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Network Analysis

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Network Analysis
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To Our Parents

Preface to the Series

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !
R. K. Kanodia
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SYLLABUS

GATE Electronics & Communications

Networks:

Network graphs: matrices associated with graphs; incidence, fundamental cut set and fundamental circuit matrices. Solution methods: nodal and mesh analysis. Network theorems: superposition, Thevenin and Norton's maximum power transfer, Wye-Delta transformation. Steady state sinusoidal analysis using phasors. Linear constant coefficient differential equations; time domain analysis of simple RLC circuits, Solution of network equations using Laplace transform: frequency domain analysis of RLC circuits. 2-port network parameters: driving point and transfer functions. State equations for networks.

IES Electronics & Telecommunication

Network Theory

Network analysis techniques; Network theorems, transient response, steady state sinusoidal response; Network graphs and their applications in network analysis; Tellegen's theorem. Two port networks; Z, Y, h and transmission parameters. Combination of two ports, analysis of common two ports. Network functions : parts of network functions, obtaining a network function from a given part. Transmission criteria : delay and rise time, Elmore's and other definitions effect of cascading. Elements of network synthesis.

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CHAPTER 3 GRAPH THEORY

CHAPTER 4 NODAL AND LOOP ANALYSIS

CHAPTER 15 TWO PORT NETWORK

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CHAPTER 5

CIRCUIT THEOREMS

5.1 INTRODUCTION

In this chapter we study the methods of simplifying the analysis of more complicated circuits. We shall learn some of the circuit theorems which are used to reduce a complex circuit into a simple equivalent circuit. This includes Thevenin theorem and Norton theorem. These theorems are applicable to linear circuits, so we first discuss the concept of circuit linearity.

5.2 LINEARITY

A system is linear if it satisfies the following two properties

Homogeneity Property

The homogeneity property requires that if the input (excitation) is multiplied by a constant, then the output (response) is multiplied by the same constant. For a resistor, for example, Ohm's law relates the input I to the output V,

$$V = IR$$

If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$kIR = kV$$

Additivity Property

The additivity property requires that the response to a sum of inputs is the sum of the responses to each input applied separately. Using the voltage-current relationship of a resistor, if

$$V_1 = I_1 R \qquad \text{(Voltage due to current } I_1\text{)}$$

and

$$V_2 = I_2 R \qquad \text{(Voltage due to current } I_2\text{)}$$

then, applying current $(I_1 + I_2)$ gives

$$V = (I_1 + I_2) R = I_1 R + I_2 R$$
$$= V_1 + V_2$$

These two properties defining a linear system can be combined into a single statement as

For any linear resistive circuit, any output voltage or current, denoted by the variable y, is related linearly to the independent sources(inputs), i.e.,

$$y = a_1 x_1 + a_2 x_2 + ... + a_n x_n$$

where $x_1, x_2 .... x_n$ are the voltage and current values of the independent sources in the circuit and a_1 through a_m are properly dimensioned constants.

Thus, a linear circuit is one whose output is linearly related (or directly proportional) to its input. For example, consider the linear circuit shown in figure 5.2.1. It is excited by an input voltage source V_s, and the current through load R is taken as output(response).

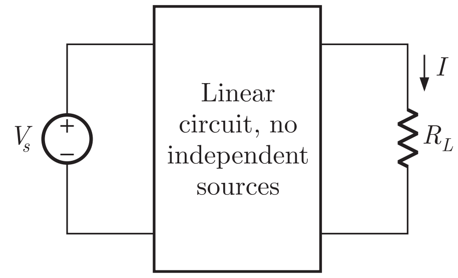


Fig. 5.2.1 A Linear Circuit

Suppose $V_s = 5\text{ V}$ gives $I = 1\text{ A}$. According to the linearity principle, $V_s = 10\text{ V}$ will give $I = 2\text{ A}$. Similarly, $I = 4\text{ mA}$ must be due to $V_s = 20\text{ mV}$. Note that ratio V_s/I remains constant, since the system is linear.

NOTE :
We know that the relationship between power and voltage (or current) is not linear. Therefore, linearity does not applicable to power calculations..

5.3 SUPERPOSITION

The number of circuits required to solve a network. using superposition theorem is equal to the number of independent sources present in the network. It states that

> In any linear circuit containing multiple independent sources the total current through or voltage across an element can be determined by algebraically adding the voltage or current due to each independent source acting alone with all other independent sources set to zero.

