Gravitation

Anybody thrown upward is attracted towards earth.

ex: Ball thrown upside falls on earth after some time.
Rain drops falling from clouds.

These attracting force known to us from long ago. Greeks like Aristotle, Archimedes etc. Indian scholars has known this also.

Aryabhata mentioned about Earth's rotation around the sun.

planetary motions are known to us, stars and their motion and positions unchanged.

ptolemy Geocentric model:

1) Earth is in center all celestial bodies like sun, stars, planents moves around the earth in circles.
Circular motion around Earth as its center.

Heliocentric model (Nicholas copernicus)

planets moves around a fixed sun. He posulated heliocentric theory which changed course of science.

* Kelpar Laws:

1) Keplar formulated his laws by analysing the austronomical analyses of his mentor 'Tycho Brahe'.

2) it improved the heliocentric theory. by replacing its circular orbits to elliptical orbits

(i) 1st Keplar law: The orbit of planet is ellipse with sun at one of two foci

2nd Kepler law: Law of Areas.

- A line segment joining planet and sun, sweeps out equal areas in equal time intervals.
- planets move slower when they are far, and move faster when they are near.
- Conservation of Angular momentum. Speed at which planet is moving constantly changes.

Angular momentum $(\vec{L}) = \vec{r} \times \vec{p}$. (or) $\boxed{L = mvr}$

3rd law:

The square of period of any planet is proportional to the cube of Semi major axis of its orbit.

$$T^2 = \frac{4\pi^2}{GM} \cdot a^3 . \rightarrow T^2 \propto a^3 \quad (a = \text{Semi major axis})$$

Actual theoretical calculation, But precisely.

$$T^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3 .$$

M_1, M_2 are Masses of sun and planet. Sun masses is far greater than planet. So, $M_1 >>> M_2 . \rightarrow (M_1 + M_2) \longrightarrow M_1$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3 . \qquad \left[\frac{4\pi^2}{GM} = K \text{ (Constant)}\right]$$

$$\boxed{T^2 = (K) a^3}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3 .$$

T= orbital Time period.
a= semi major axis

$\downarrow$

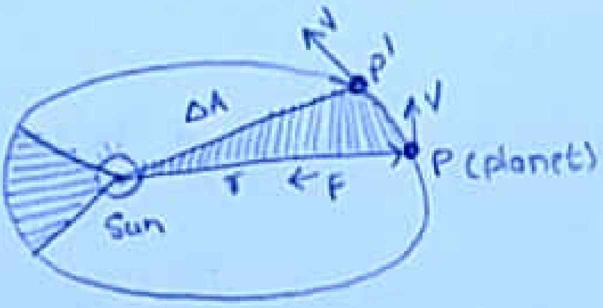
$\simeq 7.496 \times 10^{-6}$.

in solar system $\rightarrow$ $\frac{T^2}{a^3}$ = constant. $\simeq 3$. (Nearly to all planets).

$\rightarrow$ By law of areas. (2^{nd} law)

line joing (joins) any planet and sun sweeps equal areas in equal time intervals.

Another form of conservation of angular momentum(L).



planet $\rightarrow$ p at two different position.

$\Delta A \rightarrow$ Area swept.

$\Delta t \rightarrow$ change in time interval.

$\Delta A = \frac{1}{2}(r \times v\Delta t) \rightarrow \frac{\Delta A}{\Delta t} = \frac{1}{2}(r \times \frac{p}{m})$. $(r \perp v)$.

$(v = P/m) \rightarrow$ p $\rightarrow$ momentum(linear)

L $\rightarrow$ angular momentum $\rightarrow$ $\boxed{L = mvr}$ $\quad p = mv$

vector form $\leftarrow$ $\boxed{L = pr}$ $\rightarrow$ relation b/w linear and angular momentum.

$\boxed{L = r \times p}$

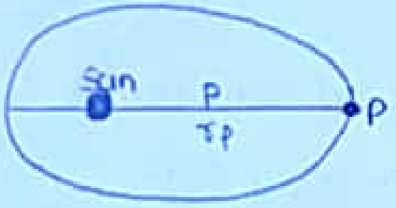
$$\frac{\Delta A}{\Delta t} = \frac{1}{2}\left(\frac{L}{m}\right).$$

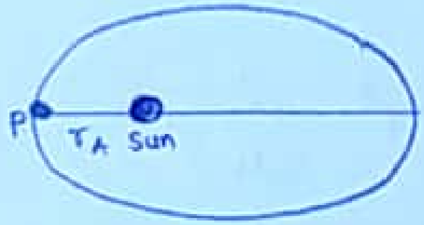
$\rightarrow$ These are central forces, means force acting on along line joining b/w two.

$$\boxed{\frac{\Delta A}{\Delta t} = \text{constant}}$$

For central forces, L is constant (or) conserved.

