Electrostatics-1

Tuesday, May 19, 2020 5:15 PM

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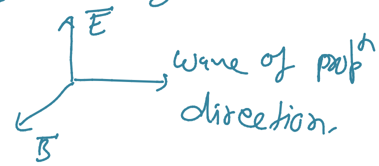
Electromagnetism

- **Electrostatics** (Charge is at Rest) or (current is at Rest)
- **Magnetism** [charge is moving with constant velocity] ⇒ $a = 0$
- **Electromagnetic wave**: accelerated charge, eg X-Ray



Properties of charge:-

1) unit: coulomb (SI unit)
   scalar quantity ✓

2) Types of charge → +ve, → −ve
   { +, + / −, − } Repel, { +, − / −, + } attraction.

3) smallest unit of charge:- is 'e'.

$$\pm e = 1.6 \times 10^{-19} C$$ ✓

By using particle accelerator ⇒ Quarks → smallest unit
electron are break
(not in syllabus)

4) Quantization of charge:- $Q = \pm ne$ → charge
   (n ↑ integer)

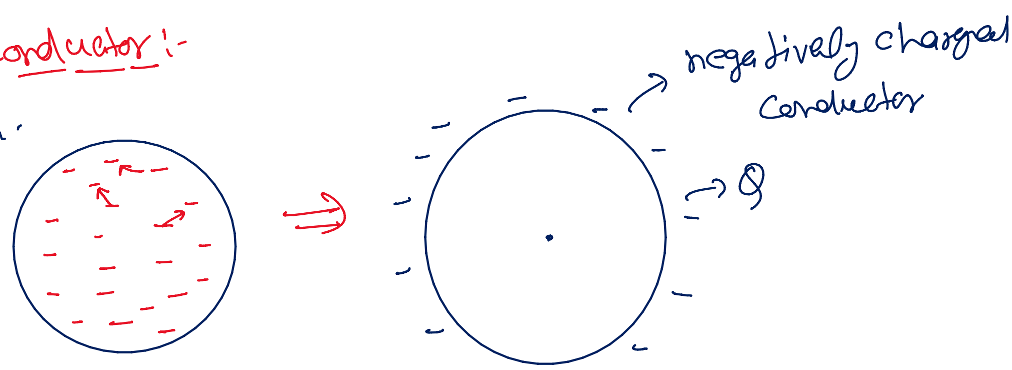
charge is exist in integer form only. 1e, 2e, 3e, 4e, - - - .
but not in fraction ⇒ 1.5e⁻ ✗, 2.5e⁻ ✗, 3.7e⁻ ✗

5) charge is independent of speed.

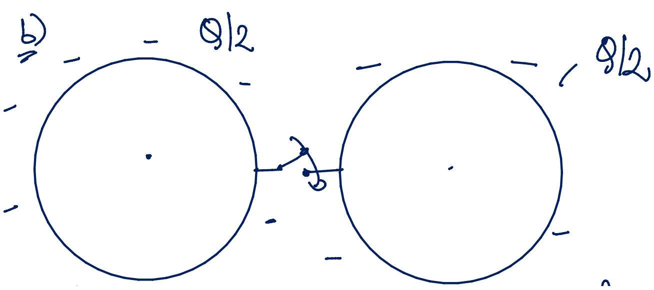
Charging of Conductor:-

i) conduction:-

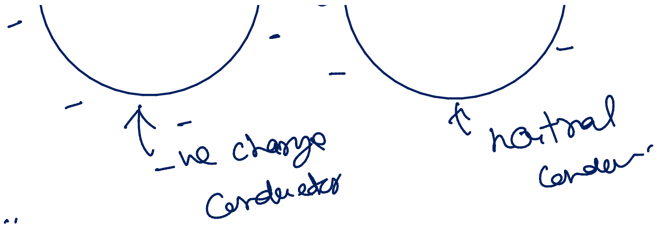
a)



