CONCEPT OF RELATION IN MATH

CARTESIAN PRODUCT OF TWO SETS

Definition : If A and B are two non-empty sets, then the Cartesian product of two sets, A and set B is the set of all ordered pairs (a, b) such that a ∈A and b ∈B which is denoted as A × B.

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

If A = {7, 8} and B = {2, 4, 6}, find A × B.

Solution:

A × B = **{**(7, 2); (7, 4); (7, 6); (8, 2); (8, 4); (8, 6)**}**

The 6 ordered pairs thus formed can represent the position of points in a plane, if A and B are subsets of a set of real numbers.

Note:
If either A or B are null sets, then A ×B will also be an empty set, i.e., if A = ∅ or B = ∅, then A × B = ∅

Problem: If A = { 1, 3, 5} and B = {2, 3}, then Find:
(i) A × B (ii) B × A (iii) A × A (iv) (B × B)

Solution:

A ×B={1, 3, 5} × {2,3} = [{1, 2},{1, 3},{3, 2},{3, 3},{5, 2},{5, 3}]

B × A = {2, 3} × {1, 3, 5} = [{2, 1},{2, 3},{2, 5},{3, 1},{3, 3},{3, 5}]

A × A = {1, 3, 5} × {1, 3, 5}= [{1, 1},{1, 3},{1, 5},{3, 1},{3, 3},{3, 5},{5, 1},
{5, 3},{5, 5}]

B × B = {2, 3} × {2, 3} = [{2, 2},{2, 3},{3, 2},{3, 3}]

If $A = \{1, 2, 3\}$

and $B = \{a, b\}$

Then $A \times B = \{(1,a)\ (1,b)\ (2,a)\ (2,b)\ (3,a)\ (3,b)\}$

Now

$(1,a) \in A \times B$ True

$\{(1,a)\} \in A \times B$ False

$\{(1,a)\} \subset A \times B$ True

The number of distinct elements in a finite set is called its cardinal number. It is denoted as n(A) and read as ‘the number of elements of the set’.

For example:

(i) Set A = {2, 4, 5, 9, 15} has 5 elements.

Therefore, the cardinal number of set A = 5. So, it is denoted as n(A) = 5.

(ii) Set B = {w, x, y, z} has 4 elements.

Therefore, the cardinal number of set B = 4. So, it is denoted as n(B) = 4.

If

The number of distinct elements in finite set $n(A)=m$

&

The number of distinct elements in finite set $n(B)=n$

Then

The number of distinct elements in finite set $n(AXB)=m.n$

If $A = \{7, 8\}$ and $B = \{2, 4, 6\}$, Then

$A \times B = \{(7, 2); (7, 4); (7, 6); (8, 2); (8, 4); (8, 6)\}$

$n(A)=m=2$ & $n(B)=n=3$ then

$n(A \times B) = m.n=2.3=6$

Number of Subsets of a given Set:

If a set contains ‘n’ elements, then the number of subsets of the set is (2^n)

Note:

Every set is a subset of itself, i.e., $A \subset A$, $B \subset B$.

Null set or Ø is a subset of every set.

Problem: If A {1, 3, 5}, then write all the possible subsets of A. Find their numbers.

Solution:

The subset of A containing no elements : { }

The subset of A containing one element each -:{1} {3} {5}

The subset of A containing two elements each -: {1, 3} {1, 5} {3, 5}

The subset of A containing three elements -: {1, 3, 5)

Therefore, all possible subsets of A are **{ }, {1}, {3}, {5}, {1, 3}, {3, 5}, {1, 3, 5}**

Therefore, number of all possible subsets of A =**2^3=8**

As we know that if the number of distinct elements in finite set n(A)=m

&

The number of distinct elements in finite set n(B)=n

Then

The number of distinct elements in finite set n(AxB)=m.n

Hence the number of subsets of the set (AxB)= 2^(m.n)

If $A = \{1, 2, 3\} \Rightarrow n(A) = 3$

And $B = \{a, b\} \Rightarrow n(B) = 2$

then $n(A \times B) = 3 \cdot 2 = 6$

Hence the number of Subset of the Set $(A \times B) = 2^6 = 64$

Every Subset of Set $(A \times B)$ is a Relation

Let $A = \{1, 2\}$, $B = \{a\}$

then $A \times B = \{(1, a)\ (2, a)\}$

$n(A \times B) = 2$

The number of subset of the set $A \times B = 2^2 = 4$, and that are

$\{\ \}, \{(1, a)\}, \{(2, a)\}, \{(1, a), (2, a)\}$

Here each subset represented a Relation

Representation of Relation in Math:

The relation in math from set A to set B is expressed in different forms.

