✡ Derivation of Distance Formula in 2D Space

Given: 2 points $A(x_1, y_1)$ and $B(x_2, y_2)$ in 2D space with $O(0,0)$ as origin.

To Derive:

$$\boxed{AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

Construction: i) Let us construct dotted lines, i.e, perpendicular lines from B to G, C to F, A to D and C to E such that AF and BE intersect at C.

ii) We have,

$OF = x_1$, $OG = x_2$, $OE = y_2$, $OD = y_1$

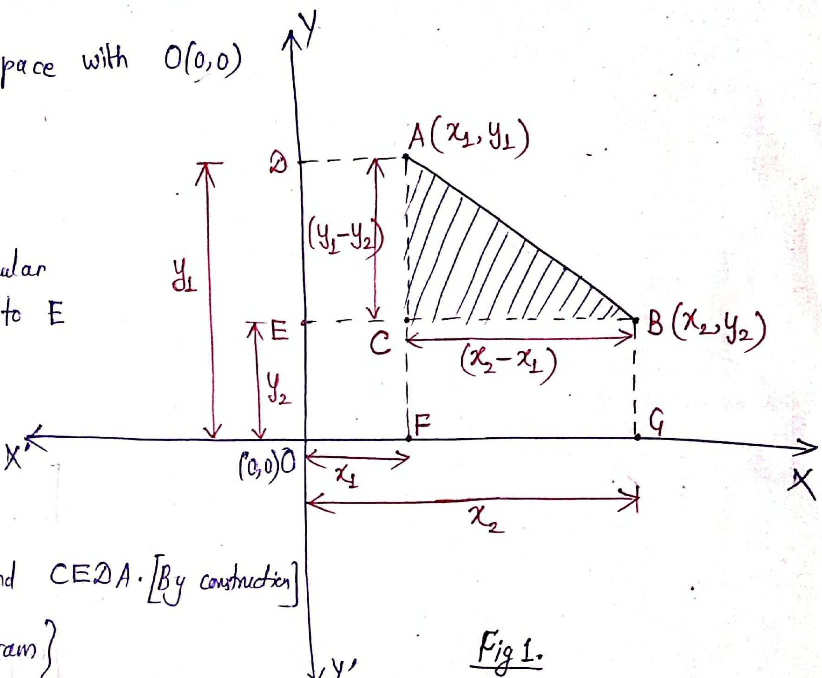


Fig 1.

Derivation:

Since, we have the parallelograms CBGF and CEDA. [By construction]

$\Rightarrow$ CB = FG
& CA = ED
$\left\{ \text{Opposite sides of parallelogram are equal} \right\}$ —— (I)

From Fig 1 :-

$$FG = OG - OF$$

$$\Rightarrow \boxed{FG = x_2 - x_1} \text{ —— (i)} \quad [\text{By construction}]$$

Also, $DE = OD - OE$

$$\Rightarrow \boxed{DE = y_1 - y_2} \text{ —— (ii)} \quad [\;\;"\;\;]$$

From (I), (i) and (ii) :-

$$CB = FG = (x_2 - x_1) \text{ —— (iii)}$$

$$\& \;\; CA = DE = (y_1 - y_2) \text{ —— (iv)}$$

So, we have:

$$\left.\begin{aligned} AC &= (y_1 - y_2) \\ \& \;\; BC &= (x_2 - x_1) \end{aligned}\right\} \text{ —— (v)}$$

Now, In fig I, We have the right angled ΔABC.

By Pythagoras Theorem :—

$$AB^2 = AC^2 + BC^2$$

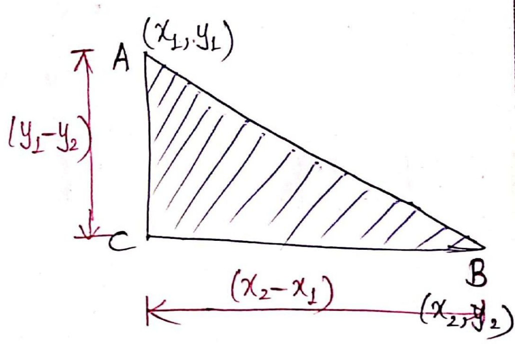
$$\Rightarrow AB = \sqrt{AC^2 + BC^2}$$

$$\Rightarrow AB = \sqrt{(y_1 - y_2)^2 + (x_2 - x_1)^2} \quad \{\text{from (v)}\}$$

$$\Rightarrow AB = \sqrt{[-(y_2 - y_1)]^2 + (x_2 - x_1)^2} \quad \{\text{Rearranging terms}\}$$

$$\Rightarrow AB = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Hence, $\boxed{AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$



Derived