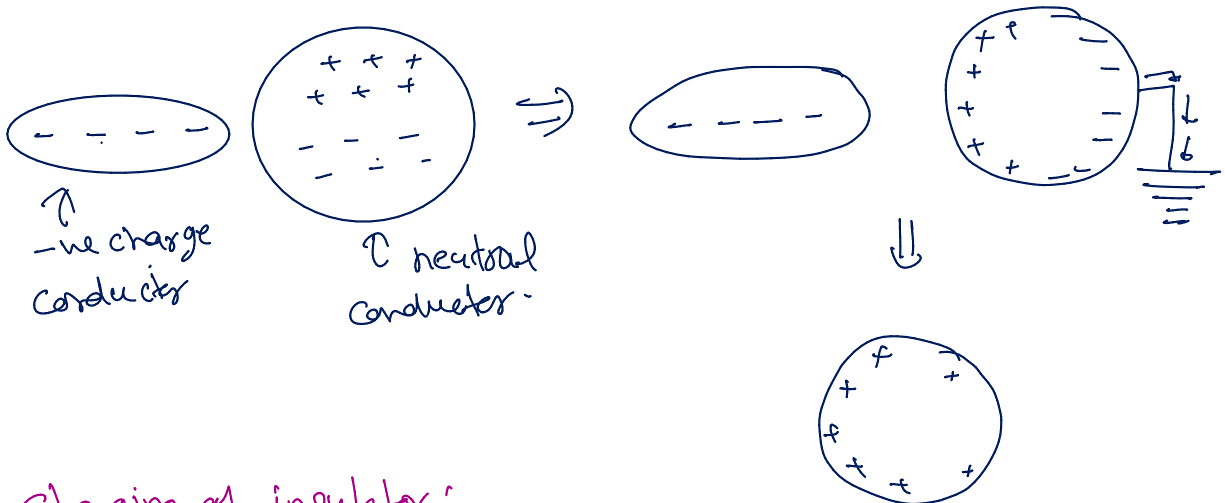
b)



"physical touch or wire. charge is distributed equally."



(ii) Induction :-



Charging of insulator :-

→ charging by friction. ⇒ one insulator & other charged body.

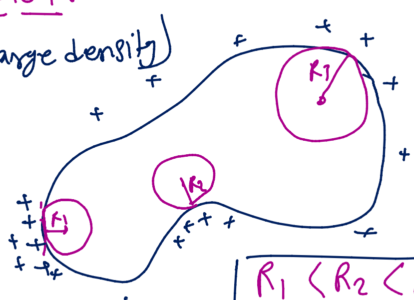
ex Rubbed scale on hair.

Charge Distribution :-

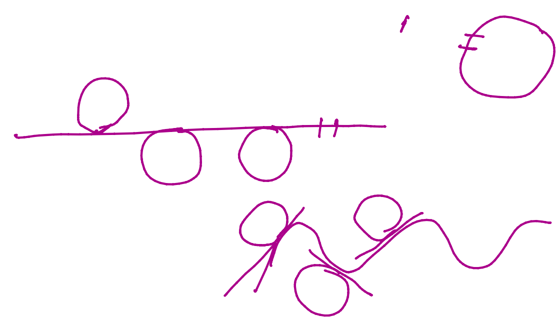
$\sigma = \frac{Q}{A}$ (σ = charge density)

$\sigma \propto \frac{1}{R}$

↑ Radius of curvature

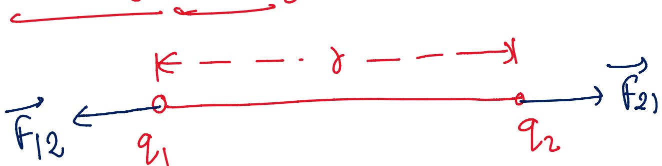


$R_1 < R_2 < R_3$
$\sigma_1 > \sigma_2 > \sigma_3$



N. charge distributed is non uniform and it is maximum at those points where Roc is minimum.

Colomb's Law :-



$|\vec{F}_{12}| = |\vec{F}_{21}|$

Electrostatic force is Acn/Rxn pair, along the line joining these two forces.

$F \propto q_1 q_2 \longrightarrow$ (i)

$F \propto \frac{1}{\;}$ $\longrightarrow$ (ii)

$K = \frac{1}{4\pi\epsilon_0}$ ← permitivity of free space

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

$F \propto q_1 q_2 \longrightarrow$ (I)

$F \propto \frac{1}{r^2} \longrightarrow$ (II)

$F \propto \frac{q_1 q_2}{r^2}$

$$\boxed{F = \frac{K q_1 q_2}{r^2}} \; *$$

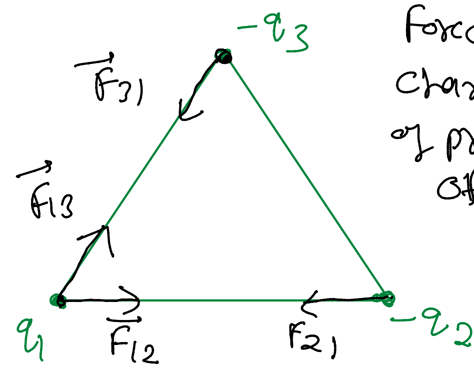
K = Coulomb's constant

$K = 9 \times 10^9 \; Nm^2/C^2$

$K = \frac{1}{4\pi\epsilon_0}$ ← permittivity ...