An independent voltage source is set to zero by replacing it with a 0 V source(short circuit) and an independent current source is set to zero by replacing it with 0 A source(an open circuit). The following methodology illustrates the procedure of applying superposition to a given circuit

METHODOLOGY

1. Consider one independent source (either voltage or current) at a time, short circuit all other voltage sources and open circuit all other current sources.
2. Dependent sources can not be set to zero as they are controlled by other circuit parameters.
3. Calculate the current or voltage due to the single source using any method (KCL, KVL, nodal or mesh analysis).
4. Repeat the above steps for each source.
5. Algebraically add the results obtained by each source to get the total response.

NOTE :
Superposition theorem can not be applied to power calculations since power is not a linear quantity.

5.4 SOURCE TRANSFORMATION

It states that an independent voltage source V_s in series with a resistance R is equivalent to an independent current source $I_s = V_s / R$, in parallel with a resistance R.

or

An independent current source I_s in parallel with a resistance R is equivalent to an independent voltage source $V_s = I_s R$, in series with a resistance R.

Figure 5.4.1 shows the source transformation of an independent source. The following points are to be noted while applying source transformation.

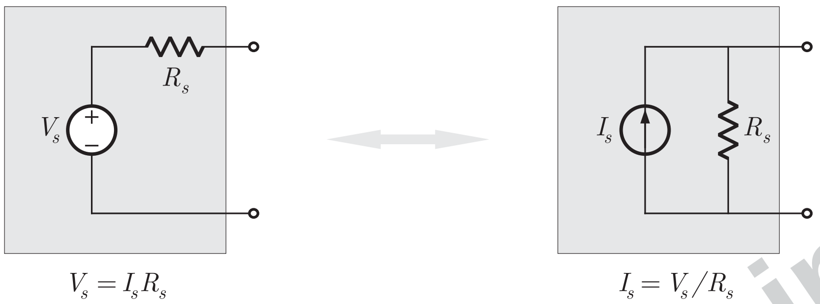


Fig. 5.4.1 Source Transformation of Independent Source

1. Note that head of the current source arrow corresponds to the +ve terminal of the voltage source. The following figure illustrates this

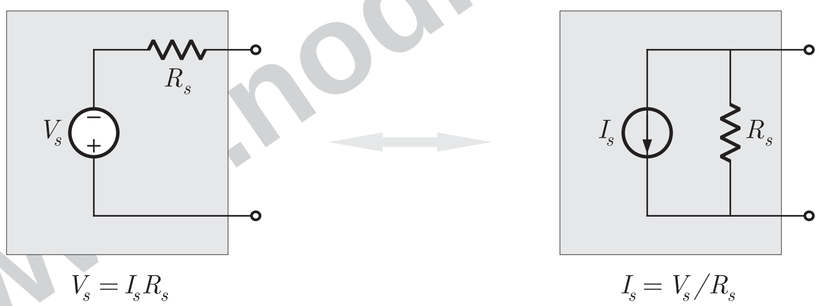


Fig. 5.4.2 Source Transformation of Independent Source

2. Source conversion are equivalent at their external terminals only i.e. the voltage-current relationship at their external terminals remains same. The two circuits in figure 5.4.3a and 5.4.3b are equivalent, provided they have the same voltage-current relation at terminals a-b

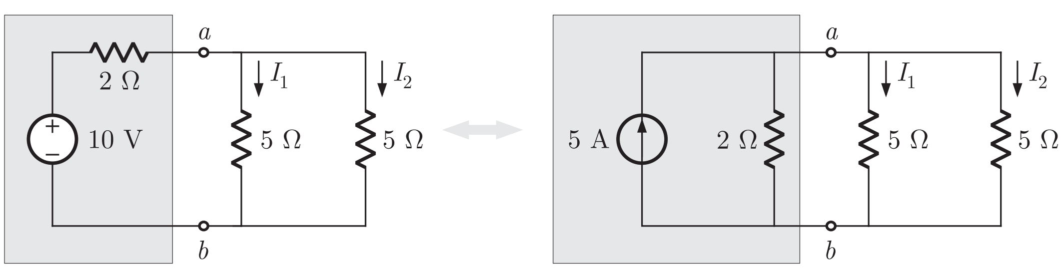


Fig. 5.4.3 An example of source transformation (a) Circuit with a voltage source (b) Equivalent circuit when the voltage source is transformed into current sources

3. Source transformation is not applicable to ideal voltage sources as $R_s = 0$ for an ideal voltage source. So, equivalent current source value $I_s = V_s / R \to \infty$. Similarly it is not applicable to ideal current source

because for an ideal current source $R_s = \infty$, so equivalent voltage source value will not be finite.

