ELECTRIC CURRENT

Electric current:

The net amount of charge flowing through a cross section in unit time is defined as current.

Mathematical Equation: If rate of flow of charge is independent of time, then the electric current is said to be steady then $I = Q/t$

If rate of flow of charge varies with time, the current at any time i.e. instantaneous current is given by $I = dQ/dt$.

If n be the number of conduction electrons crossing cross –section in time t then $I = ne/t$.

SI unit of current is ampere.

One ampere is defined as If one coulomb of charge flows across any of its cross-section in one second.

Note: Electric current has direction as well as magnitude but it is not a vector quantity. This is because currents do not add like vectors.

Ohm's Law:

The current which flows in conductor is proportional to the potential difference which causes its flow at constant temperature and pressure.

Thus $V = IR$.

where the constant R is the resistance of conductor.

Unit of resistance is Ohm denoted as Ω .

Resistance:

Formula for resistance is $R = \rho \dfrac{l}{A}$

Here l is length of conductor ., A is area of cross-section of conductor and ρ is resistivity of conductor .

Resistivity of conductor is independent of size and shape of conductor, it depends on material of conductor and temperature and pressure.
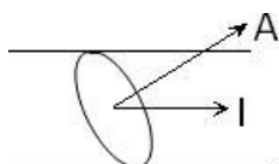
Unit of resistivity is Ohm-m.

Inverse of resistivity is conductivity denoted by σ unit is $(Ohm\text{-}m)^{-1}$

Current density:

From equation $V = IR$

$$V = I\rho\frac{l}{A} = \rho J l$$

Here J is current density defined as current per unit area (taken normal to the current) $J = I/A$. SI unit of current density is A/m^2.

If there is angle between direction of current and area vector of cross section is θ Then current density $J = I / A\cos\theta$

Relation between Current density and Electric field .
For formula V = IR

$$V = I\rho\frac{l}{A} = \rho Jl$$
$$\frac{V}{l} = \rho J$$

But $E = \frac{V}{l}$

$$E = \rho J \quad \text{or} \quad J = \sigma E$$

Origin of resistivity:

In metallic conductors, the electrons in the outer shells are less bounded with the nucleus. Due to thermal energy at room temperature, such valence electrons are liberated from atom leaving behind positively charged ions. These ions are arranged in a regular geometric arrangement on lattice points. These liberated electrons collide with the ions. Or constantly gets scattered from its path causing resistivity of metallic conductors.

Drift of electrons:

In absence of electric field electrons move in randomly after colliding with ions and direction and velocities of electrons after collisions is such that sum of velocities is zero and net charge passing through any cross section is zero causing no electric current.
In presence of electric field (E) electrons experiences electric force of magnitude Ee in the direction opposite to the direction of electric field thus the acceleration of electron is opposite to direction of electric field. Now F = ma thus acceleration of electron a = Ee/m this acceleration is momentary and becomes zero after collision, since electrons are continuously colliding with ions. And electrons get accelerated again and process goes of repeating.

As a result electrons are dragged in the opposite to the electric field. Now average time period between two successive collisions is known as relaxation time τ. And corresponding average velocity of electrons is known as drift velocity v_d.
Now $v_d = a\tau$

$$v_d = \frac{Ee}{m}\tau$$

Relation between drift velocity and current density

Let us consider a cylindrical conductor of cross section A . Let E be the electric filed exists in conductor.
 If v_d is drift velocity of electrons then volume of the electrons passing through a cross-section in one second = v_dA .
 If n is the number of electrons per unit volume then nv_dA is the number of electron in passing through a cross-section in one second.
 Net charge passing through a cros-section in one second = $neAv_d$ = I .
Now I = $neAv_d$.
Thus J = nev_d

Relation between resistivity and relaxation time.

We know that $J = \sigma E$, here σ is conductivity

And $J = nev_d$

$nev_d = \sigma E$

Substituting value of $v_d = \dfrac{Ee}{m}\tau$ in above equation we get

$$\sigma E = ne\frac{Ee}{m}\tau$$

$$\sigma = \frac{ne^2}{m}\tau$$

Since $\sigma = 1/\rho$

$$\rho = \frac{m}{ne^2\tau}$$

Here we have assumed that τ and n are constant.