$\epsilon_0 = 8.85 \times 10^{-12} \; C^2/Nm^2$

N.



Force b/w two charges is independent of presence or absence of other charges.

Q. q —— r —— q $\qquad$ $2q$ —— $2r$ —— $2q$

$F_1 = ?$ $\qquad$ $F_2 = ?$

$F_1 = \frac{kq^2}{r^2}$ $\qquad$ $F_2 = \frac{k\,4q^2}{4r^2} = F_2 = \frac{kq^2}{r^2} = F_1$

$\Rightarrow \boxed{F_1 = F_2}$

Q. find min force possible b/w two charges place at separation of 1m?

q_1 ←— 1m —→ q_2

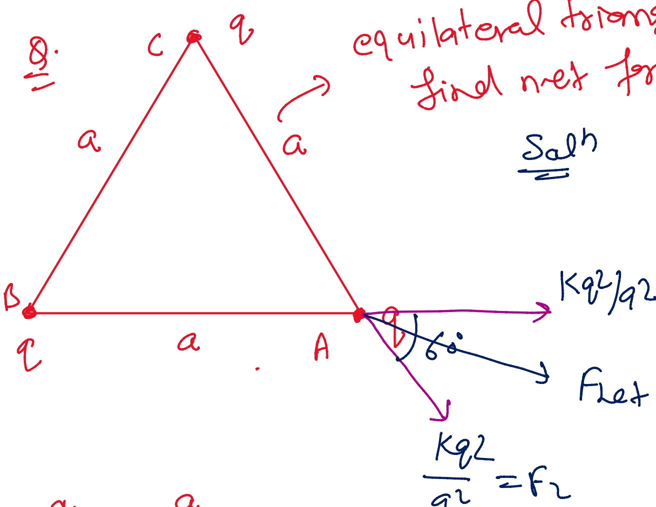
$q_1 = q_2 = e$ $\qquad$ $F = \frac{ke^2}{1^2}$

$\boxed{F = ke^2}$

Q. ←— 2m —→ $2c$ —— $4c$ $\qquad$ $F = ?$

$\boxed{F = 2K}$

Q. equilateral triangle find net force on point 'A'?



Soln

$F_{net} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60}$

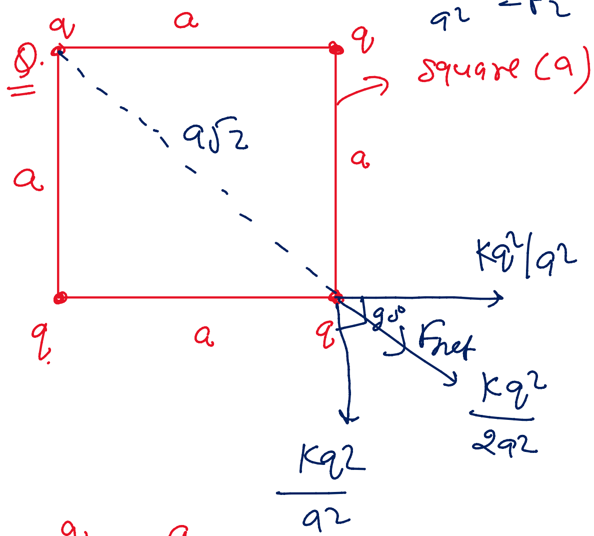
$= \sqrt{\left(\frac{kq^2}{a^2}\right)^2 + \left(\frac{kq^2}{a^2}\right)^2 + 2\left(\frac{kq^2}{a^2}\right)^2 \times \frac{1}{2}}$

$$\boxed{F_n = \sqrt{3}\,\frac{Kq^2}{a^2}} \text{ Ans}$$

Q. q —— a —— q $\qquad$ find force on any vertex?

$\frac{kq^2}{a^2} = F_2$

square (a)

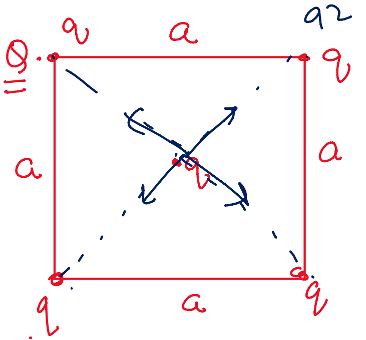
Q. 