5.4.1 Source Transformation For Dependent Source

Source transformation is also applicable to dependent source in the same manner as for independent sources. It states that

An dependent voltage source V_x in series with a resistance R is equivalent to a dependent current source $I_x = V_x / R$, in parallel with a resistance R, keeping the controlling voltage or current unaffected.

or,

A dependent current source I_x in parallel with a resistance R is equivalent to an dependent voltage source $V_x = I_x R$, in series with a resistance R, keeping the controlling voltage or current unaffected.

Figure 5.4.4 shows the source transformation of an dependent source.

Fig. 5.4.4 Source Transformation of Dependent Sources

5.5 THEVENIN'S THEOREM

It states that any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source, V_{Th}, in series with an equivalent resistance, R_{Th} as illustrated in the figure 5.5.1.

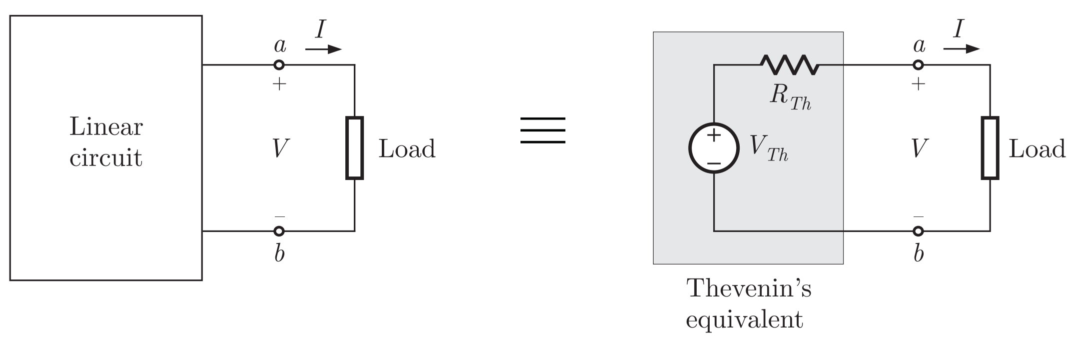


Fig. 5.5.1 Illustration of Thevenin Theorem

where V_{Th} is called Thevenin's equivalent voltage or simply Thevenin voltage and R_{Th} is called Thevenin's equivalent resistance or simply Thevenin resistance.

The methods of obtaining Thevenin equivalent voltage and resistance are given in the following sections.

5.5.1 Thevenin's Voltage

The equivalent Thevenin voltage (V_{Th}) is equal to the open-circuit voltage present at the load terminals (with the load removed). Therefore, it is also denoted by V_{oc}

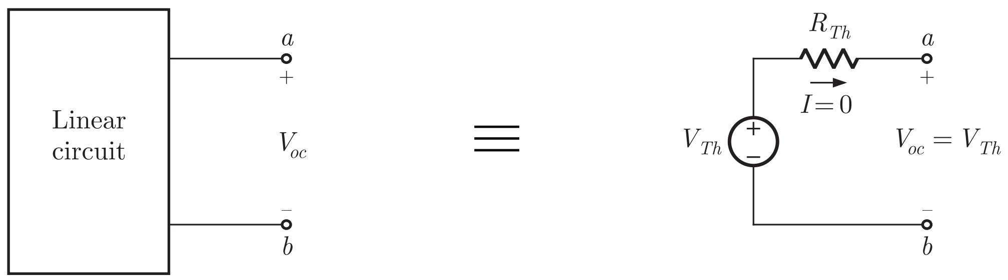


Fig. 5.5.2 Equivalence of Open circuit and Thevenin Voltage

Figure 5.5.2 illustrates that the open-circuit voltage, V_{oc}, and the Thevenin voltage, V_{Th}, must be the same because in the circuit consisting of V_{Th} and R_{Th}, the voltage V_{oc} must equal V_{Th}, since no current flows through R_{Th} and therefore the voltage across R_{Th} is zero. Kirchhoff's voltage law confirms that

$$V_{Th} = R_{Th}(0) + V_{oc} = V_{oc}$$

The procedure of obtaining Thevenin voltage is given in the following methodology.

METHODOLOGY 1

1. Remove the load i.e open circuit the load terminals.
2. Define the open-circuit voltage V_{oc} across the open load terminals.
3. Apply any preferred method (KCL, KVL, nodal analysis, mesh analysis etc.) to solve for V_{oc}.
4. The Thevenin voltage is $V_{Th} = V_{oc}$.

