ACUMEN

JEE MAIN - MATHEMATICS

Test - 2D Geometry and Circles

1. A triangle has a vertex at (1, 2) and the mid-points of the two sides through it are (-1, 1) and (2, 3). Then, the centroid of this triangle is

   a) $\left(\frac{1}{3}, 1\right)$ b) $\left(\frac{1}{3}, 2\right)$
   c) $\left(1, \frac{7}{3}\right)$ d) $\left(\frac{1}{3}, \frac{5}{3}\right)$

2. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines 3x + 4y = 9 and y = mx + 1 is also an integer, is

   a) 4 b) 0
   c) 2 d) 1

3. If length of the tangent drawn from each and every point on the curve y = $\sqrt{\lambda - x^2}$ to the circle x^2 + y^2 = 36 is 8 units, then λ is :

   a) 30 b) 50
   c) 100 d) 90

4. If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to -4, then a value of k is:

   a) $\sqrt{15}$ b) -4
   c) -2 d) $\sqrt{14}$

5. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is

   a) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$
   c) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

6. If the tangents are drawn from any point on the line x + y = 3 to the circle x^2 + y^2 = 9, then the chord of contact passes through the point :

   a) (3, 2) b) (3, 5)
   c) (3, 3) d) (5, 3)

7. The normal at a point P on the parabola y^2 = 8x meet the x-axis in G. If S is the focus and the triangle SPG is equilateral then the abscissa of the point P is

   a) 2/3 b) 3
   c) 6 d) $2\sqrt{3}$

8. The length of the chord cut by the circle x^2 + y^2 = 2 on the line y - 2x - 1 = 0, is :

a) $\frac{6}{\sqrt{5}}$

b) $\frac{4}{9}$

c) 0

d) $\frac{2}{9}$

9. The region represented by $|x - y| \leq 2$ and $|x + y| \leq 2$ is bounded by a

a) rhombus of area $8\sqrt{2}$ sq units

b) square of area 16 sq units

c) rhombus of side length 2 units

d) square of side length $2\sqrt{2}$ units

10. Two circles whose radii are equal to 4 and 8 intersect at right angles. The length of their common chord is :

a) $\frac{8\sqrt{5}}{5}$

b) $\frac{16}{\sqrt{5}}$

c) 8

d) $4\sqrt{6}$

11. A point P $(\sqrt{3}, 1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of circle in anticlockwise leaves it tangentially. The equation of a line along which the point moves after leaving the circle is :

a) $y = \sqrt{3}x + 4$

b) $y = \sqrt{3}y - 4$

c) $\sqrt{3}y = x - 4$

d) $\sqrt{3}y = x + 4$

12. Minimum distance between the circles $x^2 + y^2 = 144$ and $x^2 + y^2 - 6x - 8y = 0$, is :

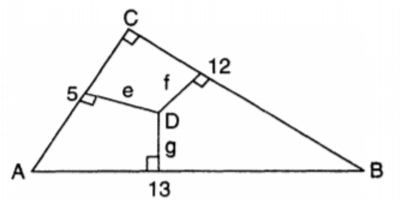
a) 7

b) 17

c) 0

d) 2

13. The sides of $\triangle$ABC are shown in given figure. Let D be any internal point and e ,f g are perpendicular distance of D from sides of triangle then the value of (5e + 12f +13g) is equal to :



a) 15

b) 30

c) 60

d) 120

14. The equation of line segment AB is y = x. If A and B lie on the same side of the line mirror 2x - y = 1, then image of AB is :

a) 7x - y - 6 = 0

b) 7x + y - 6 = 0

c) 7x - y + 6 = 0

d) 7x + y + 6 = 0

15. If P = (1,0), Q = (-1,0) and R = (2,0) are three given points, then locus of the points satisfying the relation $SQ^2 + SR^2 = 2SP^2$, is

a) a circle passing through the origin

b) a straight line parallel to Y-axis

c) a straight line parallel to X-axis

d) a circle with the centre at the origin

16. Total number of common tangents of $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ is equal to :

a) 4

b) 1

c) none of these d) 2

17. A ray of light along x + $\sqrt{3}$y = $\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is:

a) y = $\sqrt{3}$x - $\sqrt{3}$ b) $\sqrt{3}$y = x - 1

c) $\sqrt{3}$y = x - $\sqrt{3}$ d) y = x + $\sqrt{3}$

18. Suppose that the points (h, k), (1, 2) and (-3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) and (4, 3) is perpendicular to L_1, then $\frac{k}{h}$ equals

a) 3 b) 0

c) $-\frac{1}{7}$ d) $\frac{1}{3}$

19. The locus of midpoints of the chords of the circle x^2 -2x + y^2 - 2y + 1 = 0 which are of unit length is :

a) $(x - 1)^2 + (y - 1)^2 = \frac{2}{3}$ b) $(x - 1)^2 + (y - 1)^2 = 2$

c) $(x - 1)^2 + (y - 1)^2 = \frac{3}{4}$ d) $(x - 1)^2 + (y - 1)^2 = \frac{1}{4}$

