CAPACITORS AND CAPACITANCE

CAPACITY OF AN ISOLATED CONDUCTOR

When charge is given to an isolated body, its potential increases and the electric field also go on gradually increasing. In this process at some stage the electric field becomes sufficiently strong to ionize the air particles around the body as a result body is not able to store any additional charge. During the process the ratio of charge Q on the body and potential (V) on the body remains constant. This ratio is called the capacity of the body

$$C = Q/V$$

In SI system, the unit of capacity is coulomb/volt and is called Farad (F)

The capacity of a body is independent of the charge given to it and depends on the shape and size only.

CAPACITOR

Capacitor is an arrangement of two conductors carrying charges of equal magnitude and opposite sign and separated by an insulating medium. The following points may be carefully noted.

(i) The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge Q, we mean that positively charged conductor has a charge +Q and the negatively charged conductor has a charge *-Q.*

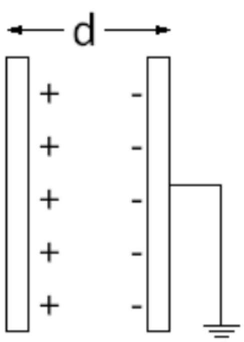
(ii) The positively charged conductor is at a higher potential than negatively charged conductor. The potential difference V between the conductors is proportional to the magnitude of charge Q and the ratio *Q/V* is known as capacitance C of the capacitor.

Q C= V

Unit of capacitance is farad (F). The capacitance is usually measured in microfarad (μF)

(iii) Circuit symbol is -||-

PARALLEL PLATE CAPACITOR



A parallel plate capacitor consists of two metal plates placed parallel to each other and separated by a distance *'d'* that is very small as compared to the dimensions of the plates. Due to this, the non-uniformity of the electric field near the ends of the plates can be neglected and in the entire region between the plates the electric field can be taken as constant. The area of each plate is *A.* Let Q be the charge on each plates. Surface charge is σ. Direction of electric field produced by both the plates is in same direction. Where outside the plates electric field is opposite in direction hence zero

Then electric field between the plates is given by

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

potential difference (V) between plates is given by V = Ed

$$V = \frac{\sigma}{\varepsilon_0} d = \frac{Q}{A\varepsilon_0} d$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

$V \quad d$

ISOLATED SPHERE AS A CAPACITOR

A conducting sphere of radius R carrying a charge Q can be treated as a capacitor with high potential conductor as the sphere itself and low potential conductor as sphere of infinite radius. The potential difference between these two spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 R} - 0$$

Hence Capacitance C = Q/V = $4\pi\varepsilon_0 R$

ENERGY STORED IN CHARGED CAPACITOR

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is = $\frac{\sigma}{2\varepsilon_0}$

Where σ $is \frac{Q}{A}$ and A is area of plate

Hence taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be = $\frac{\sigma}{2\varepsilon_0} d$

The potential energy of the second plate will be = (potential) (charge Q on it)

Potential energy stored in capacitor =$\frac{\sigma}{2\varepsilon_0} dQ$

$$U_E = \frac{\sigma dQ}{2\varepsilon_0} = \frac{Q}{A}\frac{d\,Q}{2\varepsilon_0} = \frac{Q^2}{2\left(\varepsilon_0 A/d\right)} = \frac{1}{2}\frac{Q^2}{C}$$

OR

$$U_E = \frac{1}{2} C V^2 = \frac{1}{2} V Q$$

ENERGY DENSITY OF A CHARGED CAPACITOR

Energy stored in capacitor is localized on the charges or the plates but is distributed in the field. Since in case of parallel plate capacitor, the electric field is only between the plates i.e. in a volume (A X d), the energy density

$$\rho_E = \frac{U_E}{volume} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

$$\rho_E = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

as E = V/d

$$\rho_E = \frac{1}{2}\varepsilon_0 E^2$$

FORCE BETWEEN THE PLATES OF A CAPACITOR

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in conservative field F = -dU/dx. In case of parallel plate capacitor

$$U_E = \frac{1}{2}\frac{Q^2}{C}$$

$$\text{But } C = \frac{\varepsilon_0 A}{x}$$

$$U_E = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A}x$$

So

$$F = -\frac{d}{dx}\left[\frac{1}{2}\frac{Q^2}{\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$

The negative sign indicates that the force is attractive

DIELECTRICS AND PLOLARISTAION

Non-conducting material are called dielectric. Dielectric materials are of two types (i) non-polar dielectric (ii) Polar dielectric

(i) Non-polar dielectric

In a non-polar molecule, **the centre of the positive and negative charge coincides with each other.** Hence they do not possess a permanent dipole moment. Now when it is placed in a uniform electric field, these centres are displaced in mutually opposite directions. Thus an electric dipole is induced in it or molecule is said to be polarized. If extent of electric field is not very strong, it is found that this dipole moment of molecule is proportional to external electric field $\vec{E}_0$

$$\therefore \vec{p} = \alpha \vec{E}_0$$

Where α is called the polarisability of the molecule

The units of α is $C^2\ m\ N^{-1}$

(ii) Polar Molecule

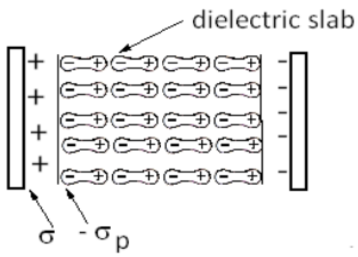
A polar molecule possesses a permanent dipole moment *p,* but such dipole moments of different molecules of the substance are randomly oriented in all possible directions and hence the resultant dipole moment of the substance becomes zero.