NOTE :
Note that this methodology is applicable with the circuits containing both the dependent and independent source.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then its open circuit voltage or Thevenin voltage will simply be zero.

NOTE :
For the Thevenin voltage we may use the terms Thevenin voltage or open circuit voltage interchangeably.

5.5.2 Thevenin's Resistance

Thevenin resistance is the input or equivalent resistance at the open circuit terminals a,b when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits).

We consider the following cases where Thevenin resistance R_{Th} is to be determined.

Case 1: Circuit With Independent Sources only

If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance or equivalent resistance of the network looking between terminals a and b, as shown in figure 5.5.3.

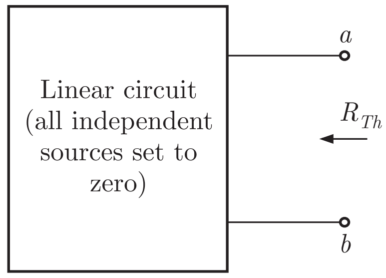


Fig 5.5.3 Circuit for Obtaining R_{Th}

Case 2: Circuit With Both Dependent and Independent Sources

Different methods can be used to determine Thevenin equivalent resistance of a circuit containing dependent sources. We may follow the given two methodologies. Both the methods are also applicable to circuit with independent sources only(case 1).

Using Test Source

METHODOLOGY 2

1. Set all independent sources to zero(Short circuit independent voltage source and open circuit independent current source).
2. Remove the load, and put a test source V_{test} across its terminals. Let the current through test source is I_{test}. Alternatively, we can put a test source I_{test} across load terminals and assume the voltage across it is V_{test}. Either method would give same result.
3. Thevenin resistance is given by $R_{Th} = V_{test}/I_{test}$.

NOTE :
We may use $V_{test} = 1\text{ V}$ or $I_{test} = 1\text{ A}$.

Using Short Circuit Current

$$R_{Th} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = \frac{V_{oc}}{I_{sc}}$$

METHODOLOGY 3

1. Connect a short circuit between terminal a and b.
2. Be careful, do not set independent sources zero in this method because we have to find short circuit current.
3. Now, obtain the short circuit current I_{sc} through terminals a, b.
4. Thevenin resistance is given as $R_{Th} = V_{oc}/I_{sc}$ where V_{oc} is open circuit voltage or Thevenin voltage across terminal a,b which can be obtained by same method given previously.

5.5.3 Circuit Analysis Using Thevenin Equivalent

Thevenin's theorem is very important in circuit analysis. It simplifies a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. The equivalent network behaves the same way externally as the original circuit. Consider a linear circuit terminated by a load R_L, as shown in figure 5.5.5. The current I_L through the load and the voltage V_L across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained.

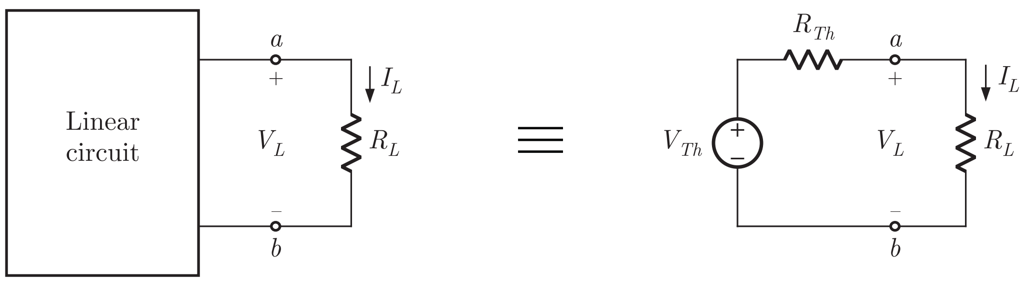


Fig. 5.5.5 A Circuit with a Load and its Equivalent Thevenin Circuit

Current through the load R_L

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Voltage across the load R_L

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

5.6 NORTON'S THEOREM

Any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source, I_N, in parallel with an equivalent resistance, R_N as illustrated in figure 5.6.1.