Total linear momentum in closed system is constant.





Fartest from sun to planet is Aphelion

(r_A, v_A)

closest distance from sun is Perihelion(PH).

(r_P, v_P)

L = angular comemtum conserved.

$$\boxed{r_A v_A m_P = r_P v_P m_P}$$

(m_P = mass of planet).

$$\frac{v_A}{v_P} = \frac{r_P}{r_A} \rightarrow \boxed{v \propto \frac{1}{r}}$$

Since, ($r_A >> r_P$) → ($v_P > v_A$).

Universally. Keplar are valid. Keplar improved heliocentric theory of Nicholas Copernicus.

Copernicus views		→	Keplar view.
planet orbit	– Circular	→	elipitical
Sun position	– center of orbit	→	present at one of foci of ellipse.
Speed planet	– Constant	→	Speed varies. at Aphelion → slow at pehillon – fast
eccentricity	– zero $e=0$	→	$0<e<1$.

if $e=0$, orbit → circular.

Keplar ellipitical orbit $\xrightarrow{e=0}$ circular orbit.

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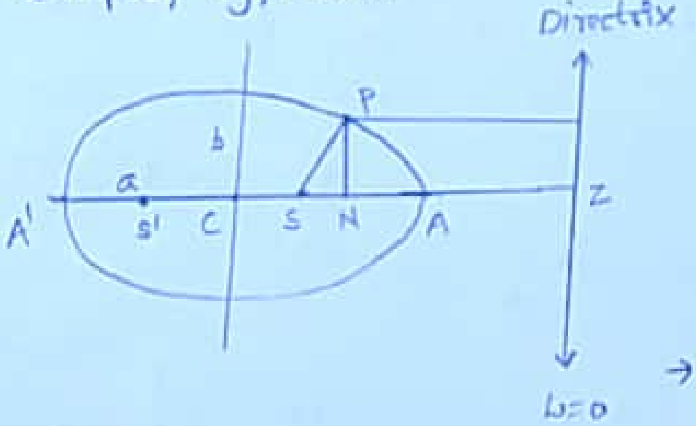
<u>Ellipse</u>

Conic section. plane cuts the cone → we have circle, parabola, ellipse, hyperbola.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$0 < e < 1.$

→ curve is symetric about axes

→ Foci $S(ae,0)$, $S'(-ae,0)$, a - semi major axis, b - semi minor axis

directrix $x = a/e$.

$P(\text{foci})_1 + P(\text{foci}_2) = 2a$, $PS + PS' = 2a$

eccentricity $(e) = \frac{c}{a} = \frac{\text{Distance from center to Foci}}{\text{Distance from center to vertex}}$.

$$\boxed{e = \sqrt{1 - b^2/a^2}}$$

Universal law of Gravitation:

every body in universe attract each other, force is directly proportional to product of masses, inversly proportional to distance b/w them.

$$|F| = G \frac{m_1 m_2}{r^2}$$

Vector form → $F = G \frac{m_1 m_2}{r^2}(-\hat{r})$ (- attractive force.

$$F = -G \frac{m_1 m_2}{|r|^3} \cdot \vec{r}$$ ↳ $-r$ along

G = Universal gravitional constant. , $\hat{r}$ = unit vector from m_1 to m_2.

$r = r_2 - r_1$.

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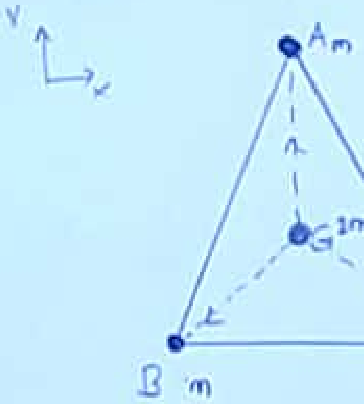
Newton III^{rd} law $F_{12} = -F_{21}$.

→ point masses like $m_1\ m_2\ m_3$.



$r = 1$

$AG = BG = CG = 1$

$F_{GA} = Gm(2m)\,\hat{j}$

$F_{GB} = Gm(2m)(-\hat{i}\cos 30 - \hat{j}\sin 30)$

$F_{GC} = Gm(2m)\,(\hat{i}\cos 30^\circ - \hat{j}\sin 30^\circ)$.