20. In a triangle ABC, sin A : sin B : sin C = 4 : 5 : 6, while cos A : cos B : cos C = x : y : 2. The ordered pair (x, y) is : [Note : All symbols used have usual meaning in triangle ABC.]

a) (12, 9) b) (9, 6)

c) (5, 4) d) (10, 5)

21. The number of intergral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is:

a) 105 b) 133

c) 190 d) 233

22. A circle of radius 10 is circumscribed about a triangle ABC. If AB = BC = 10, then the area of the triangle is :

a) 40 b) $25\sqrt{3}$

c) 50 d) $25\sqrt{2}$

23. The chords of contact of the pair of tangents drawn from points on the line 2x + y = 4 to the circle x^2 + y^2 = 1 passes through a fixed point M (a, b). The value of $\left(\frac{1}{a}+\frac{1}{b}\right)$, is equal to :

a) 3 b) 4

c) 6 d) 5

24. Let A (1, 5), B (3, 4) and C (1, 1) be vertices of a $\triangle$ABC with O as its orthocentre. If orthocentre of $\triangle$OAB be (α, β), then $|\alpha - \beta|$ is equal to :

a) 4 b) 0

c) 2 d) 1

25. If the line y cos α = x sin α + a cos α be a tangent to the circle $x^2 + y^2 = a^2$, then

a) $\cos^2\alpha = a^2$ b) $\sin^2\alpha = a^2$

c) $\cos^2\alpha = 1$ d) $\sin^2\alpha = 1$

26. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if:

a) $r > 8$ b) $2 < r < 8$

c) $2 \leq r \leq 8$ d) $r < 2$

27. a, b > 0. The length of the common chord of the circles $(x - a)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = b^2$ is

a) $\sqrt{a+b}$ b) $\frac{2ab}{a+b}$

c) $\frac{2ab}{\sqrt{a^2+b^2}}$ d) $\frac{a+b}{2}$

28. If A_0, A_1, A_2, A_3, A_4 and A_5 be a regular hexagon inscribed in a circle of unit radius. Then, the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

a) $\frac{3}{4}$ b) $\frac{3\sqrt{3}}{2}$

c) $3\sqrt{3}$ d) 3

29. Let the combined equation of a pair of tangents to a circle drawn from the origin O be $xy - y^2 = (2 + \sqrt{3})(x^2 - xy)$. If the radius of the circle is 3 units and centre is in the first quadrant, then the length OA (where A is one of the points of contact) is

a) $3(2 - \sqrt{3})$ b) $\frac{3}{2}(2+\sqrt{3})$

c) $3(2 + \sqrt{3})$ d) $\frac{\sqrt{3}}{2}(2+\sqrt{3})$

30. If the length of the chord of the circle, $x^2 + y^2 = r^2 (r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to:

a) $\frac{9}{5}$ b) $\frac{24}{5}$

c) 12 d) $\frac{12}{5}$

31. A circle touching the X-axis at (3, 0) and making an intercept of length 8 on the Y-axis passes through the point

a) (2, 3) b) (3, 10)

c) (1, 5) d) (3, 5)

32. The area of triangle formed by the tangent, normal drawn at $(1, \sqrt{3})$ to the circle $x^2 + y^2 = 4$ and the positive x-axis, is

a) $5\sqrt{3}$ b) $4\sqrt{3}$

c) $2\sqrt{3}$ d) $\sqrt{3}$

33. If the straight line $\frac{2x}{a} + \frac{y}{b} = 2\sqrt{2}$ touches the circle $x^2 + y^2 = 2ab$, a, b > 0, then:

a) a = b b) 2a = b

c) None of these d) a = 2b

34. The point (1, 4) lies inside the circle ω having equation $x^2 + y^2 - 6x - 8y + k = 0$, where k is an arbitrary constant. The circle ω neither touches the coordinate axes nor cuts them. The possible values of k are between

a) 9 and 16
b) 9 and 25
c) 9 and 21
d) 16 and 21

35. Let L_1 be a straight line passing through the origin and L_2 be the straight line x + y = 1. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1?

