PERMUTATIONS

1. The total number of 6-digit numbers in which the sum of the digits is is divisible by 5
   (A) 21000 (B) 20000 (C) 18000 (D) 16000
2. The number of five digit numbers which are even formed by using the digits 0,1,2 and 3 is
   (A) 265 (B) 482 (C) 681 (D) 384
3. Between two junctions A and B there are 12 intermediate stations. The number of ways in which a train can halt at 4 of these stations so that no two of these halts are consecutive
   (A) C (12,4) (B) C (9,4) (C) C (11,5) (D) C (9,3)
4. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is
   (A) 150 (B) 240 (C) 126 (D) 225
5. The total number of 9 digit numbers which have all different digits is
   (A) 10! (B) 9! (C) 9 x 9! (D) 9999
6. Set A contains $n \geq 3$ elements. The number of ttriplets (x,y,z) of the elements of set A in which at least two coordinates are equal is
   (A) ${}^{n}P_3$ (B) $n^3 - {}^{n}P_3$ (C) $n^3 - 2.{}^{n}P_3$ (D) $n^3 - 2.{}^{n}P_3 + {}^{n}P_1$
7. The number of times the digit 3 will be written when listing all the intergers between 1 and 1000
   (A) 333 (B) 331 (C) 301 (D) 300
8. Let Set $P = \{x : x \text{ is prime number and } x < 30\}$. The number of different rational numbers whose numerator and denominator belong to P is
   (A) 100 (B) 31 (C) 91 (D) 90.
9. The number of arrangments of the letters of the word BHARAT taken 3 at a time
   (A) 68 (B) 72 (C) 98 (D) 84
10. The letters of the word PROMPT are arranged all possible ways. The words so formed are arranged as in a dictionary. Then the rank of the word PROMPT is
    (A) 198 (B) 199 (C) 197 (D) 200.

Permutations(Hints and solutions)

1. Ans:C Hint: In the 6 digit number the first five places can be filled in 9 x10x10x10x10 ways.
The sum of the first five digts can be of the form 5m,5m+1,5m+2,5m+3 or 5m+4
The possible digits taken by the last digits can be .

Sum of the digits	last digits
5m	0 or 5
5m+1	4 or 9
5m+2	3 or 8
5m+3	2 or 7
5m+4	1 or 6

Therefore corresponding to each way of filling first five places there are two ways of filling the last digit. Hence the required number of ways is 9 x 10 x10x10x10x2 = 18000

2. An even number will have 0 or 2 in the units place. So the units place can be filled in 2 ways. The remaining four places with four digits can be filled in 3 x 4 x 4 x 4 ways. Here the first place can be filled in 3 ways only. Hence the totalnumber of numbers is 2 x 3 x 4 x 4 x 4 = 384
3. Ans:B Hint: The train stops at 4 of the stations. So it does not stop at 8 of the stations. So there are 9 halts in which it has to stop at 4 stations. This can be done in C(9,4) ways.
4. Ans: 150. Here the distribution is equal to the number of onto functions that can be formed from set containing 5 elements to set containing 3 elements.

$= 3^5 - {}^3C_1.2^5 + {}^3C_2.1^5 = 243 - 96 + 3 = 246 - 96 = 150$

5. Ans: C. The first can be filled with any one of the 9 non zero digits in 9 ways. The remaining 8 places with 9 remaingin digits in P(9,8) ways. Hence the total number of ways is
9 x P (9,8) = 9 x 9!.
6. Ans: B number of triplets = total number of triplets - number of triplets in which no two are equal.
7. Ans: D . Since 1000 does not contain 3 we have count between 1 and 999. any nunber between 1 and 999 is of the form xyz where $0 \le x, y, z \le 9$. The number of times in which 3 occupies exactly one place is 3 x 9 x 9 = 243 ways. Now 3 occurs exactly in two places in C(3,2) x 9 ways. 3 three occurs three times is only once.
Total 1 x 243 + 2 x 3 x 9 + 3 x1 = 243+54+3 = 300.
8. Ans:C. The prime numbers less than 30 are 2,3,5,7,11,13,17,19,23,29. To from a fraction we need two digits . So from these 10 2 an be picked in P(10,2) ways. Total is 90+1 (including 1)
9. Ans: B . The word contains 6 letters out of which 2A's,B,H,R,T.
To form 3 letter word the possibilities are

(i) 2A's and one from remaining 4 letters $= 4 \text{ x } \frac{3!}{2!} = 12$

(ii) one A and two from the remaining $4 = {}^4P_2 \times 3! = 36$

(iii) zero A's and all the three from B,H,R,T = 24

total = 12 = 36+ 24 = 72

10. Ans: B.

Words starting with M =60, starting with 0 =60, starting with PM ,PO,PP = 3 X 24 =72
Words starting with PRM = 3! = 6. Next word is PROMPT.
Total = 60+60+72+6+1= 199