find force on any vertex?

$$F_{net} = \frac{kq^2}{2a^2} + \left\{\sqrt{\left(\frac{kq^2}{a^2}\right)^2 + \left(\frac{kq^2}{a}\right)^2 + 0}\right\}$$

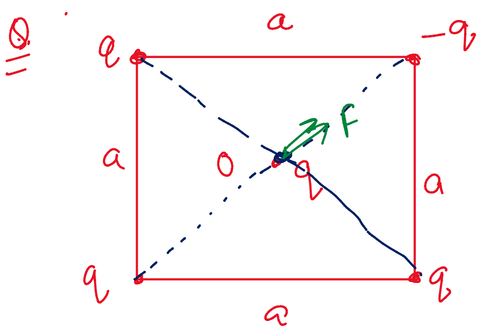
$$= \frac{kq^2}{2a^2} + \sqrt{2}\,\frac{kq^2}{a^2}$$

$$\boxed{F_{net} = \frac{kq^2}{a^2}\left[\frac{1}{2} + \sqrt{2}\right]}$$

Q. 

force on central 'q' ?

$$\boxed{F_{net} = 0}$$

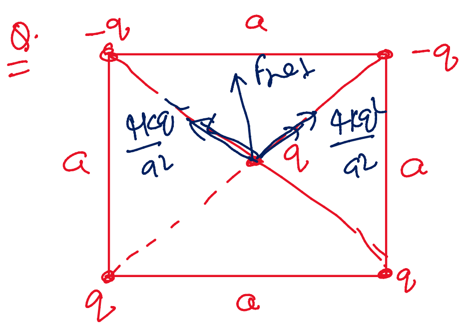
Q. 

find force on point 'O' ?

$$F = \frac{kq^2}{(a/\sqrt{2})^2} + \frac{kq^2}{(a/\sqrt{2})^2}$$

$$\boxed{F = \frac{4kq^2}{a^2}}$$

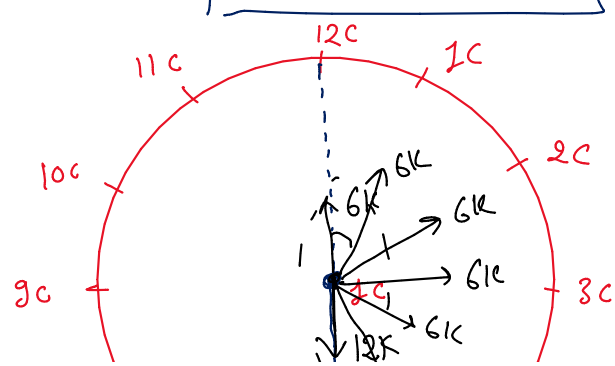
Ans.

Q. 

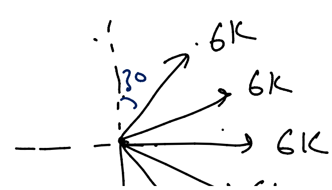
find net force on central 'q' charge ?

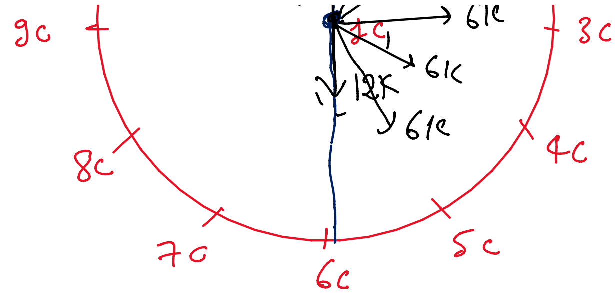
$$F_{net} = \sqrt{\left(\frac{4kq^2}{a^2}\right)^2 + \left(\frac{4kq^2}{a^2}\right)^2}$$

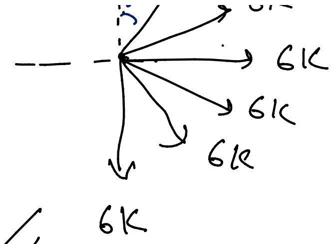
$$\boxed{F_{net} = 4\sqrt{2}\,\frac{kq^2}{a^2}}$$



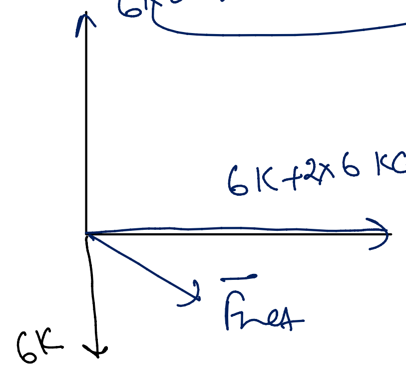
find net force on central 1C, if Radius is 1m.