(i) Roster form

(ii) Set builder form

(iii) Arrow diagram

i. Roster form:

● In this, the relation (R) from set A to B is represented as a set of ordered pairs.

● In each ordered pair 1st component is from A; 2nd component is from B.

For Example:

1. If $A = \{p, q, r\}$ $B = \{3, 4, 5\}$

then $R = \{(p, 3), (q, 4), (r, 5)\}$

Hence, $R \subseteq A \times B$

ii. Set builder form:

In this form, the relation R from set A to set B is represented as R = {(a, b): a ∈ A, b ∈ B, a...b}, the blank space is replaced by the rule which associates a and b.

Let R = {(2, 4), (4, 6), (6, 8), (8, 10) then R in the set builder form, it can be written as

R = {a, b} : a ∈ A, b ∈ B, *a* is 2 less than *b*}

iii. Arrow diagram:

● Draw two circles representing Set A and Set B.

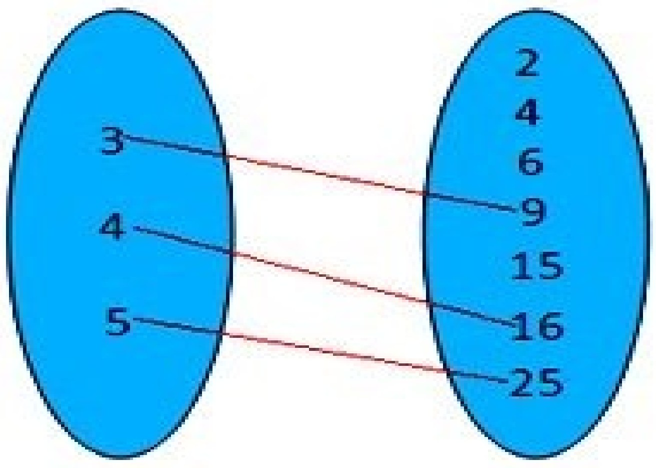
● Write their elements in the corresponding sets, i.e., elements of Set A in circle A and elements of Set B in circle B.

● Draw arrows from A to B which satisfy the relation and indicate the ordered pairs.

For Example:

1. If A = {3, 4, 5} B = {2, 4, 6, 9, 15, 16, 25}, then relation R from A to B is defined as 'is a positive square root of' and can be represented by the arrow diagram as shown.

Here R = {(3, 9); (4, 16); (5, 25)}



3 R 9 is called as '**3 related as R with 9**'

Here **9** is called **image** of **3**, and

3 is called **preimage** of **9**

Problem: If A = {2, 3, 4, 5} and B = {1, 3, 5} and R be the relation 'is less than' from A to B Then represented the Relation R in (i) Roster form (ii) Set builder form (iii) Arrow diagram

Given that $A = \{2, 3, 4, 5\}$, $B = \{1, 3, 5\}$

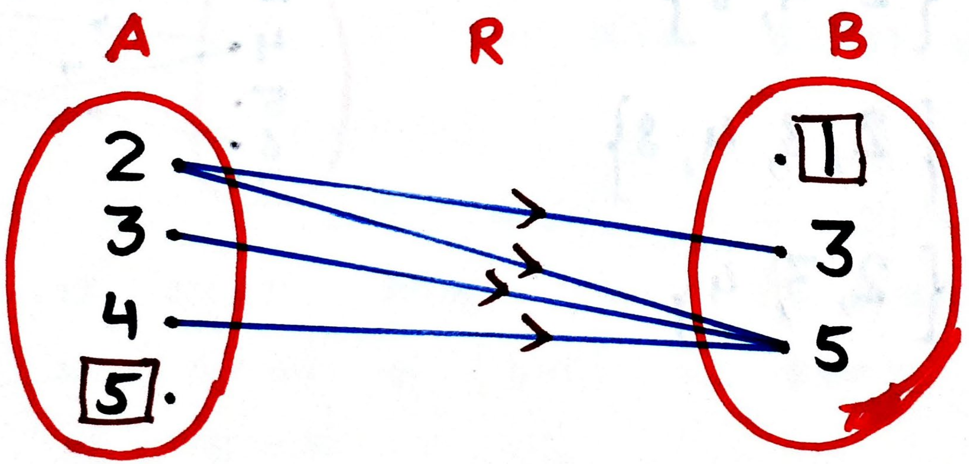
And Relation R 'is Less then' from $A \rightarrow B$