On increasing temperature of conductor n does not change appreciable. The oscillations of ions increases with temperature and becomes more erratic. As a result, the relaxation time (τ) decreases and resistivity of conductor increases with increase in temperature

Mobility

In case of conductors free electrons are mobile charge carriers. In case of electrolyte positive and negative ions are mobile charge carriers. In case of semiconductors holes and electrons are mobile charge carriers.

Mobility is defined as drift velocity of charge per unit electric field

$$\mu = \frac{|v_d|}{E}$$

S.I. unit of mobility is m^2/Vs and is 10^4 of mobility in practical units (cm^2 /Vs)

By substituting $v_d = \dfrac{Ee}{m}\tau$ in above equation we get

$$\mu = \frac{e\tau}{m}$$

Limitations of Ohm's Law

(1) V-I relations are not linear Example: diode transistor
(2) The relation between V and I depends on the sign of V. If we change the polarity of supply voltage magnitude of current changes
(3) The relation between V and I is not unique. There may be more values of potential for same current I. example tunnel diode, material Ga,As

Classification of materials based on resistivity :

Materials are classified as conductors, semiconductors and insulators depending on their resistivity.

Conductors have resistivity of order $10^{-8}\Omega m$ to $10^{-6}\Omega m$

Insulator have resistivity 10^{18} times greater than metals or more

Resistivity of semiconductors decreases with increase in temperature because covalent bond between adjacent atoms breaks which creates free electrons and holes causing decrease in resistivity. Or its conductivity increases.

Commercially produced resistors for domestic use or in laboratory are of two major types: wire bound resistors and carbon resistors

Wire bound resistors are made by winding the wires of an alloy viz. manganin, constantan, nichrome or similar ones. These alloys are relatively insensitive to temperature. These resistances are typically in the range of a fraction of an ohm or to a few hundred ohms

Resistors, in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuit. Carbon resistors are small in size and hence their values are given using colour code

Temperature dependence of resistivity:

The resistivity of materials is found to be dependent on temperature. Over a limited range of temperatures, that is not too large, the resistivity of metallic conductor is approximately given by

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

Where ρ_T is the resistivity at temperature T and ρ_0 is resistivity at temperature T_0, α is called the temperature co-efficient of resistivity, dimension of α is [temperature]$^{-1}$ units are $(^\circ C)^{-1}$ or $(K)^{-1}$

Note that temperature coefficient of Carbon and, semiconductors germanium and silicon are negative, indicating with increase in temperature resistivity decreases.

Equation of resistivity shows a linear relation between temperature and resistivity however graph of resistivity – temperature is not linear for copper.

Since alloys like manganin, constantan, nichrome are relatively insensitive to temperature there graph is straight line intercepting on Y axis

Resistivity of semiconductors decreases with temperature there graph is non linear and have negative slope indicating resistivity decreases with increase in temperature

Temperature T
Copper

Temperature T
nichrome

Temperature T
semiconductor
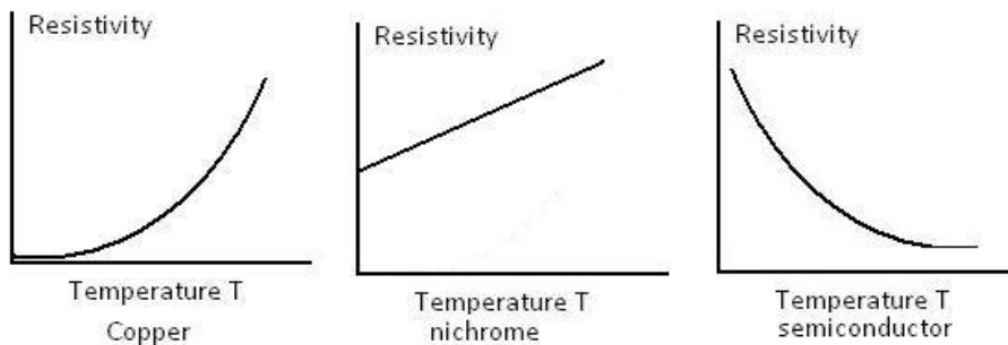
Cells, emf and internal resistance
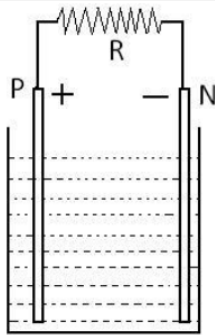
Cell is a simple device which maintains a steady current in an electrical circuit. Cell consists of electrolyte and electrodes. Electrodes are metallic plate dipped in electrolyte. Due to exchange of electrodes between electrolyte and electrode, one electrode develop positive potential deference (V_+) between electrolyte and electrode and such electrode acts as positive terminal (P). While other electrode develop negative potential difference (V_-) and acts as negative terminal (N). When there is no current, the electrolyte has the same potential throughout,

