TRIGONOMETRY

Addition & Subtraction Formulae

$\cos(A\pm B) = \cos A.\cos B \mp \sin A.\sin B$

$\sin(A\pm B) = \sin A.\cos B \pm \cos A.\sin B$

$\tan(A\pm B) = \dfrac{(\tan A \pm \tan B)}{(1\mp \tan A.\tan B)}$, $\cot(A\pm B) = \dfrac{(\cot B.\cot A \mp 1)}{(\cot B \pm \cot A)}$

$\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B)$

$\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$

$\tan\left(\dfrac{\pi}{4}\pm\theta\right) = \left(\dfrac{1\pm\tan\theta}{1\mp\tan\theta}\right)$, $\cot\left(\dfrac{\pi}{4}\pm\theta\right) = \left(\dfrac{\cot\theta\mp 1}{\cot\theta\pm 1}\right)$

$$\sin a + \sin(b+a) + \sin(a+2b) + \ldots + \sin[a+(n-1)b] = \frac{\sin\left[a+(n-1)\frac{b}{2}\right].\sin\left(\frac{nb}{2}\right)}{\sin\left(\frac{b}{2}\right)}$$

$$\cos a + \cos(a+b) + \cos(a+2b) + \ldots + \cos[a+(n-1)b] = \frac{\cos\left[a+(n-1)\frac{b}{2}\right].\sin\left(\frac{nb}{2}\right)}{\sin\left(\frac{b}{2}\right)}$$

Transformation Formulae

Products into sum or difference :

$2\cos A.\cos B = \cos(A-B) + \cos(A+B)$

$2\sin A.\sin B = \cos(A-B) - \cos(A+B)$

$2\sin A.\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A.\sin B = \sin(A+B) - \sin(A-B)$

Trigonometric Ratios of Multiple ∠'s :

$\sin 2\theta = 2\sin\theta.\cos\theta = \dfrac{2\tan\theta}{(1+\tan^2\theta)}$

$\cos 2\theta = (\cos^2\theta - \sin^2\theta) = (2\cos^2\theta - 1) = (1 - 2\sin^2\theta) = \left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\tan 2\theta = \dfrac{2\tan\theta}{(1-\tan^2\theta)}$, $\cot 2\theta = \dfrac{(\cot^2\theta - 1)}{2\cot\theta}$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \left(\dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right)$, $\cot 3\theta = \left(\dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}\right)$

Sum & Difference into Products :

$\sin C + \sin D = 2\sin\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\dfrac{C+D}{2}\right).\sin\left(\dfrac{C-D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\dfrac{C+D}{2}\right).\cos\left(\dfrac{C-D}{2}\right)$

$\cos C - \cos D = -2\sin\left(\dfrac{C+D}{2}\right).\sin\left(\dfrac{C-D}{2}\right)$

$\tan C \pm \tan D = \dfrac{\sin(C\pm D)}{\cos C.\cos D}$, $\cot C \pm \cot D = \dfrac{\sin(D\pm C)}{\sin C.\sin D}$

Trigonometric Ratios of Submultiple ∠'s

$\sin\theta = 2\sin\left(\dfrac{\theta}{2}\right).\cos\left(\dfrac{\theta}{2}\right) = 2\tan\left(\dfrac{\theta}{2}\right)\Big/\left[1+\tan^2\left(\dfrac{\theta}{2}\right)\right]$

$\cos\theta = \cos^2\left(\dfrac{\theta}{2}\right) - \sin^2\left(\dfrac{\theta}{2}\right) = 2\cos^2\left(\dfrac{\theta}{2}\right) - 1 = 1 - 2\sin^2\left(\dfrac{\theta}{2}\right) = \dfrac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$

$1+\cos\theta = 2\cos^2\left(\dfrac{\theta}{2}\right)$, $1-\cos\theta = 2\sin^2\left(\dfrac{\theta}{2}\right)$, $\left(\dfrac{1-\cos\theta}{1+\cos\theta}\right) = \tan^2\left(\dfrac{\theta}{2}\right)$

$\tan\theta = \dfrac{2\tan\left(\frac{\theta}{2}\right)}{1-\tan^2\left(\frac{\theta}{2}\right)}$, $\cot\theta = \dfrac{\cot^2\left(\frac{\theta}{2}\right) - 1}{2\cot\left(\frac{\theta}{2}\right)}$, $\left(\dfrac{1+\cos\theta}{1-\cos\theta}\right) = \cot^2\left(\dfrac{\theta}{2}\right)$

$\left(\dfrac{1-\cos\theta}{\sin\theta}\right) = \tan\left(\dfrac{\theta}{2}\right)$, $\left(\dfrac{1+\cos\theta}{\sin\theta}\right) = \cot\left(\dfrac{\theta}{2}\right)$

Trigonometric Ratios of Some Special ∠'s

$\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $\tan 15^\circ = (2-\sqrt{3})$, $\cot 15^\circ = (2+\sqrt{3})$

$\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$, $\cos 18^\circ = \left(\dfrac{\sqrt{10+2\sqrt{5}}}{4}\right)$, $\tan 18^\circ = \dfrac{\sqrt{25-10\sqrt{5}}}{5}$

$\cos 36^\circ = \dfrac{(\sqrt{5}+1)}{4}$, $\sin 36^\circ = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$, $\tan 36^\circ = \sqrt{5-2\sqrt{5}}$

$\sin 22\dfrac{1^\circ}{2} = \dfrac{1}{2}\left(\sqrt{2-\sqrt{2}}\right)$, $\cos 22\dfrac{1^\circ}{2} = \dfrac{1}{2}\left(\sqrt{2+\sqrt{2}}\right)$, $\tan 22\dfrac{1^\circ}{2} = (\sqrt{2}-1)$

Greatest & Least Values of the expression. (a sin Θ + b cos Θ)

Let $a = r\cos\alpha$, $b = r\sin\alpha$, then $a^2+b^2 = r^2 \Rightarrow r = \sqrt{a^2+b^2}$

$\therefore$ $(a\sin\theta + b\cos\theta) = r\sin(\theta+\alpha)$

But $-1 \le \sin(\theta+\alpha) \le 1$

$-r \le r\sin(\theta+\alpha) \le r$

or $-\sqrt{a^2+b^2} \le (a\sin\theta + b\cos\theta) \le \sqrt{a^2+b^2}$

Thus, the greatest & least value of $(a\sin\theta + b\cos\theta)$ are :

$\sqrt{a^2+b^2}$ & $-\sqrt{a^2+b^2}$ respectively.