$F_R = F_{GA} + F_{GB} + F_{GC}$

$= 2Gm^2\hat{j} + 2Gm^2(-\hat{i}\cos 30 - \hat{j}\sin 30)$

$+ 2Gm^2(\hat{i}\cos 30 - \hat{j}\sin 30)$

$\boxed{F_R = 0}$ → based on symmetry.

Hallow sphere. point mass

→ All point, forces, are resolved in to components. Compents which are perpendicular cancel.

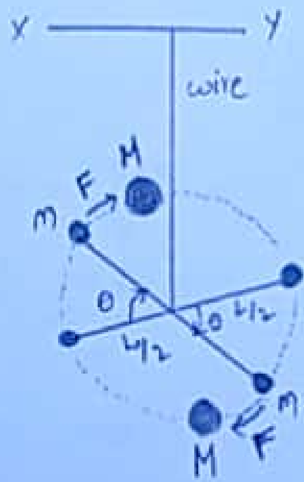
last only result force, line joining point to center.

point mass outside →

inside

All force cancel out due to symetry (Zero = F).

Gravitational Constant: Henry cavendish in 1798.



$\tau_1 = L/2\, F_G$

$\tau_2 = L/2\, F_G$

$\tau = L F_G$

$\tau = L \cdot \frac{GMm}{d^2}$

Force b/w small ball and large ball is gravitational.

→ Two force equal and opposite acting on body of Length generates torque. and causes rotation

$\tau = \tau_1 + \tau_2$.

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This torque, genelates a twist in wire.

From Hooke's law. $\tau_{wire} = K\theta$.

$$\boxed{\tau_G = \tau_{wire}}$$

when θ = known, G = can be find.

$$L \cdot \frac{G m m}{r^2} = K\theta .$$

angle Rotated.

From $\boxed{G = 6.67 \times 10^{-11}\ Nm^2/kg^2}$

Accleration due to gravity: (g)

When Force act on a body, accleration is produced in the direction of Force. Earth is uniform sphere.

$$F = G\frac{m_1 m_2}{r^2} . \qquad m_2 >>>>> m_1 \quad (m_2 - \text{earth},\ m_1 - \text{body})$$

$$F = \frac{G m_e m_b}{r^2} , \qquad 2^{nd} \text{ law Newton} \rightarrow \boxed{F = ma = mg}$$

here, body mass(m_b) is attracted towards earth. earth attraction is greter than body attraction. since both attracted to each other.

body to earth → high attracted

earth attracted to body → low.

$$F = mg \Rightarrow \frac{F}{m} = g$$

$$\rightarrow g = \left(\frac{G m_e m_b}{r^2}\right) \Big/ \frac{1}{m_b} = \frac{G m_e}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}}$$

$g = 9.8 m/sec^2$ → constant.

→ vector form $g = -\frac{GM}{r^2}\hat{r}$

$\hat{r}$ – unit vector in direction to mass.

– → sign attractive.

Earth → not perfect sphere → bluges at equator.

not Symmetrical. → cause deviation to 'g' value

flat at poles.

earth is rotating

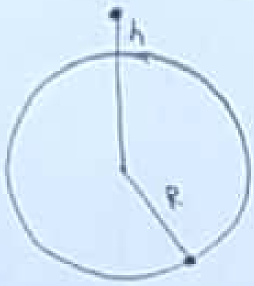
equator → bulge at equator → Radius is large → 'g' low.

$R_e \to$ large:

poles → small radius → 'g' is ~~low~~ high

equator → 'g' low, poles → 'g' high.

$g_e < g_p$.



(a) effect of 'g' based on height

$$g_h = g\left(1 - 2h/R_E\right)$$

at height 'h' → g → decreases

when going up → g decreases.

at some height (h)

$$F = \frac{G m_E m}{(R_E+h)^2}, \quad g_h = \frac{G m_E}{(R_E+h)^2}$$

$\boxed{h <<<< R_E}$ →

① $g = \dfrac{G m_E}{R_E^2}$, $\dfrac{g_h}{g} = \left[\dfrac{G m_E}{(R_E+h)^2}\right] \Big/ \left[\dfrac{G m_E}{R_E^2}\right]$.

