Fourier Series

trigonometric functions

{ Sin x, Cos x } circular functions

they can represent a circular motion

spring mass system ⟷ vibration / SHM

equation $x = A \sin/\cos \omega t$

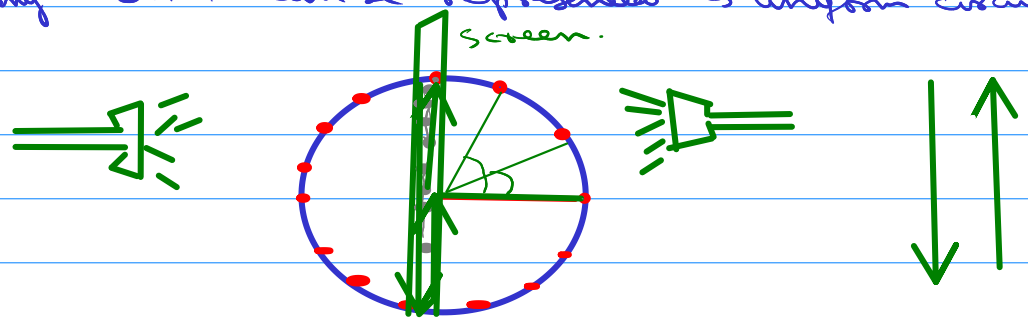
$\sin$ (input is angle)

$x = A \sin(\omega t + \phi)$ or $x = A \cos(\omega t + \phi)$

these represent SHM. ϕ phase angle

$\sin 30°$ angle.

any SHM can be represented as uniform circular motion



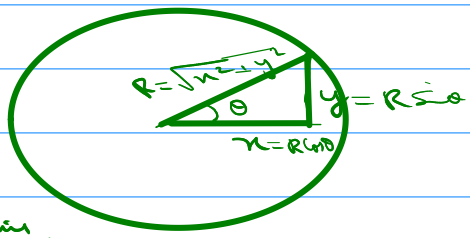
Simple harmonic motion can be treated as shadow or projection of a uniform circular motion

$x^2 + y^2 = R^2$] equation of circle. circle

$R^2 \cos^2\theta + R^2 \sin^2\theta = R^2$

$\sin^2\theta + \cos^2\theta = 1$

this represents a circle whose radius = 1



$\sqrt{x^2+y^2}$ = Radius of circle

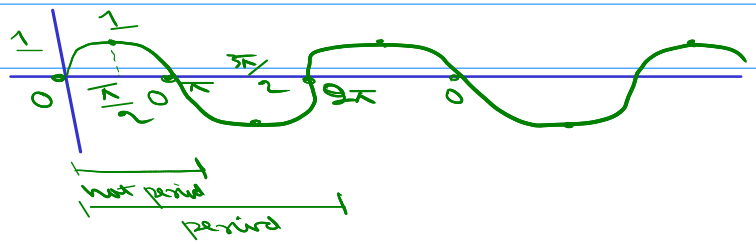
$x^2 + y^2 = R^2$

∴ Sin & Cos are called circular functions

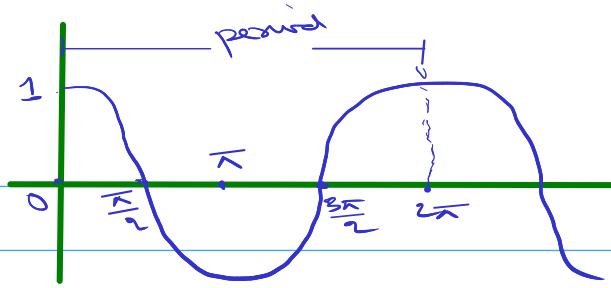
Fourier Series expansion of any periodic function in terms of infinite series of Sin and Cos functions

Periodic function

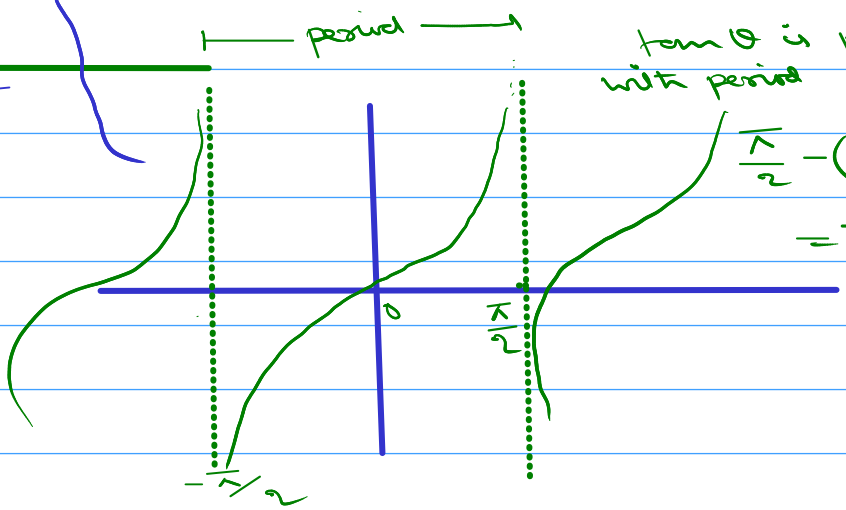
any function which repeats its value after equal intervals c/u period of function



Sin θ is a periodic function with period 2π



$\cos\theta$ is periodic with period 2π.



