PROBABILITY- STUDY NOTES

PROBABILITY- *the method of finding the likelihood of an event.*

1. For example let us throw a dice,an unbiased dice that is all faces have equal chances of turning up - 1,2,3,4,5,6 on six faces. What is the probability that the number up will be an even number?
   sample space=1,2,3,4,5,6=> n(s)=6
   even number or event space= 2,4,6=> n(e)=3
   n()=number of elements
   therefore required probability= n(e)/n(s)=3/6=1/2.

   P(E)=N(E)/N(S)

2. Now unbiased 2 dies are thrown simultaneously. What is the probability that sum of their faces is 8?
   we know that the minimum value on each die is 1 and maximum value is 6.
   we also know that we can get a sum of 8 with integers from 1 to 6 as=3+5/4+4
   so the number of ways in which the required sum of 8 can face up is=

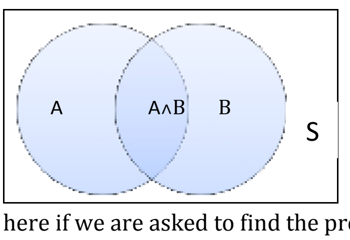
Dice1	Dice2
4	4
5	3
3	5

   so the required number of cases for or event= 4/4,3/5,5/3= 3 cases.
   total number of cases (from method of counting)=6*6=36
   P(E)=3/36=1/12

   2nd method- what is the probability that 4 will turn up in die 1? It is 1/6 as n(e)=1 & n(s)=6. Similarly for every other number on an unbiased die.
   A sum of 8 will be obtained when we get –{(4 on d1) &(4 on d2)} OR{ (3 on d1) &(5 on d2)} OR {(5 on d1) & (3 on d2)} as listed above.
   Therefore the required probability is obtained when we replace & by X (multiplication) and OR by + (addition). And we write the probability for every individual event in the bracket terms.
   so P(e)= { 1/6*1/6}+{1/6*1/6}+{1/6*1/6)=1/12.

3. Consider again an unbiased die .
   n(even)=3 since cases{2,4,6)
   n(odd)=3 since cases (1,3,5)
   As set of cases of odd number and set of cases of even number has no element common even if their number of elements is same –these two sets are said to be mutually exclusive.
   Hence we say that the two events of getting odd numbers and even numbers when an unbiased dice is thrown is mutually exclusive to each other.

4. *N(AUB)=N(A)+N(B)-N(A∧B)* from SET theory : S = sample space ,in this case represents all possible outcomes- the whole box.



   here if we are asked to find the probability that either A or B occurs (it means atleast one of A or B) , we will take help of the above concept of *vein diagram* as given above.
   P(e)= n(aUb)/n(s)

5. Consider the case of an unbiased die again.
Let A be the case of getting a prime ∴ N(A)=(2,3,5)=3 ∴P(A)=N(A)/N(S)=3/6=1/2.
Let B be the case of getting an even no ∴ N(B)=(2,4,6)=3 ∴P(B)=N(B)/N(S)=3/6=1/2.
What if we are asked to find out the probability of event A in case it is given that event B has already occurred?
In this case we find the the outcomes of B are the that is n(b)=3. In these outcomes we are to find the prime numbers that are in set A to get our event space. So our event space n(e)=n(A∧B)={2}=1 and our sample space n(b)=3.
So the req P(e)= N(A∧B)/N(B)= 1/3.
This is written as P(A/B)- probability that A will occur given B has already occurred.
Now as req event space is A∧B; We write **P(A/B)=P(A∧B)/P(B).** This known as *conditional probability*.
This can be derived as follows- We know P (A/B)=N(A∧B)/N(B)= {N(A∧B)N(S)}/{N(B)/N(S)}=
P(A/B)=P(A∧B)/P(B).

6. Consider set S ={1,2,3,4………47,48,…..50}
A= Event of getting a multiple of 5 from S= { 5,10,15,……,50}
B= Event of getting a multiple of 2 from S ={2,4,6,8………..50}
now P(A)=10/50 , P(A/B)=5/25
As we see that <u>numerically P(A)=P(A/B)</u> that is the probability of event A has not changed given the condition that B has already occurred , we say that A & B are **independent events.** As we know that P(A/B)=P(A∧B)/P(B). Putting the condition for **independent events that is P(A)=P(A/B) we get P(A).P(B)=P(A∧B)**
As A & B are independent events as proven we can also show P(A/$\overline{B}$)=P(A).