② $\dfrac{g_h}{g} = \dfrac{\cancel{G m_E}}{(R_E+h)^2} \times \dfrac{R_E^2}{\cancel{G m_E}}$

③ $g_h = g\left[\dfrac{1}{\left(\dfrac{R_E+h}{R_E}\right)^2}\right] = g\left[\dfrac{1}{(1+h/R_E)^2}\right]$

$$g_h = g\left[1/(1+h/R_E)^2\right]$$

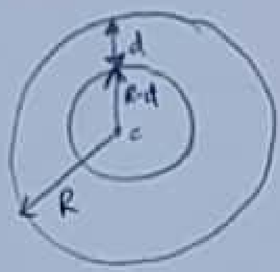
Since $h <<< R_E$

$h/R_E <<< 1$

↳ expand by bionomical expansion.

$$\boxed{g_h = g\left(1 - 2h/R_E\right)}$$

acceleration due to gravity change due to depth (d)



$g = Gm/R^2$

density $(\rho) = \frac{mass}{volume} = m/V = \rho$

$m = (\rho V) = \frac{4}{3}\pi R^3 \times \rho$ (sphere volue $= V = 4/3 \pi r^3$)

$$g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \times \rho = \frac{4}{3}\pi G R \rho, \quad \boxed{g = \frac{4}{3}\pi G R \rho}$$

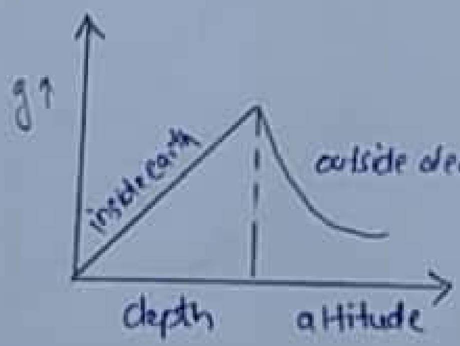
$$g_d = \frac{4}{3}\pi G (R-d)\rho.$$

$$\frac{g_d}{g} = \frac{R-d}{R} = (1 - d/R) \Rightarrow \boxed{g_d = g(1 - d/R)}$$

As depth $\uparrow$ → g decreases. $(d\uparrow, g\downarrow)$

→ As height increases → 'g' decreases faster than depth.

g_h decrease fast than g_d.



Gravitional potential Energy: (GPE)

Potential energy → energy stored in the body by virtue of its position / height.

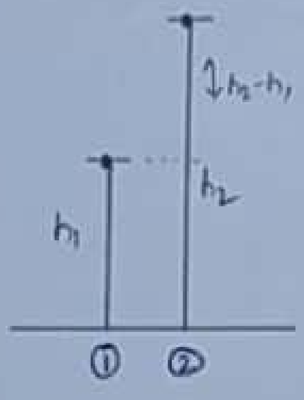
Change in PE → change in position of particle based on account of forces acting on it.

↓

Amount of work done on it.

→ gravity → Conservative force → workdone is independent of path.

Force is conservative. Find P.E of body due to gravitional force. (GPE).



$\omega_{12} = mg(h_2-h_1)$ From ① to ② position.

$\omega_h = mgh + \omega_0$ (at height 'h').

$\omega_{12} = \omega(h)_2 - \omega(h_1)$.

$\omega_{h} = \omega_0$ (at $\omega_0 \rightarrow$ Surface of earth)
$(h=0)$

$$\omega_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = -GM_E m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

at distance $\rightarrow r$, $\omega(r) = \dfrac{-GM_E m}{r^2} + \dfrac{GM_E m}{r_1}$

$$\omega(r) = \frac{-GM_E m}{r^2} + \omega_1 \qquad (r > R).$$

$\omega(r\rightarrow\infty) = \omega_1$ $\quad 1/r \rightarrow 0$.

$\boxed{V = \dfrac{-GMm}{r^2}} \rightarrow (V=0,\ r\rightarrow\infty)$.

Escape Speed : minimum speed needed for a body to escape from gravitional influence of earth / massive body. (free, non propelled body)

Rocket $\rightarrow$ sufficent propellent, suitable mode of propulsion $\rightarrow$ escapes.

Suppose body reaches infinity at $V_f \rightarrow$.

ω_1 – GPE at ∞.

$$T\cdot E = KE + PE = \left[T_\infty = \omega_1 + \frac{mv_f^2}{2}\right].$$

body $\rightarrow$ thrown to speed 'v' from distance $(h+R_E)$.

$$E(h+R_E) = \frac{1}{2}mv_i^2 - \frac{GMm}{(h+R_E)} + \omega_1$$

energy conserved.

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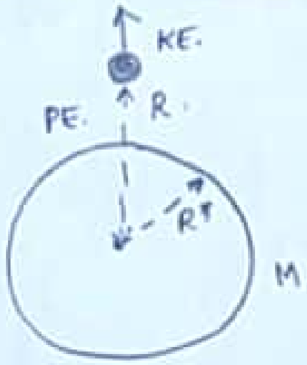
Easy method: escape velocity



Espace velocity is minimum velocity, required, to escape from gravitional force/field of a body.

