

# Triangles





- Introduction
- Congruence of triangles
- Criteria for congruence of triangles
- Some properties of a triangle
- Some more criteria for congruence of triangles
- Inequalities in a triangle

Why study triangles?









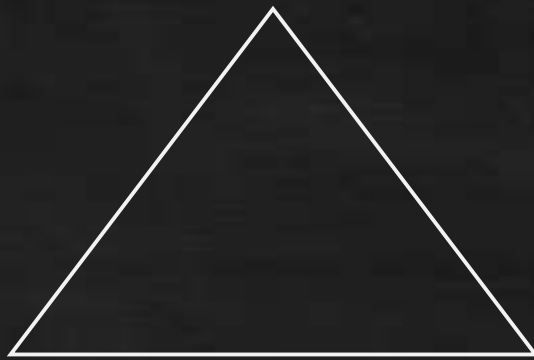






## What is a Triangle?

- A closed figure formed by three intersecting lines.
- A triangle has three sides, three angles and three vertices.





# What is Congruency?

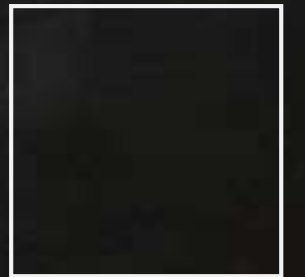
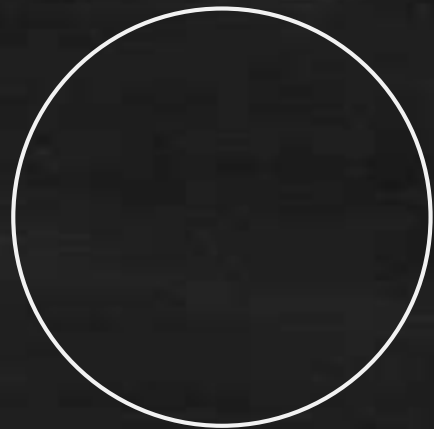
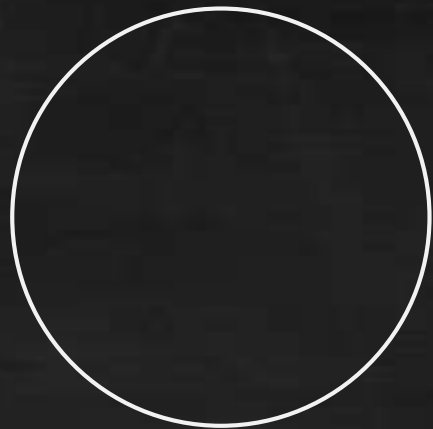
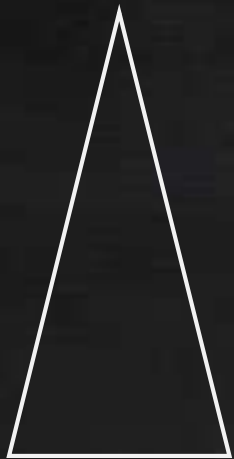
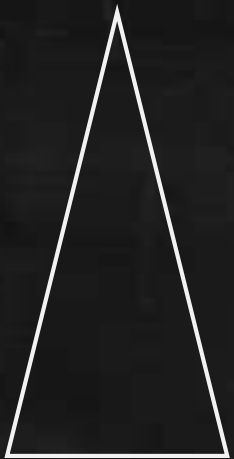


## What is Congruency?

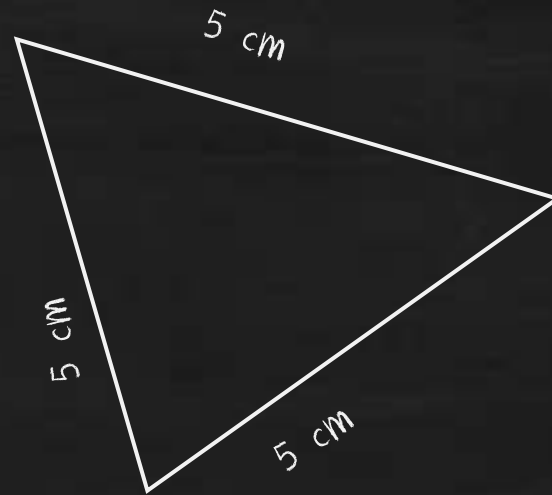
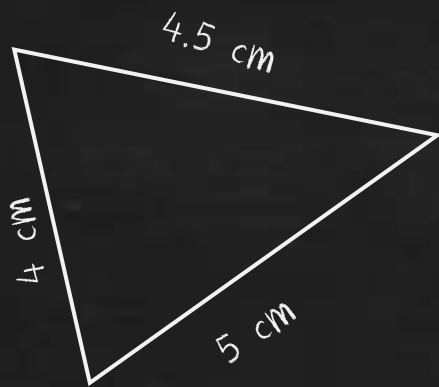
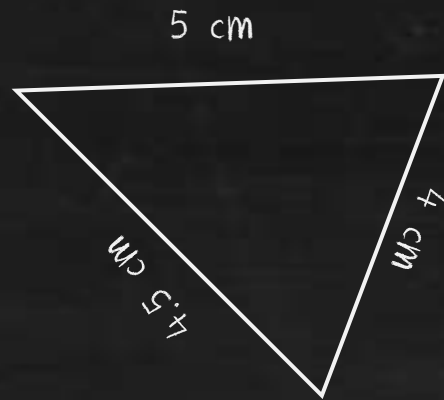
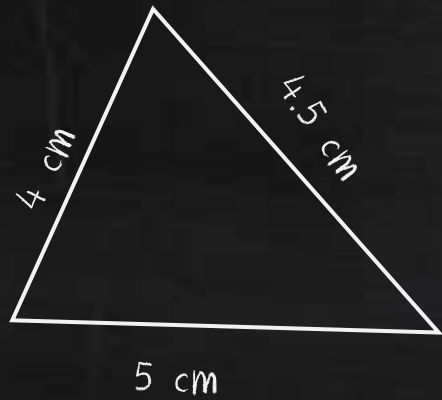
- Congruent means equal in all respects
- Figures whose shapes and sizes are both the same