On applying an external electric field a torque acts on every molecular dipole. Therefore, it rotates and tries to become parallel to the electric field. Thus a resultant dipole moment is produced. In this way the dielectric made up of such molecules is said to be polarised.

EFFECT OF DIELECTRIC ON CAPACITANCE

Capacitance of a parallel plate capacitor in vacuum is given by charge density on plates is σ

$$C_0 = \frac{\varepsilon_0 A}{d}$$



Consider a dielectric inserted between the plates of capacitor, the dielectric is polarized by the electric field, the effect is equivalent to two charged sheets with surface charge densities σ_P and $-\sigma_P$. The electric field in the dielectric will be

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0}$$

So the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0}\mathrm{d}$$

For linear dielectric, we expect σ_P to be proportional to electric field due to plates E_0
Thus $(\sigma - \sigma_P)$ is proportional to σ and can be written as
$(\sigma - \sigma_P) = \frac{\sigma}{K}$
Where K is a constant characteristic of the dielectric. Then we have

$$V = \frac{\sigma}{K\varepsilon_0}\mathrm{d} = \frac{\mathrm{Qd}}{\mathrm{A}K\varepsilon_0}$$

The capacitance C, with dielectric between the plates , is then

$$C = \frac{Q}{V} = \frac{K\varepsilon_0 A}{d} = \mathrm{KC_0}$$

The product $K\varepsilon_0$ is called the **permittivity** of the medium denoted by ε , $\varepsilon = K\varepsilon_0$
Or $K = \frac{\varepsilon}{\varepsilon 0}$
For vacuum K = 1 and for other dielectric medium K >1.

INTRODUCTION OF A DIELECTRIC SLAB OF DILECTRIC CONSTANT K BETWEEN THE PLATES

(a) When battery is disconnected

Let q_0, C_0, V_0, E_0 and U_0 represents the charge, capacity, potential difference, electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant *K* between the plates and the battery disconnected.

(i) Charge remains constant, i.e., $q = q_0$, as in an isolated system charge is conserved.

(ii) Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes *K* times.

(iii) Potential difference between the plates decreases,

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K}\ [\because q = q_0\ and\ C = KC_0]$$
$$V = \frac{V_0}{K}$$

(iv) As Field between the plates decreases,

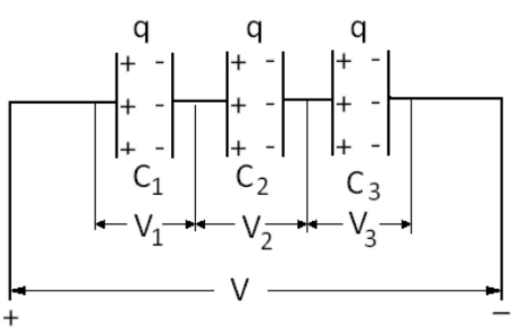
$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K}\left[As\ V = \frac{V_0}{K}\right]$$
$$E = \frac{E_0}{K}$$

(v) Energy stored in the capacitor decreases

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}\quad [as\ q = q_0\ and\ C = KC_0]$$

(b) When battery remains connected (potential held constant)
(i) Potential remains constant i.e $V = V_0$
(ii) Capacity increases i.e $C = KC_0$
(iii) Charge on the capacitor increases i.e $q = Kq_0$
(iv) Electric field remains unchanged $E = E_0$
(v) Energy stored in the capacitor increases

SERIES COMBINATION OF CAPACITORS

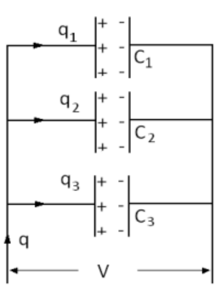


Capacitor are said to be connected in series if charge on each individual capacitor is same. In this situation
$V = V_1 + V_2 + V_3$
If C is the effective capacitance of combination then we know that $V = q/C$ and $V_1 = q/C_1$, $V_2 = q/C_2$, $V_3 = q/C_3$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

OR

PARALLEL COMBINATION OF CAPACITORS



When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1, C_2 and C_3 is different i.e q_1, q_2 and q_3
The total charge q is given by
$q = q_1 + q_2 + q_3$
potential across each capacitor is same thus $q_1 = C_1V$
$q_2 = C_2V$ and $q_3 = C_3V$
If C is equivalent capacitance then

