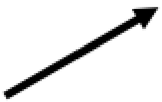
Vector: Physical quantities having magnitude, direction and obeying laws of vector algebra.

Geometrically, a vector is represented by an arrow; the arrow defines the direction of the vector and the magnitude of the vector is represented by the length of the arrow.



Example:

Displacement, velocity, acceleration, momentum, force, impulse, weight, thrust, torque, angular momentum, angular velocity *etc*.

If a physical quantity in addition to magnitude :

(a) Has a specified direction.

(b) Obeys the law of parallelogram of addition, i.e., $R = (A^2 + B^2 + 2AB\cos\theta)^{1/2}$

(c) And its addition is commutative, i.e., $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

then and then only it is said to be a vector. If any of the above conditions is not satisfied the physical quantity cannot be a vector.

Types of Vector

(1) Equal vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(2) Parallel vector: $\vec{A}$ and $\vec{B}$ are said to be parallel If,

(i) Both have same direction.

(ii) One vector is scalar (positive) non-zero multiple of another vector.

(3) Anti-parallel vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when,

(i) Both have opposite direction.

(ii) One vector is scalar non-zero negative multiple of another vector.

(4) Collinear vectors: When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear.

(5) Zero vector ($\vec{0}$): A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

(6) Unit vector: A vector divided by its magnitude is a unit vector.

Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap or A hat).

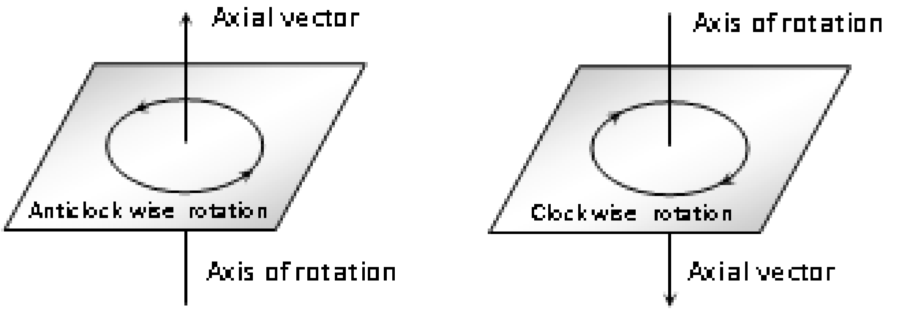
Since, $\hat{A} = \frac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

(7) Polar vectors: These have starting point or point of application. Example displacement and force *etc*.

Types of Vector

(8) Axial Vectors: These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.

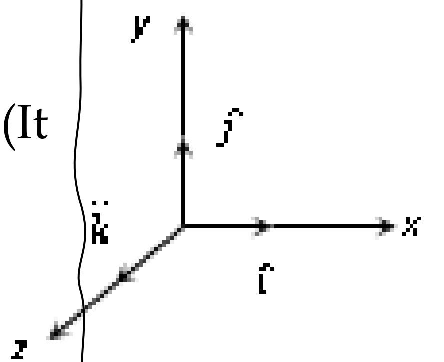


(10) Coplanar vector: Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

(9) Orthogonal unit vectors: $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right-Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb).

$$\hat{i} = \frac{\vec{x}}{x}, \hat{j} = \frac{\vec{y}}{y}, \hat{k} = \frac{\vec{z}}{z}$$

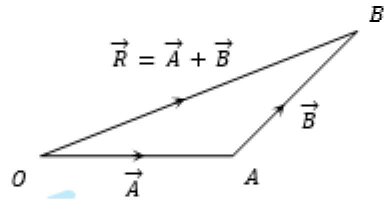
$$\therefore \quad \vec{x} = x\hat{i},\ \vec{y} = y\hat{j},\ \vec{z} = z\hat{k}$$



Triangle Law of Vector Addition of Two Vectors

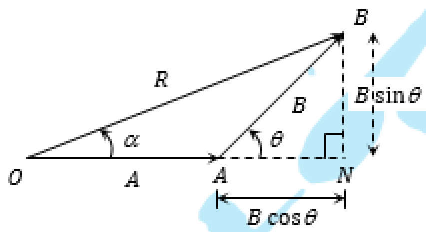
If two non-zero vectors are represented by the two sides of a triangle taken in same order, then the resultant is given by the closing side of triangle in opposite order. *i.e.*,

$$\vec{R} = \vec{A} + \vec{B}$$



$$\because \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

(1) Magnitude of resultant vector



In ΔABN, $\cos\theta = \frac{AN}{B}$ $\therefore AN = B\cos\theta$

$\sin\theta = \frac{BN}{B}$ $\therefore BN = B\sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$$\Rightarrow R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$\Rightarrow R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$
$$\Rightarrow R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$
$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$
$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

(2) Direction of resultant vectors:

If θ is angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

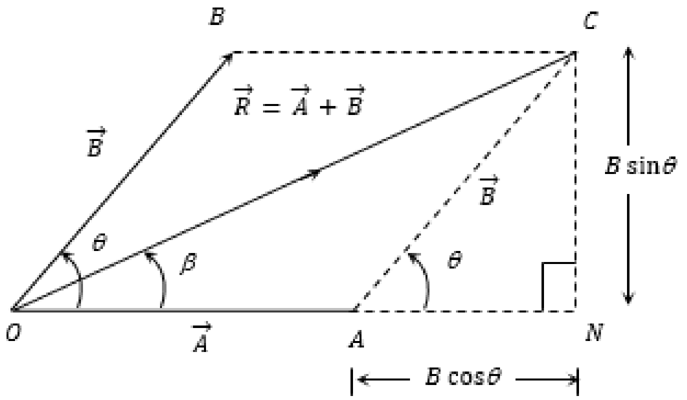
$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Parallelogram Law of Vector Addition

If two non-zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(1) Magnitude



Since, $R^2 = ON^2 + CN^2$

$\Rightarrow R^2 = (OA + AN)^2 + CN^2$

$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$

$\therefore\ R = |\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

Special cases:

$R = A + B$ when $\theta = 0°$

$R = A - B$ when $\theta = 180°$

$R = \sqrt{A^2 + B^2}$ when $\theta = 90°$

(2) Direction:

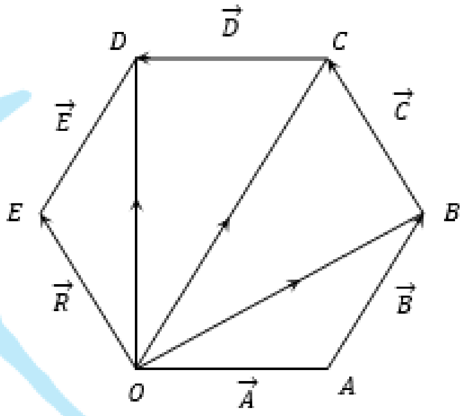
$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

Polygon Law of Vector Addition

If a number of non-zero vectors are represented by the $(n - 1)$ sides of an n-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order.

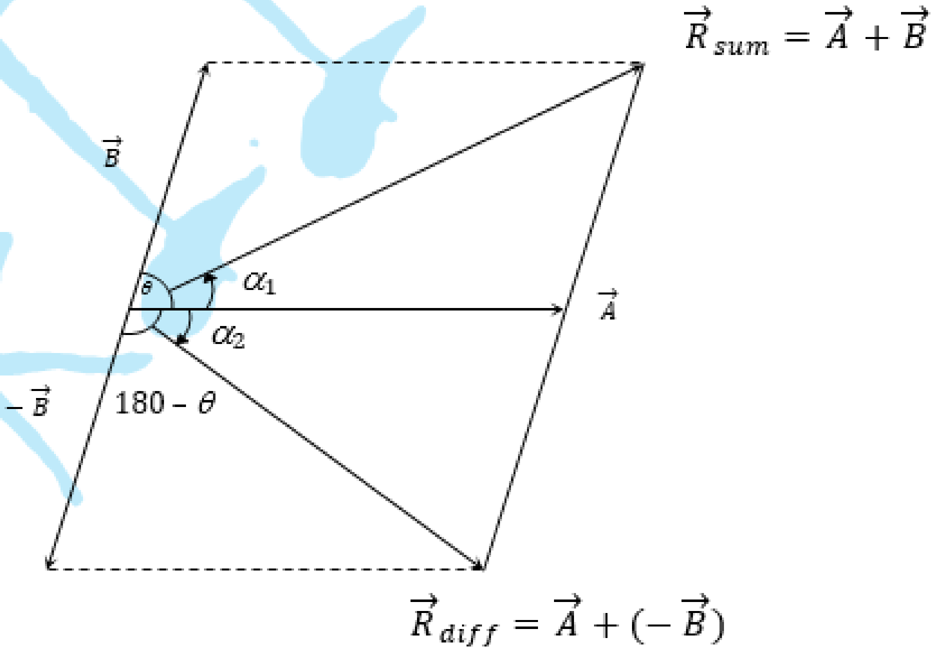
So,

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$
$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$$



- ★ Resultant of two unequal vectors cannot be zero.
- ★ Resultant of three co-planar vectors may or may not be zero.
- ★ Resultant of three non-coplanar vectors cannot be zero.

Subtraction of vectors



Since, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and

$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^o - \theta)}$

Since, $\cos(180 - \theta) = -\cos\theta$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta}$$

and $\tan\alpha_2 = \dfrac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$

But $\sin(180-\theta) = \sin\theta$ and $\cos(180-\theta) = -\cos\theta$

$$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$