ASSIGNMENT : Trigonometry

Q.1   Find the diameter of Sun in km supposing that it subtends an angle of 32° at the eye of an observer. Distance of the Sun is 91×10^6 km. (Ans: 847407.4 km)

Q.2   If angular diameter of moon is 30°, how far from the eye a coin of diameter 2.2 cm can be kept to hide the mood.(Ans: 252 cm)

Q.3   If sec x + tan x = p, find the value of sec x, tan x, sin x in term of p.

Q.4   Prove that:

(i) $\cos 24 + \cos 55 + \cos 125 + \cos 204 + \cos 300 = 1/2$

(ii) $\sin 780 . \sin 120 + \cos 240 . \sin 390 = 1/2$

(iii) $\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right) = 2$

(iv) $\tan 15 + \tan 30 + \tan 15 . \tan 30 = 1$

(v) $\frac{\cos 11 + \sin 11}{\cos 11 - \sin 11} = \tan 56$

(vi) $\frac{\tan 69 + \tan 66}{1 - \tan 69 . \tan 66} = -1$

(vii) $\tan 70 = \tan 20 + 2\tan 50$

Q.5   If A+B = π/4 show that (1+tanA)(1+tanB) = 2

Q.6   If tanA = 5/6 tanB = 1/11, show that A+B= π/4

Q.7   If tanA = m/m-1 tanB = 1/2m-1, prove that A-B = π/4

Q.8   If tan A = x tan B prove that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$

Q9   Prove that:

i) $\sin 20 . \sin 40 . \sin 60 . \sin 80 = 3/16$
ii) $\cos 20 . \cos 40 . \cos 60 . \cos 80 = 1/16$
iii) $\tan 20 . \tan 40 . \tan 60 . \tan 80 = 3$
iv) $\sin 10 . \sin 30 . \sin 50 . \sin 70 = 1/16$

Q.10   If cosA + cosB = 1/2 and sinA + sinB = 1/4, prove that tan(A+B/2)=1/2

Q.11   If tanθ/2=$\sqrt{a-b/a \; a+b \; tan \; \varphi/2}$, Prove that $cos\theta = \frac{a \cos \varphi + b}{a + b \cos \varphi}$

Q.12   Prove that:

i) $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2\cos\theta$

ii) $\sqrt{3} \; \text{cosec} 20 - \sec 20 = 4$

iii) $\cot\frac{\pi}{24} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

iv) $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 2\sec 2\theta$

v) $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right) = \frac{1}{8}$

vi) $\frac{\cos\theta}{1+\sin\theta} = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

vii) $\frac{1-\cos2\theta+\sin2\theta}{1+\cos2\theta+\sin2\theta} = \tan\theta$

viii) $\cos^2 A + \cos^2\left(A + \frac{2\pi}{3}\right) + \cos^2\left(A - \frac{2\pi}{3}\right) = \frac{3}{2}$

ix) $\sin^2\left(\frac{\pi}{8} + \frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{A}{2}\right) = \frac{1}{\sqrt{2}\sin A}$

x) $\frac{1+\sin2\theta+\cos2\theta}{1+\sin2\theta-\cos2\theta} = \cot\theta$

xi) $\frac{\sec8\theta-1}{\sec4\theta-1} = \frac{\tan8\theta}{\tan2\theta}$

xii) $\frac{\sin\theta+\sin2\theta}{1+\cos\theta+\cos2\theta} = \tan\theta$

Q.13 Find the value of:

a) $\sin\frac{\pi}{8}$ b) $\cos\frac{\pi}{8}$ c) $\tan\frac{\pi}{8}$

d) $\sin\frac{\pi}{12}$ e) $\cos\frac{\pi}{12}$ f) $\tan\frac{\pi}{12}$

g) $\sin\frac{\pi}{24}$ h) $\cos\frac{\pi}{24}$ i) $\tan\frac{\pi}{24}$

j) $\sin 18$ k) $\cos 18$ l) $\tan 18$

m) $\sin 36$ n) $\cos 36$ o) $\tan 36$

p) $\sin 54$ q) $\cos 54$ r) $\tan 54$

Q.14 Prove: $\tan A + \tan(60+A) - \tan(60-A) = 3\tan3A$

Q.15 If $\tan^2\theta = 2\tan^2\varphi + 1$, Prove that: $\cos2\theta + \sin^2\varphi = 0$

Q.16 If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -3/2$, prove that

$\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$

Q.17 If $\tan\beta = \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$, prove that $\tan(\alpha-\beta) = (1-n)\tan\alpha$.

Q.18 Solve the equations: (General Solutions)

i) $\cos\theta + \cos3\theta - 2\cos2\theta = 0$ $\theta = \frac{(2n+1)\pi}{4}$ or $2m\pi$

ii) $2\cos^2\theta + 3\sin\theta = 0$ $\theta = \frac{n\pi + (-1)^{n+1}\pi}{6}$

iii) $\tan\theta + \tan2\theta + \tan\theta\tan2\theta = 1$ $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$

iv) $7\cos^2\theta + 3\sin^2\theta = 4$ $\theta = \frac{n\pi\pm\pi}{3}$

v) $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ $\theta = \frac{2n\pi+5\pi}{12}$

vi) $\sqrt{2}\sec\theta + \tan\theta = 1$ $\theta = \frac{2n\pi-\pi}{4}$

vii) $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$ $\theta = \frac{2n\pi\pm5\pi}{6}$