$q = CV$

Thus $CV = C_1V + C_2V + C_3V$
Or $C = C_1 + C_2 + C_3$
Charge on capacitor
Let two capacitors are connected in parallel, let Q be the total charge , Let Q_1 be the charge on capacitor of capacity C_1 and Q_2 be the charge on capacitor of capacity C_2
Since both capacitor have same potential from formula V = Q/C we get

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

$$\frac{Q_1 + Q_2}{Q_2} = \frac{C_1 + C_2}{C_2}$$

Since $Q = Q_1+Q_2$

$$\frac{Q}{Q_2} = \frac{C_1 + C_2}{C_2}$$

$$Q_2 = \frac{C_2}{C_1 + C_2}Q$$

Similarly

$$Q_1 = \frac{C_1}{C_1 + C_2}Q$$

REDISTRIBUTION OF CHARGES

If there are two spherical conductors of radius R_1 and R_2 at potential V_1 and V_2 respectively Far apart from each other (so that charge on one does not affect the other). The charges on them will be
$Q_1 = C_1V_1$ and $Q_2 = C_2V_2$
The total charge on the system is $Q = Q_1 + Q_2$
The capacitance $C = C_1 + C_2$
Now if they are connected through a wire, charge will flow from conductor at higher potential to lower potential till both acquires same potential let charge on first becomes q_1 and charge on second sphere becomes q_2
Since potential is same

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

We know that capacity of sphere $C = 4\pi\varepsilon_0 R$. Thus $C \propto R$

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}$$

$$\frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\frac{q_1 + q_2}{q_2} = \frac{R_1 + R_2}{R_2}$$

But $Q = q_1 + q_2$

$$\frac{Q}{q_2} = \frac{R_1 + R_2}{R_2}$$

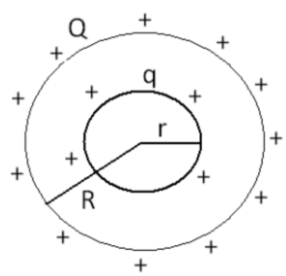
$$q_2 = \frac{R_2}{R_1 + R_2} Q$$

similarly

$$q_1 = \frac{R_1}{R_1 + R_2} Q$$

VAN DE GRAFF GENERATOR

Principle:



Suppose there is a positive charge Q, on an insulated conducting spherical shell of radius R, as shown in figure. At the centre of this shell, there is a conducting sphere of radius r (r<R), having a charge q.

Here electric potential on the shell of radius R is,

$$V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

And the electric potential on the spherical shell of radius r is,

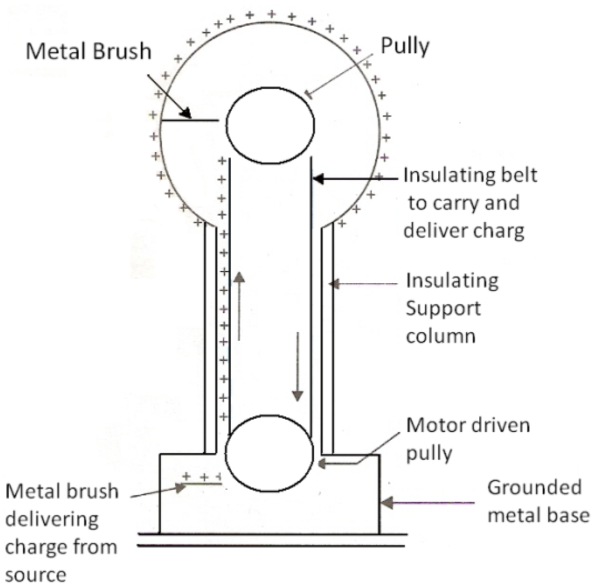
$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

It is clear from these two equations that the potential on the smaller sphere is more and the potential difference between them is

$$V_r - V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Hence if the smaller sphere is brought in electrical contact with bigger sphere then the charge goes from smaller to bigger sphere. Thus charge can be accumulated to a very large amount on the bigger sphere and there by its potential can be largely increased



As shown in the figure a spherical shell of a few meter radius, is kept on an insulated support, at a height of a few meters from the ground.
A pulley is kept at the centre of the big sphere and another pulley is kept on the ground. An arrangement is made such that a non-conducting belt moves across two pulleys. Positive charges are obtained from a discharge tube and are continuously sprayed on the belt using a metallic brush (with sharp edges) near the lower pulley. This positive charge goes with the belt towards the upper pulley.
There it is removed from the belt with the help of another brush and is deposited on the shell (because the potential on the shell is less than that of the belt on the pulley.) Thus a large potential difference (nearly 6 to 8 million volt) is obtained on the big spherical shell.

Uses : With the help of this machine, a potential difference of a few million (1 million = 10^6 = ten lac) volt can be established. By suitably passing a charged particle through such a high potential difference it is accelerated (to very high velocity) and hence acquires a very high energy ($\frac{1}{2}mv^2$). Because of such a high energy they are able to penetrate deeper into the matter. Therefore, fine structure of the matter can be studied with the help of them.