WAVES

Wave, propagation of disturbances from place to place in a regular and organized way.

Type of waves

- **Mechanical waves** : These waves require material medium for their propagation.

  For example : sound waves, waves in stretched string etc.

- **Non-mechanical waves or electromagnetic waves** : These waves do not require any material medium for their propagation.

  For example : light waves, x-rays etc.

There are two types of mechanical waves. Transverse wave and Longitudinal wave.

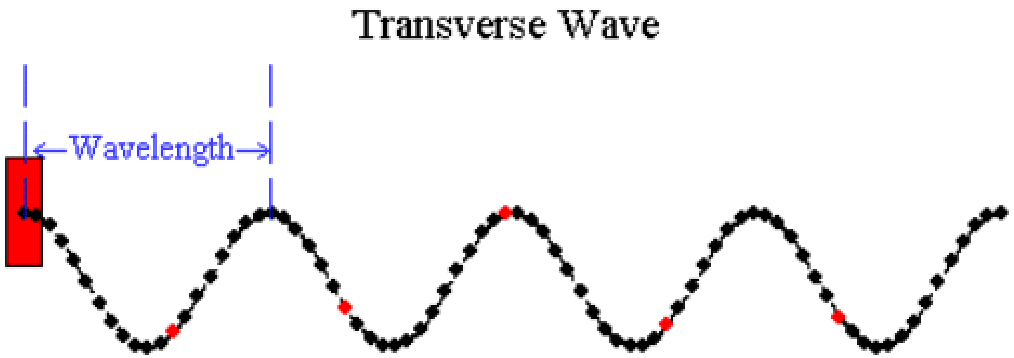
- **Transverse waves** : In the transverse wave, the particles of medium oscillate in a direction perpendicular to the direction of wave propagation.

  Example :-
  Waves in stretched string, waves on the water surface are transverse in nature.

Note

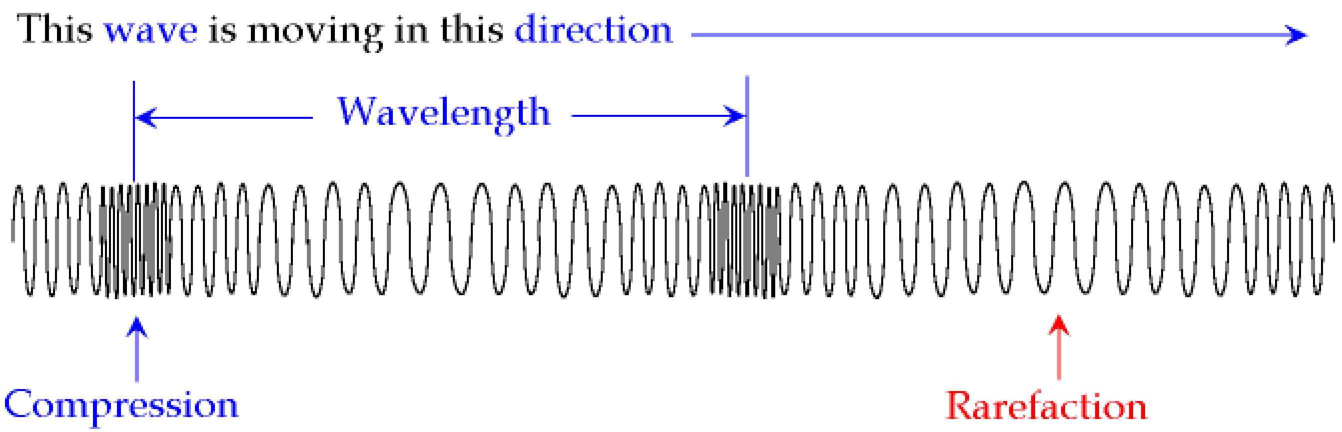
- Transverse wave can travel only in solids and surface of liquids.
- Transverse waves propagate in the form of crests and troughs.

All electromagnetic waves are transverse in nature.



- **Longitudinal waves** : In longitudinal waves particles of medium oscillate about their mean position along the direction of wave propagation.

Sound waves in air are longitudinal. These waves can travel in solids, liquids and gases.



EQUATION OF A HARMONIC WAVE

Harmonic waves are generated by sources that execute simple harmonic motion.

A harmonic wave travelling along the positive direction of x-axis is represented by

$$y = A\sin(\omega t - kx)$$

$$y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

where,
y = displacement of the particle of the medium at a location x at time t
A = amplitude of the wave
λ = wavelength
T = time period
$\omega = \frac{2\pi}{T}$ angular frequency
$k = \frac{2\pi}{\lambda}$ propagation constant or angular wave number

A harmonic wave travelling along the positive direction of x-axis is represented by

$$y = A\sin(\omega t + kx)$$

$$y = A\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

DIFFERENTIAL EQUATION OF WAVE MOTION

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

RELATION BETWEEN WAVE VELOCITY AND PARTICLE VELOCITY

The equation of a plane progressive wave is

$$y = A\sin(\omega t - kx)$$

The particle velocity

$$v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$$

Slope of displacement curve or strain

$$\frac{dy}{dx} = -Ak\cos(\omega t - kx)$$

Problem

The displacement y (in cm) produced by a simple harmonic wave is given by $y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right)$

What will be the periodic time and maximum velocity of the particles in the medium?

Given

$$y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right) \qquad y = \frac{10}{\pi}\sin 2\pi\left(1000t - \frac{x}{17}\right) --- (1)$$

General form of simple harmonic wave is $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) --- (2)$

On comparing the equation (1) and (2)

We get $T = \frac{1}{1000}$

$$T = 10^{-3}$$

For finding the particle maximum velocity differentiate given equation with respect to ‘t’

$$y = \frac{10}{\pi}\sin\left(2000\pi t - \frac{\pi x}{17}\right)$$

$$v_{\max} = \frac{dy}{dt}$$

$$= \frac{10}{\pi} \times 2000\pi \cos\left(2000\pi t - \frac{\pi x}{17}\right)$$

$$= 10 \times 2000 cm / s$$

$$= 20,000 cm / s$$

Therefore maximum velocity is Vmax = 20,000cm/s