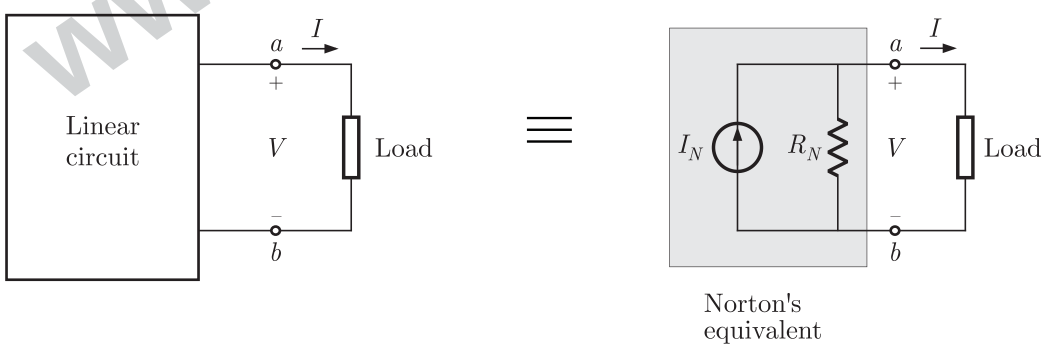


Fig. 5.6.1 Illustration of Norton Theorem

where I_N is called Norton's equivalent current or simply Norton current and R_N is called Norton's equivalent resistance. The methods of obtaining Norton equivalent current and resistance are given in the following sections.

5.6.1 Norton's Current

The Norton equivalent current is equal to the short-circuit current that would flow when the load replaced by a short circuit. Therefore, it is also called short circuit current I_{sc}.

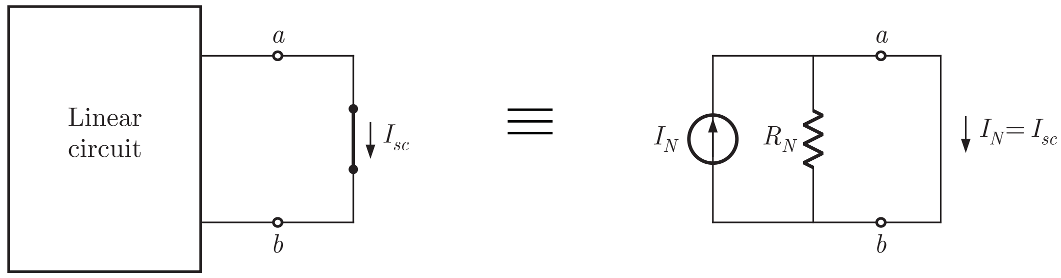


Fig 5.6.2 Equivalence of Short Circuit Current and Norton Current

Figure 5.6.2 illustrates that if we replace the load by a short circuit, then current flowing through this short circuit will be same as Norton current I_N

$$I_N = I_{sc}$$

The procedure of obtaining Norton current is given in the following methodology. Note that this methodology is applicable with the circuits containing both the dependent and independent source.

METHODOLOGY

1. Replace the load with a short circuit.
2. Define the short circuit current, I_{sc}, through load terminal.
3. Obtain I_{sc} using any method (KCL, KVL, nodal analysis, loop analysis).
4. The Norton current is $I_N = I_{sc}$.

If a circuit contains dependent sources only, i.e. there is no independent source present in the network then the short circuit current or Norton current will simply be zero.

5.6.2 Norton's Resistance

Norton resistance is the input or equivalent resistance seen at the load terminals when all independent sources are set to zero(voltage sources replaced by short circuits and current sources replaced by open circuits) i.e. Norton resistance is same as Thevenin's resistance

$$R_N = R_{Th}$$

So, we can obtain Norton resistance using same methodologies as for Thevenin resistance. Dependent and independent sources are treated the same way as in Thevenin's theorem.

NOTE :
For the Norton current we may use the term Norton current or short circuit current interchangeably.

5.6.3 Circuit Analysis Using Norton's Equivalent

As discussed for Thevenin's theorem, Norton equivalent is also useful in circuit analysis. It simplifies a circuit. Consider a linear circuit terminated by a load R_L, as shown in figure 5.6.4. The current I_L through the load and the voltage V_L across the load are easily determined once the Norton equivalent of the circuit at the load's terminals is obtained,