(i) Roster Form

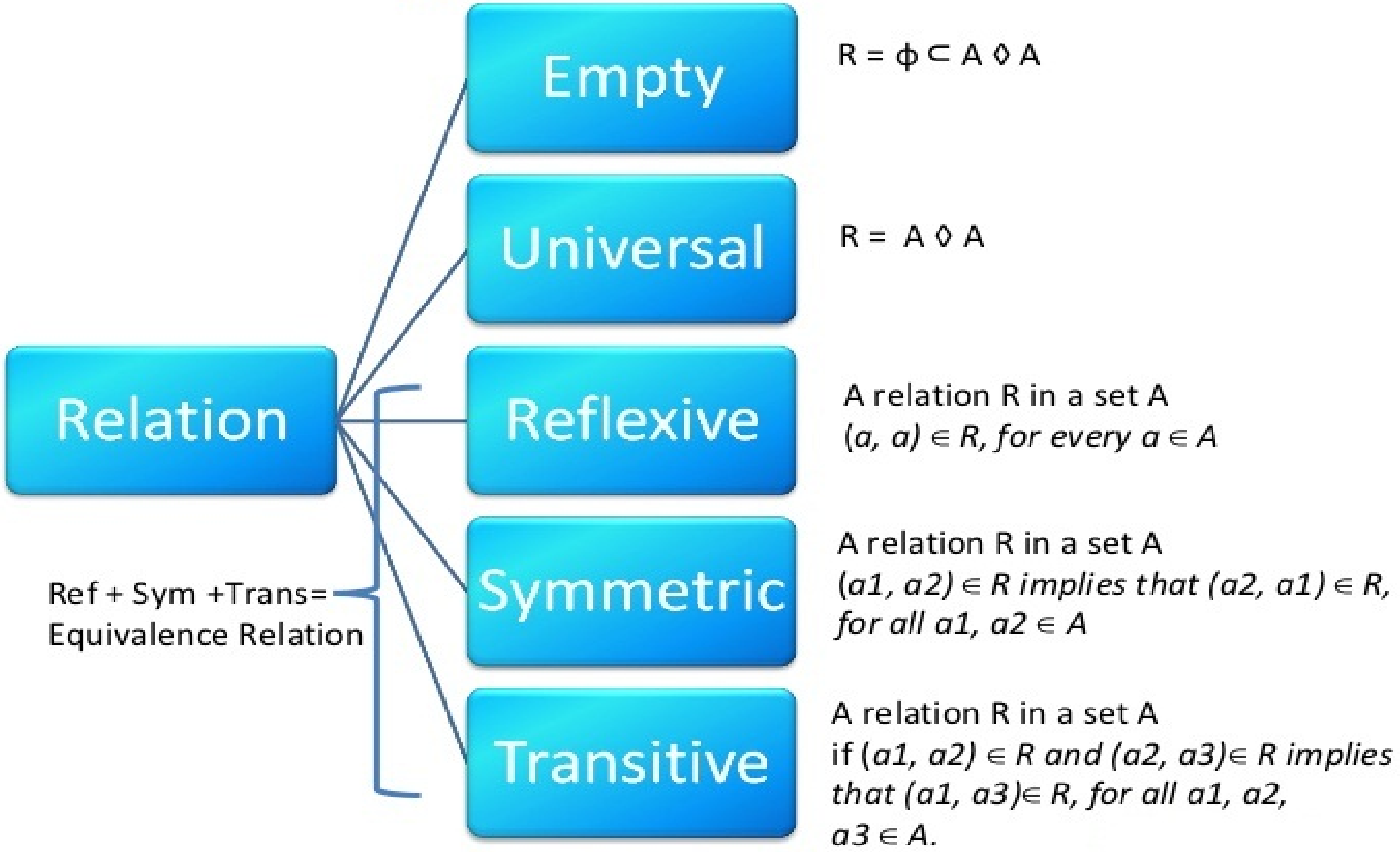
$$R = \{(2,3), (2,5), (3,5), (4,5)\}$$

(ii) Set builder Form $R = \{(a,b) : a \in A, \& \ b \in B, \ a < b\}$

(iii)



Types of Relations



Reflexive Relation:

Reflexive relation on set is a binary element in which every element is related to itself.