$6K\sin 30 + 6K\sin 60$ = cancel.



$6K + 2\times 6K\cos 30 + 6K\cos 30 \times 2$

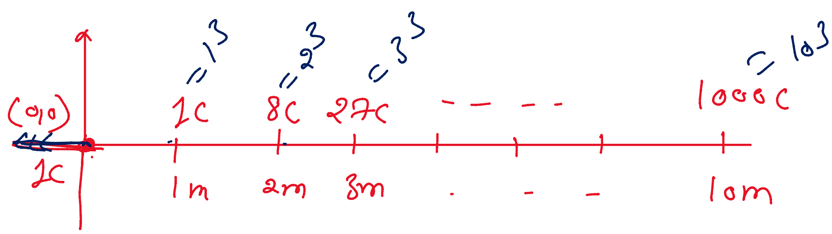
$= 6K + 12K \times \frac{\sqrt{3}}{2} + 6K$

$= 12K + 6\sqrt{3}K$

$6K\sin 30 + 6K\sin 30$

$$\boxed{\vec{F}_{net} = (12K + 6\sqrt{3}K)\hat{i} + KK(-\hat{j})}$$

Q. Find out the net force at origin charge?



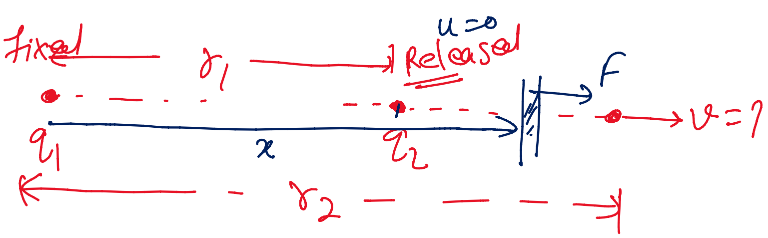
Formula: $1+2+\dots = \frac{n(n+1)}{2}$

$$(F_{net})_{origin} = \frac{K\cdot 1\cdot 1^3}{1^2} + \frac{K\cdot 1\cdot 2^3}{2^2} + \frac{K\cdot 1\cdot 3^3}{3^2} + \dots$$

$$= K[1+2+\dots 10]$$

$$\boxed{F_n = 55K}$$

Q.



Find v of q_2 at r_2 distance.

$F = \frac{Kq_1q_2}{x^2}$

$ma = \frac{Kq_1q_2}{x^2}$

$\frac{v^2}{2} = \frac{Kq_1q_2}{m}\left[-\frac{1}{x}\right]_{r_1}^{r_2}$

$v^2 = \frac{2Kq_1q_2}{\dots}\left[\frac{1}{\dots} - \frac{1}{\dots}\right]$

$$ma = \frac{Kq_1q_2}{x^2}$$

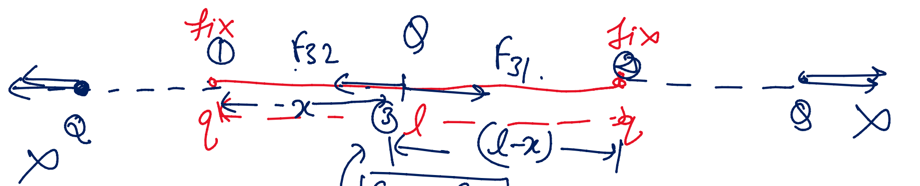
$$mv\frac{dv}{dx} = \frac{Kq_1q_2}{x^2}$$

$$\int_0^v v\,dv = \frac{Kq_1q_2}{m}\int_{r_1}^{r_2}\frac{1}{x^2}dx$$

$$v = \sqrt{\frac{2Kq_1q_2}{m}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]}$$ **Ans.**

:- Equilibrium Problems :-

Q. Find out location of 3rd charge, so that it will be in equilibrium?



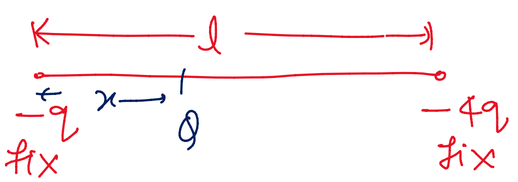
$F_{32} = F_{31}$ So charge is in equilibrium