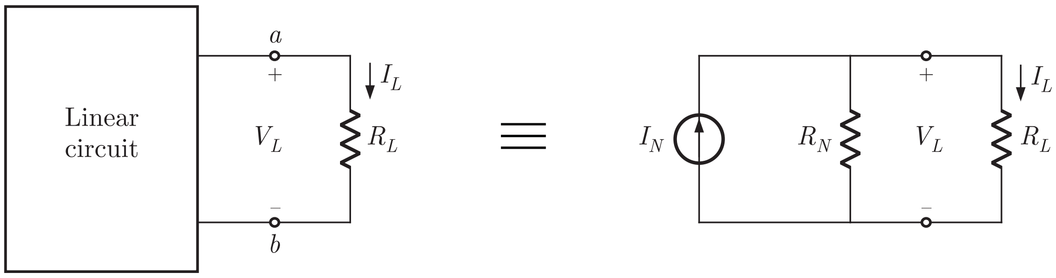


Fig. 5.6.4 A circuit with a Load and its Equivalent Norton Circuit

Current through load R_L is,

$$I_L = \frac{R_N}{R_L + R_L} I_N$$

Voltage across load R_L is,

$$V_L = R_L I_L = \frac{R_L R_N}{R_{Th} + R_L} I_N$$

5.7 TRANSFORMATION BETWEEN THEVENIN & NORTON'S EQUIVALENT CIRCUITS

From source transformation it is easy to find Norton's and Thevenin's equivalent circuit from one form to another as following

Fig. 5.7.1 Source Transformation of Thevenin and Norton Equivalents

5.8 MAXIMUM POWER TRANSFER THEOREM

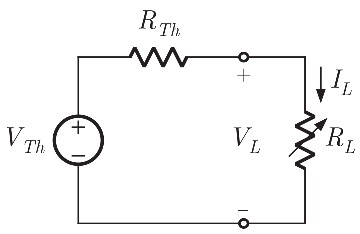
Maximum power transfer theorem states that a load resistance R_L will receive maximum power from a circuit when the load resistance is equal to Thevenin's/Norton's resistance seen at load terminals.

i.e. $R_L = R_{Th}$, (For maximum power transfer)

In other words a network delivers maximum power to a load resistance R_L when R_L is equal to Thevenin equivalent resistance of the network.

PROOF :

Consider the Thevenin equivalent circuit of figure 5.8.1 with Thevenin voltage V_{Th} and Thevenin resistance R_{Th}.



Fiig. 5.8.1 A Circuit Used for Maximum Power Transfer

We assume that we can adjust the load resistance R_L. The power absorbed by the load, P_L, is given by the expression

$$P_L = I_L^2 R_L \qquad (5.8.1)$$

and that the load current is given as,

$$I_L = \frac{V_{Th}}{R_L + R_{Th}} \qquad (5.8.2)$$

Substituting I_L from equation (5.8.2) into equation (5.8.1)

$$P_L = \frac{V_{Th}^2}{(R_L + R_{Th})^2} R_L \qquad (5.8.3)$$

To find the value of R_L that maximizes the expression for P_L (assuming that V_{Th} and R_{Th} are fixed), we write

$$\frac{dP_L}{dR_L} = 0$$

Computing the derivative, we obtain the following expression :

$$\frac{dP_L}{dR_L} = \frac{V_{Th}^2 (R_L + R_{Th})^2 - 2V_{Th}^2 R_L (R_L + R_{Th})}{(R_L + R_{Th})^4}$$

which leads to the expression

$$(R_L + R_{Th})^2 - 2R_L (R_L + R_{Th}) = 0$$

or $$R_L = R_{Th}$$

Thus, in order to transfer maximum power to a load, the equivalent source and load resistances must be matched, that is, equal to each other.

$$R_L = R_{Th}$$

The maximum power transferred is obtained by substituting $R_L = R_{Th}$ into equation (5.8.3)

$$P_{max} = \frac{V_{Th}^2 R_{Th}}{(R_{Th} + R_{Th})^2} = \frac{V_{Th}^2}{4R_{Th}} \qquad (5.8.4)$$

or, $$P_{max} = \frac{V_{Th}^2}{4R_L}$$

If the Load resistance R_L is fixed :

Now consider a problem where the load resistance R_L is fixed and Thevenin resistance or source resistance R_s is being varied, then

$$P_L = \frac{V_{Th}^2}{(R_L + R_s)^2} R_L$$

To obtain maximum P_L denominator should be minimum or $R_s = 0$. This can be solved by differentiating the expression for the load power, P_L, with respect to R_s instead of R_L.

The step-by-step methodology to solve problems based on maximum power transfer is given as following :

METHODOLOGY

1. Remove the load R_L and find the Thevenin equivalent voltage V_{Th} and resistance R_{Th} for the remainder of the circuit.
2. Select $R_L = R_{Th}$, for maximum power transfer.
3. The maximum average power transfer can be calculated using $P_{max} = V_{Th}^2 / 4R_{Th}$.

