

Quadratic Equations - An Example

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Question

Solve the following equation

$$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}$$

Smart Simplification

$$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3} \quad (1)$$

$$\frac{x-3+1}{x-3} + \frac{x-5+1}{x-5} = \frac{10}{3} \quad (2)$$

$$1 + \frac{1}{x-3} + 1 + \frac{1}{x-5} = \frac{10}{3} \quad (3)$$

$$\frac{1}{x-3} + \frac{1}{x-5} = \frac{10}{3} - 2 \quad (4)$$

$$\frac{1}{x-3} + \frac{1}{x-5} = \frac{4}{3} \quad (5)$$

Method I: 1 / 3

$$\frac{1}{x-3} + \frac{1}{x-5} = \frac{4}{3} \quad (6)$$

$$\frac{1}{x-4+1} + \frac{1}{x-4-1} = \frac{4}{3} \quad (7)$$

$$\frac{x-4+\cancel{1} + x-4-\cancel{1}}{(x-4)^2-1} = \frac{4}{3} \quad (8)$$

$$\frac{2(x-4)}{(x-4)^2-1} = \frac{4}{3} \quad (9)$$

Method I: 2 / 3

$$\frac{\cancel{2}(x-4)}{(x-4)^2-1} = \frac{\cancel{4}^2}{3} \quad (10)$$

$$\frac{3(x-4)}{2} = (x-4)^2 - 1 \quad (11)$$

$$(x-4)^2 - 1 = \frac{3}{2}(x-4) \quad (12)$$

$$(x-4)^2 - 2 \cdot \frac{3}{4}(x-4) + \left(\frac{3}{4}\right)^2 = 1 + \left(\frac{3}{4}\right)^2 \quad (13)$$

Method I: 3 /3

$$\left((x - 4) - \frac{3}{4}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2 \quad (14)$$

$$(x - 4) - \frac{3}{4} = \pm \frac{5}{4} \quad (15)$$

$$x = \frac{19}{4} \pm \frac{5}{4} \quad (16)$$

$$\boxed{x_1 = 6, x_2 = \frac{7}{2}} \quad (17)$$

The next upload will have the conventional method of solving this equation.

Stay tuned ...