Let A be a set and R be the relation defined in it.

R is set to be reflexive, if (a, a) ∈ R for all a ∈ A that is, every element of A is R-related to itself, in other words aRa for every a ∈ A.

A relation R in a set A is not reflexive if there be at least one element a ∈ A such that (a, a) ∉ R.

For example A relation R is defined on the set Z by "aRb if a – b is divisible by 5" for a, b ∈ Z.

Let a ∈ Z. Then a – a is divisible by 5. Therefore a R a holds for all a in Z i.e. R is reflexive.

Symmetric Relation:

Let A be a set in which the relation R defined. Then R is said to be a symmetric relation, if $(a, b) \in R \Rightarrow (b, a) \in R$, that is, $aRb \Rightarrow bRa$ for all $(a, b) \in R$.

Consider, for example, the set A of natural numbers. If a relation A be defined by “$x + y = 5$”, then this relation is symmetric in A, for $a + b = 5 \Rightarrow b + a = 5$

But in the set A of natural numbers if the relation R be defined as ‘x is a divisor of y’, then the relation R is not symmetric as 3R9 does not imply 9R3; for, 3 divides 9 but 9 does not divide 3.

Transitive Relation:

Let A be a set in which the relation R defined. R is said to be transitive, if

$$(a, b) \in R \text{ and } (b, a) \in R \Rightarrow (a, c) \in R,$$

That is aRb and bRc $\Rightarrow$ aRc where $a, b, c \in A$.

The relation is said to be non-transitive, if

$$(a, b) \in R \text{ and } (b, c) \in R \text{ do not imply } (a, c) \in R.$$

For example, in the set A of natural numbers if the relation R be defined by ‘x less than y’ then

$$a < b \text{ and } b < c \text{ imply } a < c, \text{ that is, } aRb \text{ and } bRc \Rightarrow aRc.$$

Hence this relation is transitive.

Equivalence Relation:

Equivalence relation on set is a relation which is reflexive, symmetric and transitive.

A relation R, defined in a set A, is said to be an equivalence relation if and only if

(i) R is reflexive, that is, aRa for all a ∈ A.

(ii) R is symmetric, that is, aRb ⇒ bRa for all a, b ∈ A.

(iii) R is transitive, that is aRb and bRc ⇒ aRc for all a, b, c ∈ A.

Problem. A relation R is defined on the set Z by "a R b if a – b is divisible by 5" for a, b ∈ Z. Examine if R is an equivalence relation on Z.

Solution:

(i) Let a ∈ Z. Then a – a is divisible by 5. Therefore aRa holds for all a in Z and R is reflexive.

(ii) Let a, b ∈ Z and aRb hold. Then a – b is divisible by 5 and therefore b – a is divisible by 5. Thus, aRb ⇒ bRa and therefore R is symmetric.

(iii) Let a, b, c ∈ Z and aRb, bRc both hold. Then a – b and b – c are both divisible by 5. Therefore a – c = (a – b) + (b – c) is divisible by 5.

Thus, aRb and bRc ⇒ aRc and therefore R is transitive.

Since R is reflexive, symmetric and transitive so, R is an equivalence relation on Z.

Problem: Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L1, L2) : L1 is parallel to L2}. Show that R is an equivalence relation.

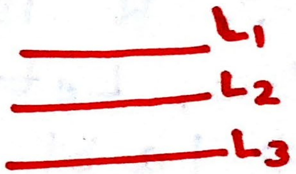
$R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$

R is reflexive as any line L_1 paralled to itself.
i.e. $(L_1, L_1) \in R$

Now Let $(L_1, L_2) \in R$

$\Rightarrow$ L_1 is parallel to L_2
$\Rightarrow$ L_2 is parallel to L_1
$\Rightarrow$ $(L_2, L_1) \in R$
$\therefore$ R is symmetric

Now Let (L_1, L_2) $(L_2, L_3) \in R$



$\Rightarrow$ L_1 is parallel to L_2 & L_2 is parallel to L_3
$\Rightarrow$ L_2 is parallel to L_1 & L_3 is parallel to L_2
$\Rightarrow$ L_1 is parallel to L_3
$\therefore$ R is transitive

Hence R is an equivalence relation

Problem: Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(*a*, *b*) : |*a* – *b*| is even}, is an equivalence relation.