5.9 RECIPROCITY THEOREM

The reciprocity theorem is a theorem which can only be used with single source circuits (either voltage or current source). The theorem states the following

5.9.1 Circuit With a Voltage Source

In any linear bilateral network, if a single voltage source V_a in branch a produces a current I_b in another branch b, then if the voltage source V_a is removed(i.e. short circuited) and inserted in branch b, it will produce a current I_b in branch a.

In other words, it states that the ratio of response(output) to excitation(input) remains constant if the positions of output and input are interchanged in a reciprocal network. Consider the network shown in figure 5.9.1a and b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \qquad (5.9.1)$$

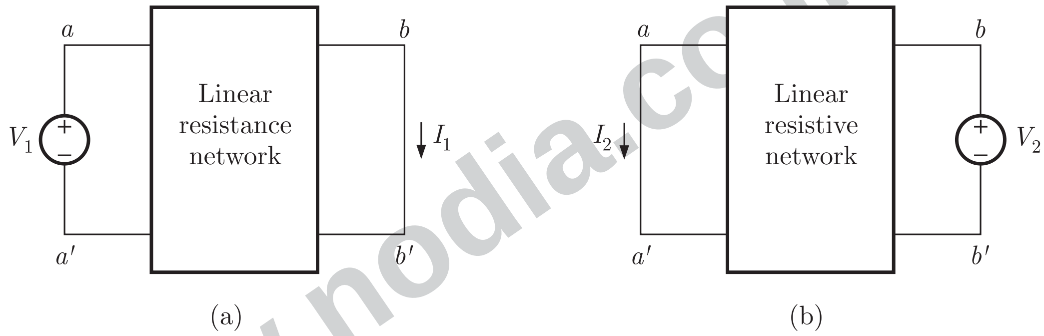


Fig. 5.9.1 Illustration of Reciprocity Theorem for a Voltage Source

When applying the reciprocity theorem for a voltage source, the following steps must be followed:

1. The voltage source is replaced by a short circuit in the original location.
2. The polarity of the voltage source in the new location have the same correspondence with branch current, in each position, otherwise a –ve sign appears in the expression (5.9.1).

This can be explained in a better way through following example.

5.9.2 Circuit With a Current Source

In any linear bilateral network, if a single current source I_a in branch a produces a voltage V_b in another branch b, then if the current source I_a is removed(i.e. open circuited) and inserted in branch b, it will produce a voltage V_b in open-circuited branch a.

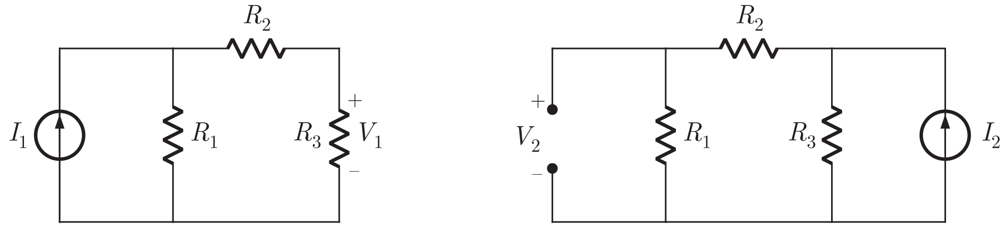


Fig. 5.9.2 Illustration of Reciprocity Theorem for a Current Source

Again, the ratio of voltage and current remains constant. Consider the network shown in figure 5.9.2a and 5.9.2b. Using reciprocity theorem we my write

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} \qquad (5.9.2)$$

When applying the reciprocity theorem for a current source, the following conditions must be met:

1. The current source is replaced by an open circuit in the original location.
2. The direction of the current source in the new location have the same correspondence with voltage polarity, in each position, otherwise a –ve sign appears in the expression (5.9.2).

5.10 SUBSTITUTION THEOREM

If the voltage across and the current through any branch of a dc bilateral network are known, this branch can be replaced by any combination of elements that will maintain the same voltage across and current through the chosen branch.

For example consider the circuit of figure 5.10.1 .

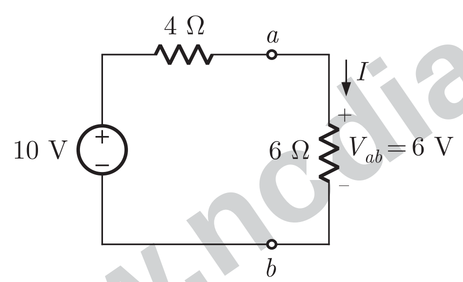


Fig 5.10.1 A Circuit having Voltage $V_{ab} = 6$ V and Current $I = 1$ A in Branch ab

The voltage V_{ab} and the current I in the circuit are given as

$$V_{ab} = \left(\frac{6}{6+4}\right)10 = 6\text{ V}$$

$$I = \frac{10}{6+4} = 1\text{ A}$$

The $6\,\Omega$ resistor in branch a-b may be replaced with any combination of components, provided that the terminal voltage and current must be the same.