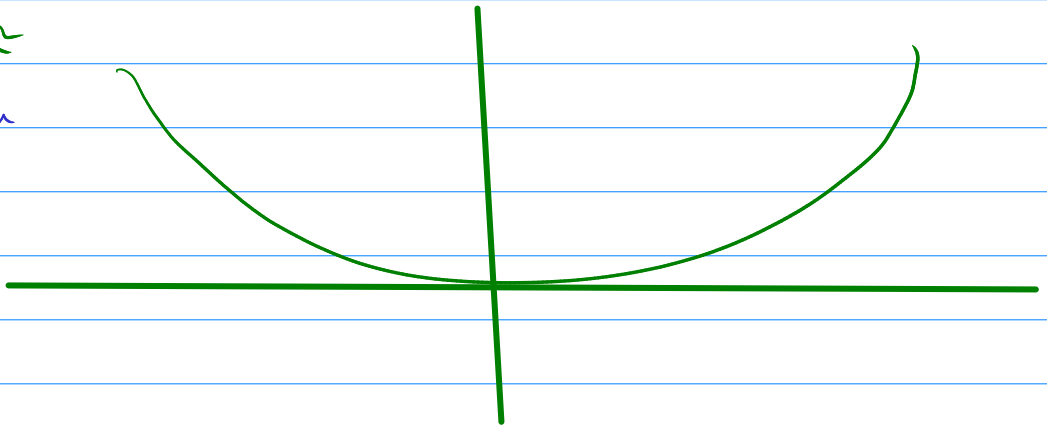
$\tan\theta$ is periodic with period

$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$

$= \pi$

$f(x) = x^2$

not periodic



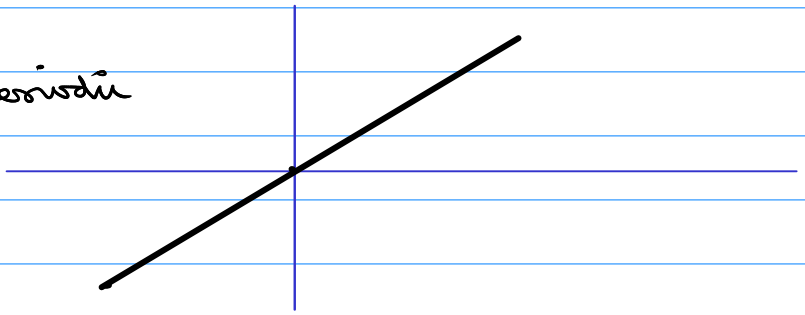
$f(x) = x$

not periodic

$y = mx + c$

$m = 1$

$c = 0$



Mathematical definition of periodic function

$$\boxed{f(x+p) = f(x)}$$

eg $\sin(x + 2\pi) = \sin x$

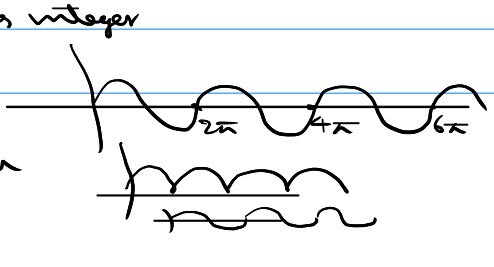
$\cos(x + 2\pi) = \cos x$

$\tan(x + \pi) = \tan x$

More general $f(x + np) = f(x)$ n is integer

$\sin(x + 4\pi)$

$= \sin(x + 2(2\pi)) = \sin x$



Generally if P is the period of a periodic function then

nP where $n = 1, 2, 3 \ldots$ is also the period

∴ period of $\sin x$ is $2\pi, 4\pi, 6\pi, 8\pi, 10\pi \ldots$

the smallest period is called the Fundamental period

* If $f(x)$ and $g(x)$ are two functions with same period P then period of $f(x) + g(x)$ is also P.

Q. find period of $\sin x + \cos x$.

Ans $f(x) = \sin x$ has period $P = 2\pi$

$g(x) = \cos x$ has period $P = 2\pi$

∴ $f(x) + g(x) = \sin x + \cos x$ has period $= P = 2\pi$

Q. What is period of $\sin(x+2)$

adding a constant will not change the period

∴ period of $\sin(x+2)$ is 2π same as of $\sin x$

If $f(x)$ is periodic with period P then $f(x+b)$ has period P

Q. What is period of $\tan(x+10)$

Ans π

Q. What is period of $\sin(2x)$

$\sin x$.
at $x = 0$, $\sin$ was 0
at $x = \pi$, $\sin$ [2π]

If $f(x)$ is periodic with period P then $f(ax)$ has period $\frac{P}{|a|}$

∴ period of $\sin 2x$ is $= \frac{2\pi}{|2|} = \pi$

General statement

If $f(x)$ is a periodic function with period P then $f(ax+b)$ has period $\frac{P}{|a|}$

Q. find period of $\sin(-2x+3)$

Ans period $= \frac{2\pi}{|-2|} = \pi$