N:B- If P(A∧B)=P(A).P(B) numerically, A &B are pairwise independent.
Similarly if P(A∧B∧C)=P(A).P(B).P(C) , A,B & C are mutually independent.

7. BAYES THEOREM of Conditional Probability
If we are given the probabilities that B occurs given A1, A2…..An occurs and we are asked to find the probability that Ai will occur given B has already occurred, when we are given or can calculate P(Ai), we use Bayes theorem.
Note that A1 A2 A3 …..An are mutually exclusive and collectively exhaustive that is individual event spaces don't have any common and all As make up A event space.
Given, Mathematically if P(B/Ai)=Qi and P(Ai)=Pi then
P(Ai/B)= Pi.Qi/ΣPi.Qi

Example-
Box 1 contains 3 white and 2 black balls. Box 2 contains 1 white and 4 black balls.A ball is picked from one of the two boxes, it turns out to be black. Find the probability that it was drawn from box 1.

We are to find probability of selecting box 1 given we have picked a black ball=P(B1/BLACK)
By BAYES THEOREM,
P(B1/BLACK)={P(B1).P(BLACK/B1)}/[{ P(B1).(BLACK/B1)}+ {P(B2).P(BLACK/B2)}]
Hence,
P(BLACK/B1)=Probability of drawing a black ball from box 1= 2/5
P(BLACK/B2)=Probability of drawing a black ball from box 2= 4/5
P(B1)=1/2
P(B2)=1/2
∴ P(B1/BLACK)= (1/2)(2/5)/[{(1/2)(2/5)}+{(1/2)(4/5)}]=1/3

™<u>Solution of Bayes theorem problem without Bayes theorem</u>
Consider the given problem above.

Note that since we are given that a black ball has been picked already that reduces out sample space probability to selecting bag 1 and black ball or selecting bag 2 and black ball while the former is our event space probability. So basically it reduces to the probability of the way or ways we can get the desired outcome or outcomes divided by the summation of all the probabilities of available outcomes , given the pre condition.

Probability of selecting box 1 given we have picked a black ball=
P(B1/BLACK)= event space probability P(e)/ sample space probability P(s) = event of selecting black ball from B1/ events of selecting black balls from B1 & B2=event of selecting B1 and black ball from it/[event of selecting B1 and black ball from it OR event of selecting B2 and black ball from it]= 1/3

8. Expected Value= E1.M1+E2.M2+…….+En.Mn
Example-
A person tosses a coin. It comes up with head he gets 10 rupees. It comes up with tail he has to pay 5 rupees. What is his expectation ?
E=P(heads)M(heads)+P(tails)M(tails)= (1/2)10+(1/2)(-5)=2.5

TUTORIAL PROBLEMS **10Questions;30minutes**

1. Two fair dice are thrown .What is the probability that one dice shows up number greater than 4 and the other shows up a number less than 3?

2. If 5 fair dice are thrown what is the probability of getting atmost 3 tails?

3. When two are cards are drawn simultaneously from a pack of cards, what is the probability that both are red or both are kings?

4. The probability of a snake being venomous is 0.5. The probability of a snake being not venomous or not an instant killer is 0.8. If it is known that a snake is venomous, find the probability that it is an instant killer.

5. The probability that a square selected at random from 8x8 chessboard is of size 3x3 is……..

6. A bag contains 9 white and 5 yellow balls. Another bag contains 6 white and 8 yellow balls.If one of the bags is selected at random and two balls are drawn at random from the bag, then probability that both the balls are white is……..

7. A) In a month of February in a non leap year , the probability that it will have 5 Saturdays is….
   B) In a month of February in a leap year , the probability that it will have 5 Saturdays is….

8. A cinema historian noted that for a brief period all movies released were either directed by Nolan or starred Bale. Also no movie directed by Nolan starred Bale. The probability that a movie is directed by Nolan is 0.5 and the probability it stars Bale is 0.5. the probability that the movie is a hit if directed by Nolan is 0.6, while the probability that the movie is a hit given that Bale acted in it is 0.4.Given that the movie is a hit, find the probability that it is directed by Nolan.

9. A game involving a biased die is such that 5rs is paid each time the die shows up a 3 while 8 is paid on every other score of a die.The die is such that the score of 3 occurs 4 times as frequently as any other score. How much a person be willing to pay each time as entry fee if he expects neither loss nor profit in the long run?

10. A and b are two positive numbers such that $a<4$,$b<4$. Find the probability that $2a+3b<14$.