so that the potential difference between P and N is $V_+ - (-V_-) = V_+ + V_-$.
This difference is called electromotive force (emf) of the cell and is denoted by ε. Thus
$\varepsilon = V_+ + V_- > 0$
emf is work done by non-electrical force in moving a positive charge from negative terminal to positive terminal of battery.

Working of cell:

Consider a wire of resistance R connected across the two terminals of the battery as shown in figure. The electric field is established in wire. As a result positive charge will move from higher potential (P) to lower potential (N) through external resistance R. The energy of the positive charge is consumed to overcome the resistance of wire. As it reaches the negative terminal N, its energy becomes zero, it is as per law of conservation of energy.

Now due to non-electrical force positive charge on the negative electrode is moved towards the positive electrode inside the cell. Thus work is done by the non-electrical force and potential energy of positive charge increases when it reaches positive electrode. Again it flows through the external resistance R and process goes on repeating

Now if external resistance is not connected then positive charge gets accumulated on the positive electrode and produces an electric field in the direction from positive electrode to negative electrode so that direction of electric force is opposite to non-electric force. When force due to electric field becomes equal to non-electric force flow of positive charge from negative terminal to positive terminal stops and potential across terminal is ε

Now during the discharge of cell positive charge has to overcome the resistance of electrolyte, such resistance is called internal resistance denoted by r.

Terminal voltage

When positive charge flows in electrolyte they have to overcome internal resistance t. if I is the current then energy lost in electrolyte is Ir. As a result voltage across P and N is less than ε in the open circuit condition. The net energy per unit charge will be ($\varepsilon - Ir$) .

Thus , during the flow of current potential across between two terminal P and N is
$V = \varepsilon - Ir$. This potential difference is called terminal voltage.
If $\varepsilon \gg Ir$, internal resistance is neglected
We also observe that since V is the potential difference across resistance R, from Ohm's law $V = IR$

Thus $IR = \varepsilon - Ir$

$$I = \frac{\varepsilon}{R + r}$$

Maximum current can be drown from cell is R = 0. However, in most of the cells maximum allowed current is much lower to prevent permanent damage to cell.

Internal resistance of electrolyte cell is very small. Thus electrolyte cell gives large value of current. Internal resistance of dry cell is higher thus it gives low current.

Electrical Energy, Power

Consider a conductor with endpoints A and B, in which a current I is flowing from A and B. The electric potential at A and B are denoted V(A) and V(B) respectively.

Since current is flowing from A to B and potential difference across AB is

V = V(A) − V(B) >0

In a time interval Δt, an amount of charge $\Delta Q = I \Delta t$ travels from A to B.

The potential energy of charge at A = $\Delta Q V(A)$ and

The potential energy of charge at B = $\Delta Q V(B)$

Thus change in potential energy

ΔU = Final potential energy – Initial potential energy

$\Delta U = \Delta QV(B) – \Delta QV(A) = \Delta Q[V(B) - V(A)]$

$\Delta U = - \Delta QV = -IV \ \Delta t < 0$

If charges moved without collisions through the conductor, the kinetic energy would also change so that the total energy is unchanged. Conservation of total energy would then imply that.