a) x - 7y = 0
b) x - y = 0, x + 7y = 0
c) 7x + y = 0
d) x + y = 0, x - 7y = 0

36. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumfeence of the circle $x^2 + y^2 - 2x + 8y - d = 0$ (c, d > 0), then maximum value of cd is:

a) 425
b) 625
c) 125
d) 25

37. Equations to the circles which touch the lines 3x - 4y + 1 = 0, 4x + 3y - 7 = 0 and pass through (2, 3) are

a) $5x^2 + 5y^2 - 12x - 24y + 31 = 0$
b) $(x - 2)^2 + (y - 8)^2 = 25$
c) $(x - 2)^2 + (y - 8)^2 = 25$ and $5x^2 + 5y^2 - 12x - 24y + 31 = 0$
d) None of these

38. Let C be the circle with center at (1, 1) and radius 1. If T is the circle centered at (0, y) passing through the origin and touching the circle C externally, then the radius of T is equal to

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) $\frac{\sqrt{3}}{2}$
d) $\frac{\sqrt{3}}{\sqrt{2}}$

39. P is a lattice point (a point having integer coordinates) in the 1^{st} quadrant. The segment joining $(\sqrt{33}, \sqrt{17})$ and $(-\sqrt{33}, -\sqrt{17})$ subtends a right angle at P. The number of points which satisfy P are

a) 4
b) 3
c) 2
d) 12

40. A circle cuts a chord oflength 4a on the X-axis and passes through a point on the Y-axis, distant 2b from the origin. Then, the locus of the centre of this circle, is

a) A hyperbola
b) A straight line
c) A parabola
d) An ellipse

41. Three circles of radii a, b, c (a < b < c) touch each other externally. If they have X-axis as a common tangent, then

a) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$
b) a, b, c are in AP
c) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$
d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in AP

42. The locus of the centres of the circles, which touch the circle, $x^2 + y^2 = 1$ externally, also touch the Y-axis and lie in the first quadrant, is

a) $x = \sqrt{1 + 4y}, y \geq 0$
b) $x = \sqrt{1 + 2y}, y \geq 0$

c) $y = \sqrt{1 + 2x}, x \geq 0$ d) $y = \sqrt{1 + 4x}, x \geq 0$

43. The equation of a tangent to the circle $x^2 + y^2 = 25$ passing through (-2, 11) is

a) 7x + 24y = 230 b) 3x + 4y = 38

c) 4x + 3y = 25 d) 24x + 7y + 125 = 0

44. If the circle $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is

a) 2 or -3/2 b) -2 or -3/2

c) 2 or 3/2 d) -2 or 3/2

45. The point P(10, 7) lies outside the circle $x^2 + y^2 - 4x - 2y - 20 = 0$. The greatest distance of P from the circle is

a) 5 b) $\sqrt{5}$

c) 15 d) $\sqrt{3}$

46. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is

a) (1/2, 3/2) b) (1/2, 1/2)

c) (3/2, 1/2) d) $(1/2, -2^{1/2})$

47. AB and CD are perpendicular chords of a circle of radius R meeting at point E. Then the expression $EA^2 + EB^2 + EC^2 + ED^2$ equals

a) R^2 b) $2R^2$

c) $4R^2$ d) $8R^2$

48. ω_1 and ω_2 are two circles passing through the points (0, a), (0, -a) and each touches the line y = mx + c. If ω_1 and ω_2 cut each other orthogonally, then

a) $a^2 = c^2(2 + m^2)$ b) $c^2 = a^2(1 + m^2)$

c) $a^2 = 2c^2(1 + m^2)$ d) $c^2 = a^2(2 + m^2)$

49. The area of the trapezium ABCD with AB||CD, AD$\perp$AB and AB = 3CD is equal to 4. A circle inside the trapezium is tangent to all of its sides. If the radius of the circle is r then the value of $4r^2$, is:

a) 3 b) 8

c) 2 d) 1

50. The square of the length of the tangent from (3, -4) on the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ is

a) 30 b) 40

c) 50 d) 20

51. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is ________.

52. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines x - y = 0 and x + y = 0, respectively. The area of the region R consisting of all points P lying in the first quadrant of

the plane and satisfying $2 \leq d_1(P) + d_2(P) \leq 4$ is ________ sq units.

53. Let point B be the reflection of point A(2, 3) with respect to the line 8x - 6y - 23 = 0. Let T_A and T_B be circles of radii 2 and 1 with centers A and B respectively. Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ________.
54. The number of integral values of k for which the line, 3x + 4y = k intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ________.
55. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?
56. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where k > 0, then the value of [k] is ________.
    NOTE: [k] denotes the largest integer less than or equal to k.
57. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is 2x + 4y = 5. If the centre of the circumcircle of the triangle OPQ lies on the line x + 2y = 4, then the value of r is ________.
58. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangents to C_1 and C passing through Pis also a common tangent to C_2 and C, then the radius of the circle C is ________
59. The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x = 3 and y = 2, is ________.
60. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, (k > 0) touch each other at a point, then the largest value of k is ________.