Location $F_{32} = F_{31}$

$$\frac{KQq}{(l-x)^2} = \frac{KQq}{x^2} \Rightarrow l-x = x$$

$$2x = l \Rightarrow \boxed{x = \frac{l}{2}}$$

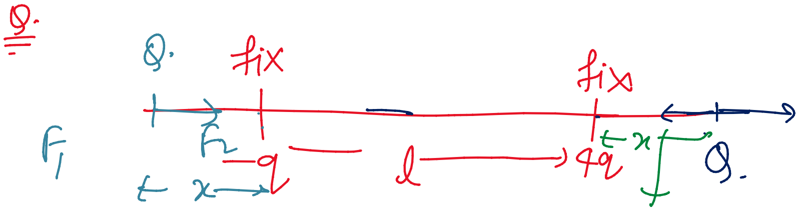
Q.



find location of 3rd charge so that system is in equilibrium.

$$\frac{KqQ}{x^2} = \frac{K\,4qQ}{(l-x)^2} \Rightarrow \boxed{x = \frac{l}{3}}$$

Q.



How many position possible for 3rd charge is in equilibrium.

$$F_1 = \frac{K4qQ}{x^2} = \frac{(4KqQ)}{x^2} \doteq Nr_1 \Rightarrow Dr_1$$

$$F_2 = \frac{KqQ}{(l+x)^2} = \frac{Nr_2}{Dr_2}$$

$F_1 = F_2$

$$\frac{K4qQ}{\ldots} = \frac{KqQ}{\ldots}$$

$F_2 < F_1$ not equilibrium.

$$\frac{K4qQ}{(l+x)^2} = \frac{KqQ}{x^2}$$

$(l+x)^2 = r_2^2$ IV

$\boxed{F_2 < F_1}$ ✗ not equilibrium

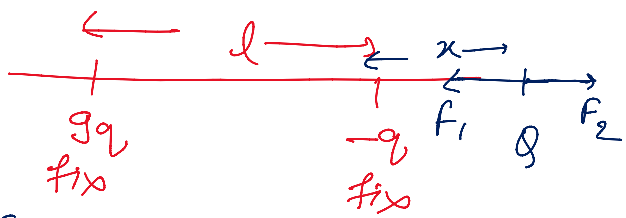
$$\left(\frac{2}{l+x}\right)^2 = \frac{1}{x^2} \Rightarrow \frac{2}{l+x} = \frac{1}{x} \Rightarrow 2x = l+x$$

$\boxed{x = l}$ one soln is possible

N: When nature of charge is same then 3rd charge lies b/w them near to smaller charge

N: When nature of charge is different then 3rd charge lies outside of the smaller charge.

Q. Location of 3rd charge for equilibrium.



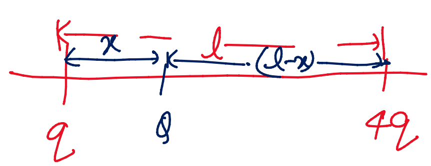
$F_1 = F_2$

$$\frac{K\,Q\,9q}{(l+x)^2} = \frac{KqQ}{x^2}$$

$l + x = 3x$

$\boxed{x = \frac{l}{2}}$

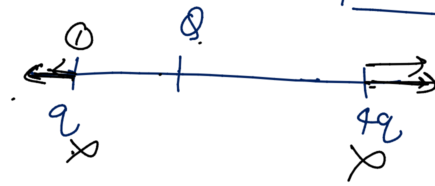
Q. Find out magnitude, nature & position of third charge s.t all the three charge are in equilibrium.



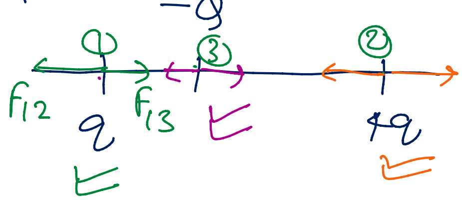
i) Location $\frac{KqQ}{x^2} = \frac{K4qQ}{(l-x)^2}$

$\Rightarrow \boxed{x = l/3}$

ii) Nature of charge (Q) :- i) if Q = +ve ✗



ii) if Q = −ve

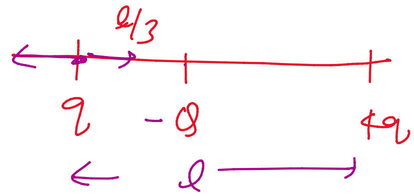


∴ nature of Q is −ve.