$\Delta K = - \Delta U$

That is $\Delta K = IV \ \Delta t > 0$

But when charges flow through the conductor they move with the steady drift velocity, because of collision with ions and atoms during transit. During collisions, energy gained by the charges thus is shared with the atoms. The atoms vibrate more vigorously, as a result conductor heats up. The amount of energy dissipated as heat in the conductor during the time interval is

$$\Delta W = IV \ \Delta t$$

The energy dissipated per unit time is the power P = $\Delta W / \Delta t$ and we have

$$P = IV$$

Using Ohm's law V = IR

$$P = I^2 R = V^2/R$$

Power loss is also called as "Ohmic loss"

Power loss in transmission lines

Consider a device of resistance R, to which power is to be delivered via transmission station. Power of device is $P = VI$ or $I = P/V$
Let resistance of transmission cable is R_C. The power dissipated in the connecting wires, which is wasted is $P_C = I^2 R_C$

$$P_C = \left(\frac{P}{V}\right)^2 R_C$$

Transmission cables from power stations are hundreds of miles and there resistance R_C is considerable. To reduce P_C voltage V is increased to very large value. Using such high voltage is dangerous, thus at user end voltage is reduced this can be achieved by transformers.

Series Combination of resistors

Two resistors are said to be connected in series, if only one end of their ends points are joined . In series connection current flowing through each resistor remains same, while sum of potential drop across resistors is equal to potential drop across combination
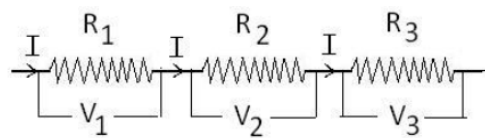
Consider three resistor $R_1 R_2$ and R_3 are connected in series as shown in figure.
Potential difference across R_1 is $V_1 = IR_1$
Potential difference across R_2 is $V_2 = IR_2$

Potential difference across R_3 is $V_3 = IR_3$
The potential difference across combination is $V = V_1 + V_2 + V_3$
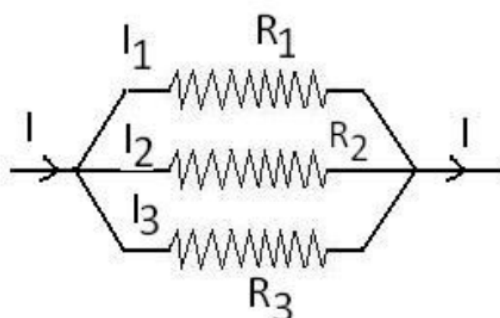$V = I (R_1 + R_2 + R_3)$
If R_{eq} is equivalent resistance and V is potential difference across combination then
$V = I (R_{eq})$
Thus $R_{eq} = R_1 + R_2 + R_3$
IF n resistance are connected in series the $R_{eq} = R_1 + R_2 + R_3 + ...+ R_n$

Parallel combination of resistors

Two or more resistors are said to be connected in parallel if one end of all the resistors is joined together and similarly the other ends are joined together as shown in figure.
In parallel combination current gets divided depending up on value of resistor, sum of current passing through the resistor is equal to current

passing through combination, but potential difference across each resistor is equal to potential difference across combination
Here I_1, I_2 and I_3 are the currents passing through the resistors as shown in figure

If I is the current passing through the combination then $I = I_1 + I_2 + I_3$
By applying Ohms law we get voltage across R_1 as $V = I_1R_1$ or $I_1 = V/R_1$
Similarly $I_2 = V/R_2$ and $I_3 = V/R_3$
By substituting values of current in above equation we get

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

If combination is replaced by equivalent resistance R_{eq}, we would have from Ohm's law
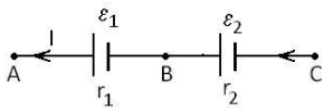$$I = V/R_{eq}$$

From above equations we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

IF n resistors are connected in parallel the equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots . + \frac{1}{R_n}$$

Cells in series

Consider first two cells in series, where one terminal of the two cells is joined together leaving the other terminal in either cell free. Let ε_1 and ε_2 be the emf of the two cell having internal resistance r_1 and r_2 respectively