Given that $A = \{1, 2, 3, 4, 5\}$

& $R = \{(a, b) : |a-b| \text{ is even}\}$

For any element $a \in A$. We have

$|a-a| = 0$ (an even number)

$\therefore$ R is reflexive

Let $(a, b) \in R$

$\Rightarrow |a-b|$ is even

$\Rightarrow |-(b-a)| = |b-a|$ is also even $\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric

Now, Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even

$\Rightarrow (a-c) = (a-b) + (b-c)$ is even

$\Rightarrow |a-c|$ is even

$\Rightarrow \therefore$ R is transitive.

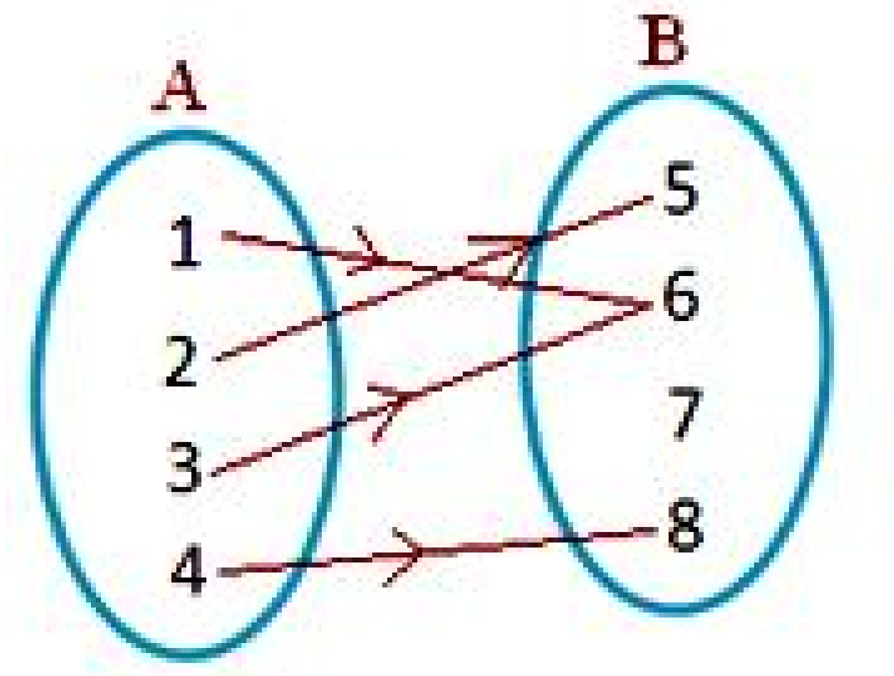
Hence R is equivalence relation

Domain Co-domain and Range of Relation

Domain: what can go *into* a Relation

Codomain: what *may possibly come out* of a Relation

Range: what *actually comes out* of a Relation



Domain = {1, 2, 3, 4}

Codomain = {5,6,7,8}

Range = {5,6,8}

Problem: Given that A = {2, 4, 5, 6, 7}, B = {2, 3,4}. R is a relation from A to B defined by R = {(a, b) : a ∈ A, b ∈ B and a is divisible by b}

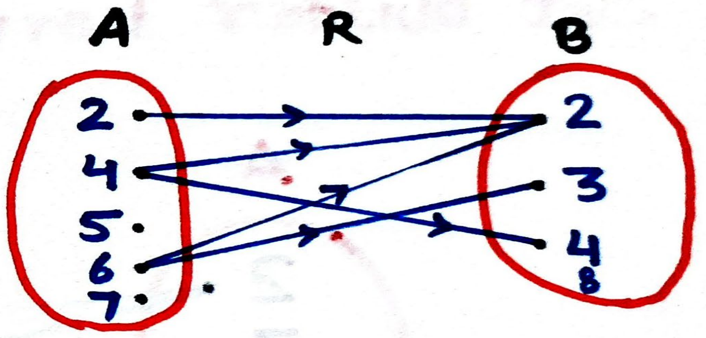
find (i) R in the roster form (ii) Represent R by arrow diagram. (iii) Domain of R (iv) Codomain of R (v) Range of R

Solution: (i) R in roster form

Given that A = {2, 4, 5, 6, 7} And B = {2, 3, 4, 8}

R = {(a, b) : a ∈ A, b ∈ B & a is divisible by b}

= {(2, 2), (4, 2), (4, 4), (6, 2), (6, 3)}



Domain of R = {2, 4, 6}

Codomain of R = {2, 3, 4, 8}

Range of R = {2, 3, 4}