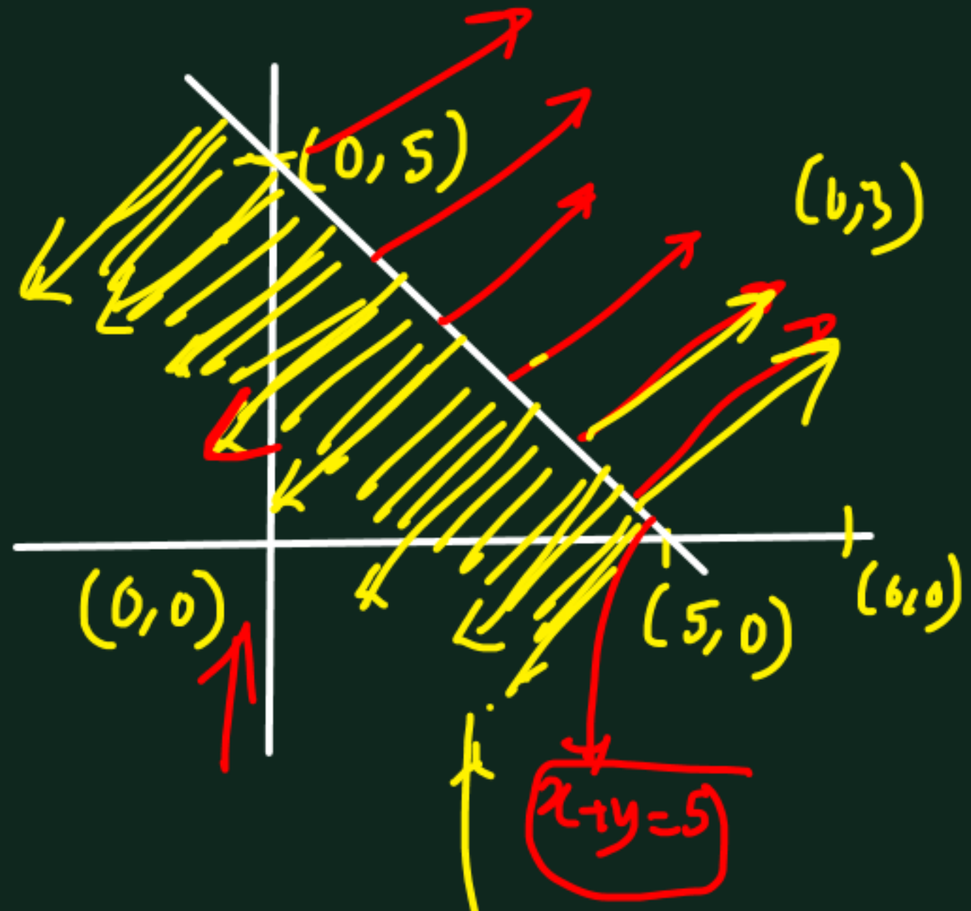
(false) $9 < 5$

① $x + y < 5$

Step 1: $x + y = 5$

when $x = 0$, $y = 5$ $(0, 5)$

when $y = 0$, $x = 5$ $(5, 0)$



$x + y < 5$

put point $(0, 0)$ in above inequality

$0 + 0 < 5$

$0 < 5$ (true)

part of solution