Let V (A), V (B), V (C) be the potentials at points A, B and C shown in figure.
Then V (A) – V (B) is the potential difference between the positive and negative terminals of the first cell.
We know that terminal voltage of first cell $V_{AB} = V(A) - V(B) = \varepsilon_1 - I r_1$
Similarly for second cell $V_{BC} = V(B) - V(C) = \varepsilon_2 - I r_2$
Hence, the potential difference between the terminals A and C of the combination is
$V_{AC} = V(A) - V(C) = V(A) - V(B) + [V(B) - V(C)] = \varepsilon_1 - I r_1 + (\varepsilon_2 - I r_2)$
$V_{AC} = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$
If we wish to replace the combination by a single cell between A and C of emf ε_{eq} and internal resistance r_{eq}, we would have
$V_{AC} = \varepsilon_{eq} - I r_{eq}$
Comparing the last two equations, we get $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ and $r_{eq} = r_1 + r_2$

If polarity of second cell is reverse then $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$ $(\varepsilon_1 > \varepsilon_2)$

Important notes-

(i)The equivalent emf of a series combination of n cells is just the sum of their individual emf's, and
(ii) The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances
If n cells of emf ε having internal resistance r each connected in series. Total current I if resistance R connected across combination

$$I = \frac{n\varepsilon}{R + nr}$$

Cells in parallel
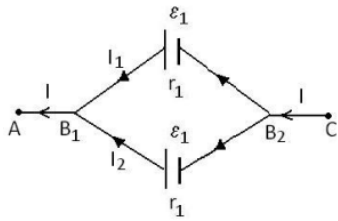
Consider a parallel combination of the cells as shown in figure I_1 and I_2 are the currents leaving the positive electrodes of the cells. At the point B_1, I_1 and I_2 flow out whereas the current I flows in B_2. Since as much charge flows in as out, we have $I = I_1 + I_2$
Let V (B_1) and V (B_2) be the potentials at B_1 and B_2, respectively. Then, considering the first cell, the potential difference across its terminals is V (B1) – V (B2). Hence
V= V(B_1)- V(B_2)= ε_1 - I r_1

$$I_1 = \frac{\varepsilon_1 - V}{r_1}$$

Points B_1 and B_2 are connected exactly similarly to the first cell. Hence considering the second cell, we also have
V= V(B1)- V(B2)= ε_2 - I r_2

$$I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Combining the last three equations we get

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

If we want to replace the combination by a single cell, between B_1 and B_2, of emf ε_{eq} and internal resistance r_{eq}, we would have
V = ε_{eq} - I r_{eq} . Thus

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

In simpler way

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

General formula for n cells

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots . + \frac{1}{r_n}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots . + \frac{\varepsilon_n}{r_n}$$

If n cells of emf ε having internal resistance r each connected in parallel. Total current I if resistance R connected across combination

$$I = \frac{\varepsilon}{R + \dfrac{r}{m}} = \frac{m\varepsilon}{mR + r}$$

Mix grouping of cell

Let n identical cells be arranged in series and let m such rows be connected in parallel.
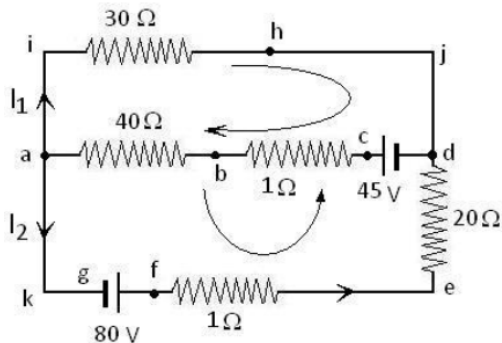Total numbers of cells are nm
Emf of system = $n\varepsilon$
Internal resistance of the system = nr/m
The current through the external resistance R

$$I = \frac{n\varepsilon}{R + \dfrac{nr}{m}} = \frac{mn\varepsilon}{mR + nr}$$

Kirchhoff's rules

Electric circuits generally consist of a number of resistors and cells interconnected sometimes in a complicated way.

Junction or Branch point: The point in network at which more than two conductors meet is called a junction or a branch point . Point a and d shown in figure are junction point .
Loop: A closed circuit formed by conductors is known as loop. As shown in figure ihjdcbai forms a closed loop

Kirchhoff's first rule or junction rule:
At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction.
The proof of this rule follows from the fact that when currents are steady, there is no accumulation of charges at any junction or at any point in a line. Thus, the total current flowing in, (which is the rate at which charge flows into the junction), must equal the total current flowing out.