At point $P_1 \rightarrow GPE = \frac{GMm}{R}$.

→ body escape gravitional force, $PE = 0$.

At point $P_2 \rightarrow GPE = 0$.

P_1 (surface)

$PE_1 = \frac{GMm}{R}$

$KE_1 = 0 \ (v=0)$.

P_2 (At point were, Gravition by Earth = 0)

$PE_2 = 0$ (No Gravition).

$KE_2 = \frac{1}{2}mv_e^2$ (V_e – escape velocity).

→ Since Energy is conserved / constant for this system.

$$T\cdot E_{P_1} = T\cdot E_{P_2}$$

$$\boxed{PE_1 + KE_1 = PE_2 + KE_2} \rightarrow$$

m = body mass

M = Earth Mass.

$$\rightarrow \frac{GMm}{R} + 0 = 0 + \frac{1}{2}mv_e^2.$$

$$V_e^2 = \frac{2GM}{R} \rightarrow \boxed{V_e = \sqrt{\frac{2GM}{R}}} \rightarrow eq\ 1$$

escape velocity depends on Earth mass/planents mass.

E.V Doesnot on mass of body.

→ Keep $\boxed{g = \frac{GM}{R^2}}$ in eq 1

$$\boxed{V_e = \sqrt{2gR}} \qquad \boxed{V_e \propto \sqrt{R}}$$

$$\boxed{\frac{V_e^2}{R} = \text{Constant}}$$

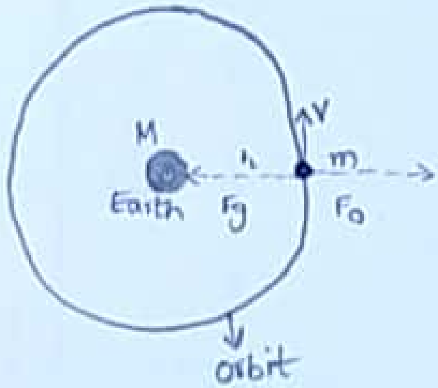
Earth satellites

Satelities revolve around earth in circular/ellipitical orbit. They follows Kepler law.

Satelities in circular orbit.

R- earth radius, h= orbit radius.

Total distance = (R+h).



Body in circular motion, two forces should be opposite & equal.

→ Body rotation in circle orbit,

so, centripetal force is produced directed to center.

→ gravitional force produces centripetal force.

$$F_c = \frac{mv^2}{r} = \frac{Mv_0^2}{(R_E+h)^2} \quad , \quad \boxed{F_c = F_g}$$

$$F_g = \frac{GM_E m}{(R_E+h)^2} \Rightarrow \boxed{v_0^2 = \frac{GM_E}{(R_E+h)}} \qquad R_E >>> h.$$

$$\boxed{v_0^2 = gR_E}$$

$$\text{Time} = T = \frac{2\pi(R_E+h)}{v} = \frac{2\pi(R_E+h)^{3/2}}{\sqrt{GM_E}}$$

circular path ⇒ . h=0, satellite close to earth.

$$T = \frac{2\pi R_E^{3/2}}{\sqrt{GM_E}} \rightarrow T_0 = 2\pi\sqrt{R_E/g}.$$

Energy of satellite! At $r \to \infty$, $PE \to 0$.

$$\text{at } (R_E+h) \rightarrow PE = -\frac{GMm}{(R_E+h)}.$$

$KE = \frac{1}{2}mv_0^2 = \frac{1}{2}m(\frac{Gm}{r})$ $\boxed{v_0^2 = \frac{GM_E}{R}.}$

$$\boxed{KE = \frac{1}{2}\frac{GmM}{r}}$$

$TE = PE + KE = \frac{-GmM}{(R_E+h)} + \frac{1}{2}\left(\frac{GMm}{(R_E+h)}\right)$ $(r = R+h)$

↳ we take account of "h".

$$\boxed{TE = \frac{-GMm}{2(R_E+h)}}$$

if $R_E >> h$ → $\boxed{TE = \frac{-GMm}{2R}}$ TE → negative.

Orbit → elliptical → KE, PE vary from point to point.

TE → +ve → object to infinity (
↘ 0 → ↗

Advanced: Newtonian mechanics deals Gravity in different way
Einstein, General theory of Relativity defined Gravity
→ curvature of spacetime. spacetime include 4D manifold three dimension, one time dimension.

→ Like, Take a big cloth, fix it all sides. Then place Ball, due to weight of Ball, some inclination is created. This way due to Massive body, Space-time is curved. This creates gravity.