# Congruent figures in geometry



# Congruency of triangles





## Note:

- Write the correspondence of vertices correctly for writing of congruence of triangles in symbolic form.
- In congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

## Criteria for congruency of triangles

- 3 sides
- 3 angles



## Criteria for congruency: SAS criteria

- Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Eg. In fig,  $OA=OB$  and  $OD=OC$ . Show that (i)  $\triangle AOD \cong \triangle BOC$  & (ii)  $AD \parallel BC$ .

## Criteria for congruency: SAS criteria

Eg.  $AB$  is a line segment and line  $l$  is its perpendicular bisector. If a point  $P$  lies on  $l$ , show that  $P$  is equidistant from  $A$  and  $B$ .



## Criteria for congruency: ASA criteria

**Theorem 7.1:** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

## Criteria for congruency: AAS criteria

Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal. We may call it as the AAS Congruence Rule.

## Criteria for congruency: AAS criteria

Eg. Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD (fig). Show that (i)  $\triangle AOB \cong \triangle DOC$  (ii) O is also the mid-point of BC.



## Exercise 7.1

Q. In quadrilateral ACBD,  $AC = AD$  and  $AB$  bisects  $\angle A$  (fig). Show that  $\triangle ABC \cong \triangle ABD$ . What can you say about  $BC$  and  $BD$ ?

## Exercise 7.1

Q. ABCD is a quadrilateral in which  $AD = BC$  and  $\angle DAB = \angle CBA$  (fig). Prove that

(i)  $\triangle ABD \cong \triangle BAC$

(ii)  $BD = AC$

(iii)  $\angle ABD = \angle BAC$ .

## Exercise 7.1

Q. In fig,  $AC = AE$ ,  $AB = AD$  and  $\angle BAD = \angle EAC$ . Show that  $BC = DE$ .



## Exercise 7.1

Q. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that  $\angle BAD = \angle ABE$  and  $\angle EPA = \angle DPB$  (fig). Show that

(i)  $\triangle DAP \cong \triangle EBP$

(ii)  $AD = BE$

## Exercise 7.1

Q. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that  $DM = CM$ . Point D is joined to point B (fig). Show that:

(i)  $\triangle AMC \cong \triangle BMD$

(ii)  $\angle DBC$  is a right angle.

(iii)  $\triangle DBC \cong \triangle ACB$

(iv)  $CM = \frac{1}{2} AB$

## Some Properties of a Triangle

**Theorem 7.2** : Angles opposite to equal sides of an isosceles triangle are equal.



## Some Properties of a Triangle

**Theorem 7.3** : The sides opposite to equal angles of a triangle are equal.

## Some Properties of a Triangle

Eg. In  $\triangle ABC$ , the bisector  $AD$  of  $\angle A$  is perpendicular to side  $BC$  (fig). Show that  $AB = AC$  and  $\triangle ABC$  is isosceles.

## Some Properties of a Triangle

Eg. E and F are respectively the mid-points of equal sides AB and AC of  $\Delta ABC$  (fig). Show that  $BF = CE$ .



## Some Properties of a Triangle:

Eg. In an isosceles triangle  $ABC$  with  $AB = AC$ ,  $D$  and  $E$  are points on  $BC$  such that  $BE = CD$  (fig). Show that  $AD = AE$ .

## Exercise 7.2

Q. In an isosceles triangle  $ABC$ , with  $AB = AC$ , the bisectors of  $\angle B$  and  $\angle C$  intersect each other at  $O$ . Join  $A$  to  $O$ . Show that :

(i)  $OB = OC$

(ii)  $AO$  bisects  $\angle A$

## Some More Criteria for Congr. of Tns. : SSS congruence

**Theorem 7.4** : If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

## Some More Criteria for Congr. of Tns. : RHS congruence

**Theorem 7.5** : If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.