Proof : Let Q_1, Q_2 ...Q_5 be electrical charges flowing through the cross-sectional area of the respective conductors in time interval t which constitute current I_1, I_2, ... I_5
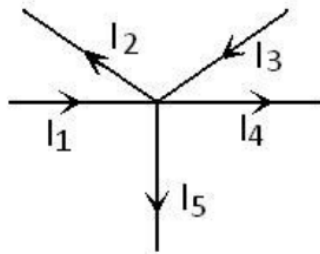
Hence $Q_1 = I_1t$, $Q_2 = I_2t$, ... $Q_5 = I_5t$

It is evident from figure that the total electric charge entering the junction is $Q_1 + Q_3$, while $Q_2 + Q_4 + Q_5$ amount of charge is leaving the junction in the same interval of time

Thus $Q_1 + Q_3 = Q_2 + Q_4 + Q_5$

$\therefore$ $I_1t + I_3t = I_2t + I_4t + I_5t$

$I_1 + I_3 - I_2 - I_4 - I_5 = 0$

$\therefore \sum I = 0$.

<u>Kirchhoff's second rule or Loop rule</u>: *The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero*

Using law of conservation of energy and the concept of electric potential any closed circuit can be analyzed. In electric circuit, the electric potential at any point is a steady circuit does not change with time.

Following are the sign convention has to be followed:

(i)   If our selected path is in the direction of current then potential drop across resistor should be taken as negative else it should be taken as positive

(ii)   The emf of a battery should be considered negative while moving from negative terminal of a battery to the positive terminal. The emf of battery is taken as positive while moving from positive terminal while moving from positive to negative terminal of battery

While analyzing the circuit we may get negative value of current indicates direction of current which arbitrarily chosen is opposite to the actual direction of current

Wheatstone bridge
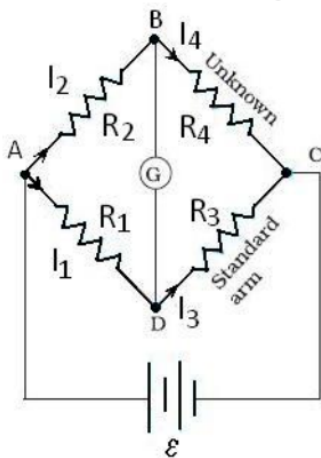
The circuit shown in figure which is called the Wheatstone bridge. The bridge has four resistors R_1, R_2, R_3 and R_4 .

Across one pair of diagonally opposite points (A and C in the figure) a source is connected. This (i.e., AC) is called the battery arm.

Between the other two vertices, B and D, a galvanometer G (which is a device to detect currents) is connected. This line, shown as BD in the figure, is called the galvanometer arm.

For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current I_g through G. Of special interest, is the case of a balanced bridge where the resistors

are such that Ig = 0.
We can easily get the balance condition, such that there is no current through G. In this case, the Kirchhoff's junction rule applied to junctions D and B (see the figure) immediately gives us the relations $I_1 = I_3$ and $I_2 = I_4$. Next, we apply Kirchhoff's loop rule to closed loops ADBA and CBDC.

The first loop gives ADBA
$-I_1 R_1 + 0 + I_2 R_2 = 0$ (Ig = 0)

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

and the second loop CBDC. And using $I_1 = I_3$ and $I_2 = I_4$

$I_2 R_4 + 0 - I_1 R_3 = 0$

$$\frac{I_1}{I_2} = \frac{R_4}{R_3}$$

Hence at balanced condition from above equation

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

 This last equation relating the four resistors is called the balance condition for the galvanometer to give zero or null deflection.
The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance. Let us suppose we have an unknown resistance, which we insert in the fourth arm; R_4 is thus not known. Keeping known resistances R_1

 and R_2 in the first and second arm of the bridge, we go on varying R_3 till the galvanometer shows a null deflection. The bridge then is balanced, and from the balance condition the value of the unknown resistance R_4 is given by,