magnitude of Q

q is ... 4q

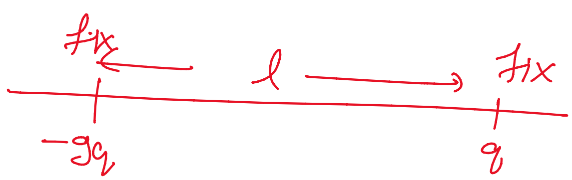
magnitude of Q.



$$\frac{kq\,4q}{l^2} = \frac{kqQ}{\left(\frac{l}{3}\right)^2}$$

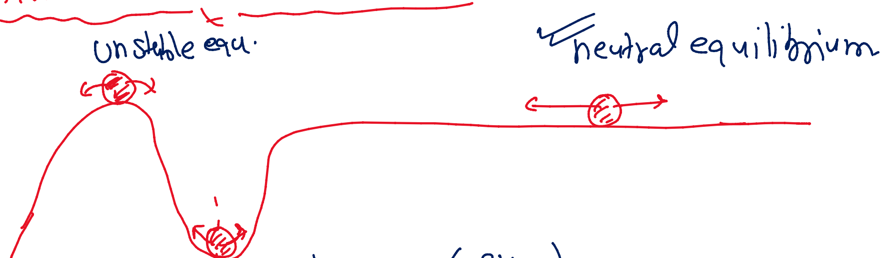
$$\frac{4q}{l^2} = \frac{9Q}{l^2} \Rightarrow \boxed{Q = -\frac{4q}{9}}$$

HW Q.



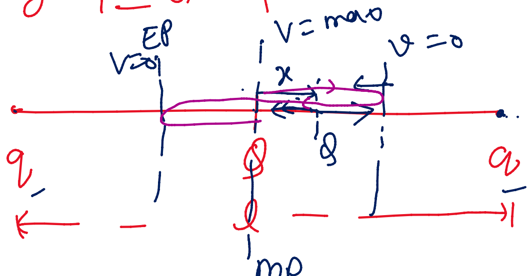
find location, nature, magnitude } of Q.

SHM + electrostatics :-



Q. find type of equilibrium?

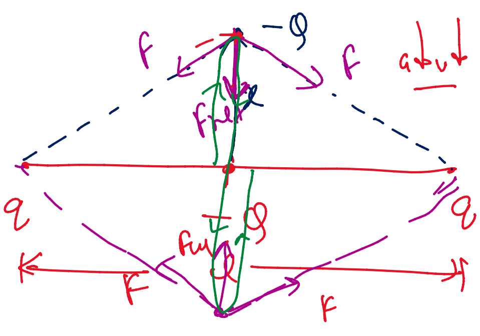
a)



if Q is displaced (slightly) along the line joining.

→ stable equilibrium.

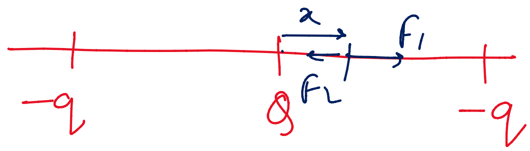
b)



if -Q is displaced ⊥r to line joining. find type of equi.?

→ stable equilibrium

c)



'Q' displaced along the line joining

$F_1 > F_2$

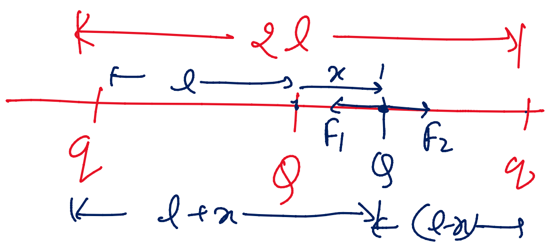
$\Rightarrow$ Unstable equilibrium.

N: Time period in SHM

(i) $F_{net} = -Kx$ SHM condn.

(ii) $T = 2\pi\sqrt{\frac{m}{K}}$

Q.



If Q charge slightly displaced along the line joining, find time period ?

Take $x <<< l$.

$$F_{net} = \frac{kqQ}{(l+x)^2} - \frac{kqQ}{(l-x)^2}$$

$$= kqQ\left[\frac{1}{(l+x)^2} - \frac{1}{(l-x)^2}\right]$$

$$= kqQ\left[\frac{(l-x)^2 - (l+x)^2}{l^4 - x^4}\right] = \frac{l^2 + x^2 - 2lx - l^2 - x^2 - 2lx}{l^4 - x^4}$$ (x^4 neglected)

$$F_{net} = \frac{-4lx}{l^4{}^3} kqQ$$

$$F_{net} = -\frac{4kqQ}{l^3}x \qquad \frac{4kqQ}{l^3} \equiv k \text{ (SHM constant)}$$