$$R_4 = R_3 \frac{R_2}{R_1}$$

Meter bridge

The meter bridge is shown in Figure
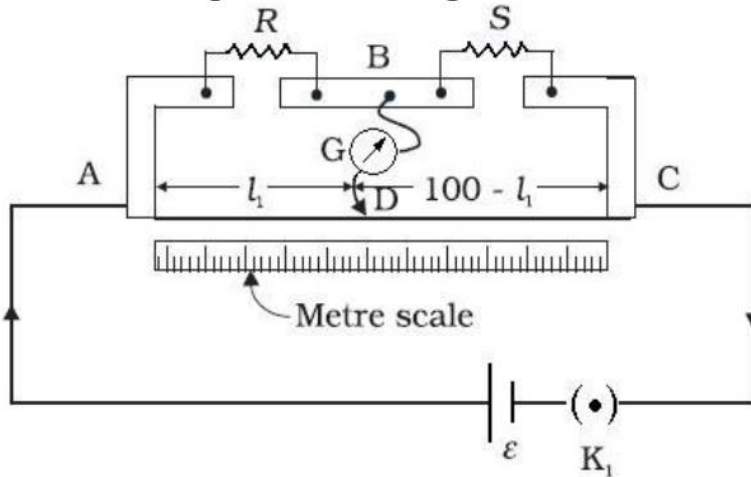
It consists of a wire of length 1m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips bent at right angles, as shown.
The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key.
 One end of a galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galvanometer is connected to a 'jockey'. The jockey is essentially a metallic rod whose one end has a knife-edge which can slide over the wire to make electrical connection.
R is an unknown resistance whose value we want to determine. It is connected across one of the gaps. Across the other gap, we connect a standard known resistance S. The jockey is connected to some point D on the wire, If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current.

Let the distance of the jockey from the end A at the balance point be $l = l_1$.
The four resistances of the bridge at the balance point then are R, S, $\rho\, l_1$ and $\rho\,(100- l_1)$.
The balance condition,

$$\frac{R}{S} = \frac{\rho l_1}{\rho(100 - l_1)} = \frac{l_1}{100 - l_1}$$

Thus, once we have found out l_1, the unknown resistance R is known in terms of the standard known resistance S by

$$R = S\frac{l_1}{100 - l_1}$$

By choosing various values of S, we would get various values of l_1, and calculate R each time. An error in measurement of l_1 would naturally result in an error in R. It can be shown

that the percentage error in R can be minimized by adjusting the balance point near the middle of the bridge, i.e., when l_1 is close to 50 cm.

Potentiometer

<u>Construction</u>

It is basically a long piece of uniform wire, sometimes a few meters in length across which a standard cell is connected. In actual design, the wire is sometimes cut in several pieces placed side by side and connected at the ends by thick metal strip.

In the figure, the wires run from A to C. The small vertical portions are the thick metal strips connecting the various sections of the wire. A current I flows through the wire which can be varied by a variable resistance (rheostat, R) in the circuit.

Since the wire is uniform, the potential difference between A and any point at a distance l from A is ε (l)=ϕ l (where ϕ is the potential drop per unit length.

Figure shows an application of the potentiometer to compare the emf of two cells of emf ε_1 and ε_2 . The points marked 1, 2, 3 form a two way key.

Consider first a position of the key where 1 and 3 are connected so that the galvanometer is connected to ε_1. The jockey is moved along the wire till at a point N_1, at a distance l_1 from A, there is no deflection in the galvanometer. We can apply Kirchhoff's loop rule to the closed loop AN_1G31A and get,

Use to compare emf
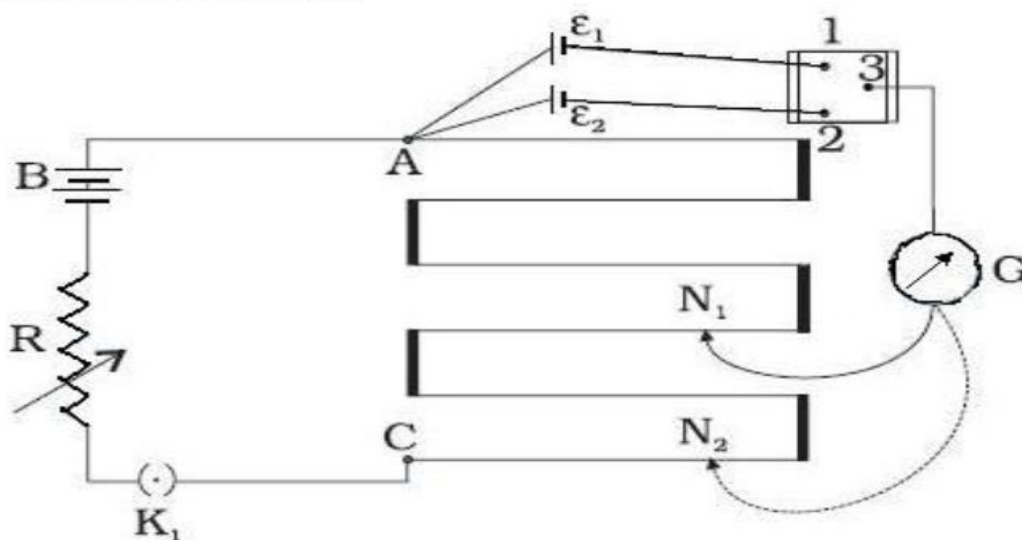
ϕ l_1 + 0 − ε_1 = 0   Similarly, if another emf ε_2 is balanced against l_2 (AN_2)
ϕ l_2 + 0 − ε_2 = 0 From the last two equations

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

Thus we can compare the emf's of any two sources. In practice one of the cells is chosen as a standard cell whose emf is known to a high degree of accuracy. The emf of the other cell is then easily calculated from above equation

use to measure internal resistance of a cell
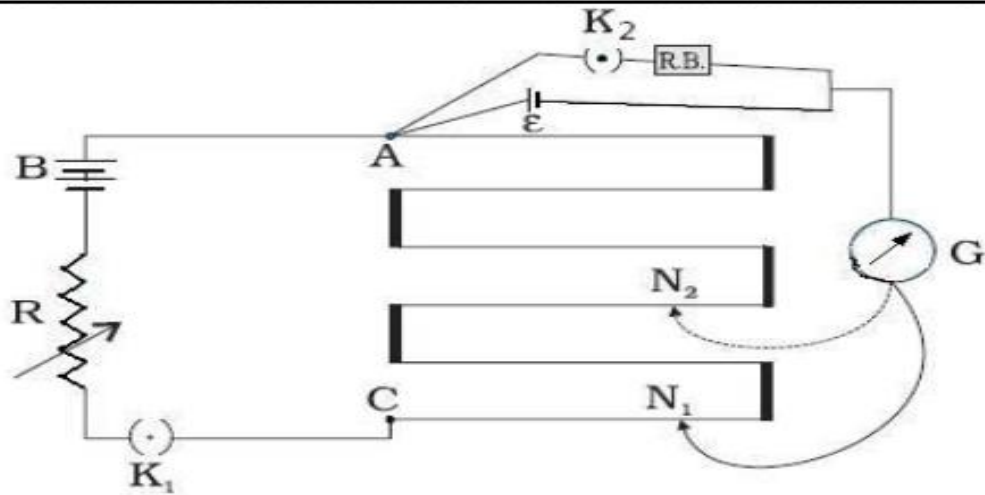
For this the cell (emf ε) whose internal resistance (r) is to be determined is connected across a resistance box through a key K_2, as shown in the figure. With key K_2 open, balance is obtained at length l_1 (AN_1). Then,

$\varepsilon = \phi\, l_1$

When key K_2 is closed, the cell sends a current (I) through the resistance box (R). If V is the terminal potential difference of the cell and balance is obtained at length l_2 (AN_2),

$V = \phi\, l_2$

So, we have

$$\frac{\varepsilon}{V} = \frac{l_1}{l_2}$$

But, $\varepsilon = I\,(r + R)$ and $V = IR$. This gives

$$\frac{\varepsilon}{V} = \frac{(r + R)}{R}$$

From above equations we have

$$\frac{(r + R)}{R} = \frac{l_1}{l_2}$$

$$r = R\left(\frac{l_1}{l_2} - 1\right)$$

Using above we can find the internal resistance of a given cell.

The potentiometer has the advantage that it draws no current from the voltage source being measured. As such it is unaffected by the internal.