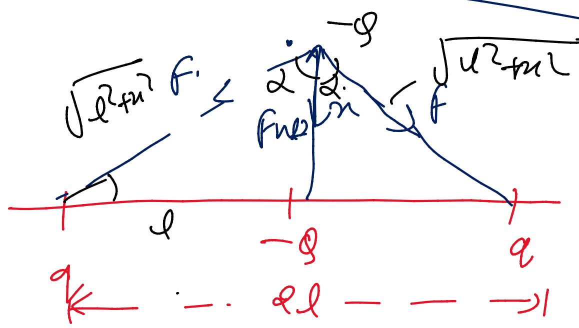
$F_{net} = -kx$ SHM condn

$$\because k = \frac{4kqQ}{l^3}$$

$$\therefore T = 2\pi\sqrt{\frac{ml^3}{4kqQ}}$$

$$T = 2\pi\sqrt{\frac{m l^3}{4kqQ}}$$

Q.



−Q is ⊥r to line joining
Find T = ?

$$\cos\alpha = \frac{x}{\sqrt{l^2+x^2}}$$

$$F_{net} = -2F\cos\alpha$$

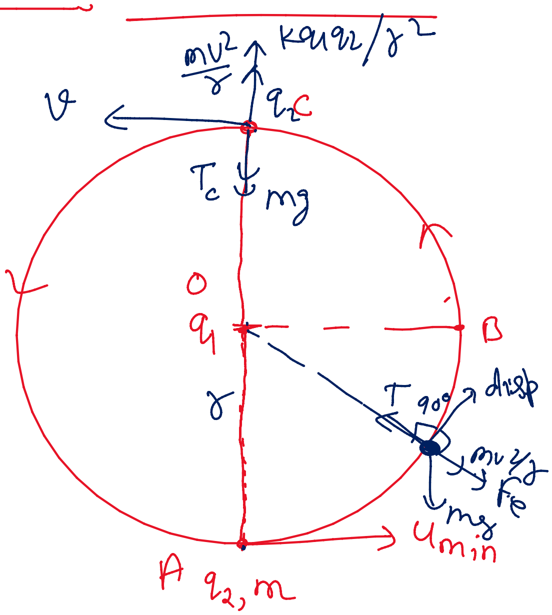
$$= -2\frac{kqQ}{(l^2+x^2)}\cdot\frac{x}{\sqrt{l^2+x^2}} = -\frac{2kqQ}{(l^2+x^2)^{3/2}}x$$

(x^2) neglected. $x \lll l$

$$F_{net} = -\left(\frac{2kqQ}{l^3}\right)x = K \Rightarrow F = -Kx$$

$$T = 2\pi\sqrt{\frac{ml^3}{2kqQ}}$$

Circular motion + electrostatics:-



Find u_{min} so, it complete vertical circular motion.

$T_c \geq 0$ for complete vertical circular motion.

i) NLM eqn point C

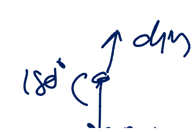
$$T_c + mg = \frac{kq_1q_2}{r^2} + \frac{mv^2}{r}$$

$$T_c = \frac{kq_1q_2}{r^2} + \frac{mv^2}{r} - mg \geq 0 \longrightarrow (1)$$

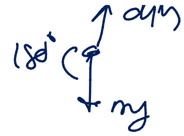
ii) Energy Equation:-

Wall forces = ΔKE

$$W_{mg} + W_T^{\,0} + W_{F_e}^{\,0} + W_N^{\,0} = K_f - K_i$$



all forces –



$$W_{mg} + \cancel{W_{Fe}}^{0} + \cancel{W_{Pseudo}}^{0} + \cancel{W_T}^{0} = K_f - K_i$$

$$-2mgr = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\frac{1}{2}mv^2 = -2mgr + \frac{1}{2}mu^2$$

$$\frac{mv^2}{r} = -4mg + \frac{mu^2}{r} \longrightarrow (11)$$

put (11) in eqn (1),

$$\frac{Kq_1q_2}{r^2} + -4mg + \frac{mu^2}{r} - mg \geq 0$$

$$\frac{mu^2}{r} \geq 5mg - \frac{Kq_1q_2}{r^2}$$

$$u \geq \sqrt{\frac{5mgr}{m} - \frac{Kq_1q_2}{mr}}$$

$$u_{min} \geq \sqrt{5gr - \frac{Kq_1q_2}{mr}}$$

N.

$$u \geq \sqrt{5gr}$$