We see that the branches of figure 5.10.2a-e are each equivalent to the original branch between terminals a and b of the circuit in figure 5.10.1.

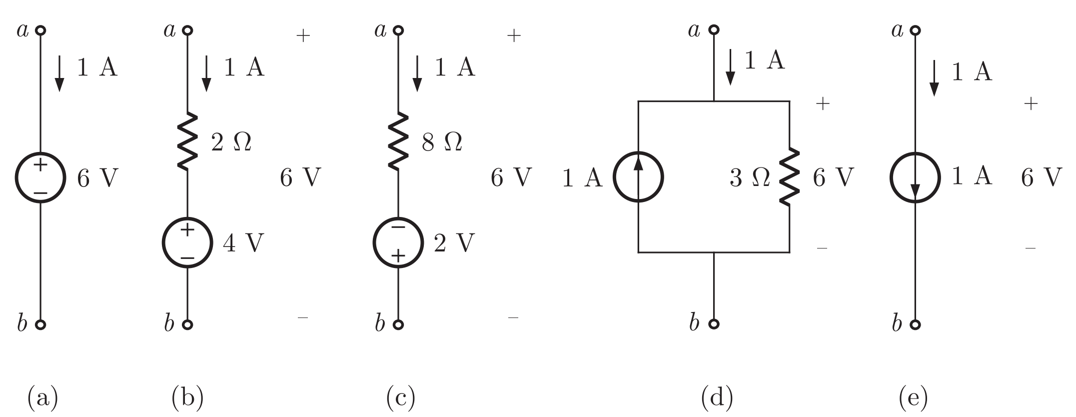


Fig. 5.10.2 Equivalent Circuits for Branch *ab*

Also consider that the response of the remainder of the circuit of figure 5.10.1 is unchanged by substituting any one of the equivalent branches.

5.11 MILLMAN'S THEOREM

Millman's theorem is used to reduce a circuit that contains several branches in parallel where each branch has a voltage source in series with a resistor as shown in figure 5.11.1.

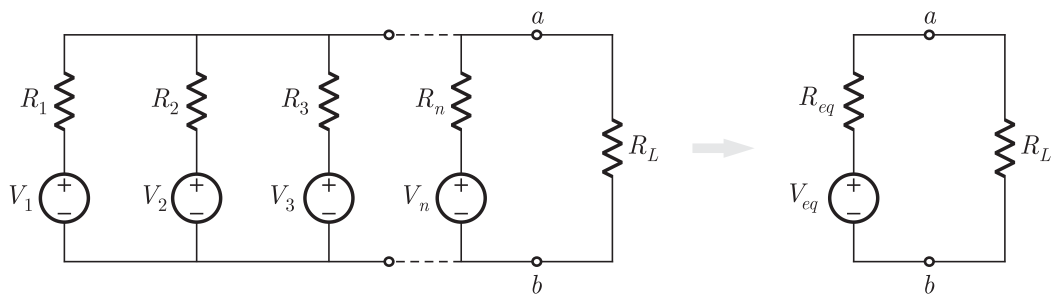


Fig. 5.11.1 Illustration of Millman's Theorem

Mathematically

$$V_{eq} = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3 + V_4 G_4 + \ldots + V_n G_n}{G_1 + G_2 + G_3 + G_4 + \ldots + G_n}$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + G_3 + \ldots + G_n}$$

where conductances

$$G_1 = \frac{1}{R_1}, G_2 = \frac{1}{R_2}, G_3 = \frac{1}{R_3}, G_4 = \frac{1}{R_4}, \ldots G_n = \frac{1}{R_n}$$

In terms of resistances

$$V_{eq} = \frac{V_1/R_1 + V_2/R_2 + V_3/R_3 + V_4/R_4 + \ldots + V_n R_n}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + \ldots + 1/R_n}$$

$$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + \ldots + 1/R_n}$$

5.12 TELLEGEN'S THEOREM

Tellegen's theorem states that the sum of the power dissipations in a lumped network at any instant is always zero. This is supported by Kirchhoff's voltage and current laws. Tellegen's theorem is valid for any lumped network which may be linear or non-linear, passive or active, time-varying or time-invariant.

For a network with n branches, the power summation equation is,

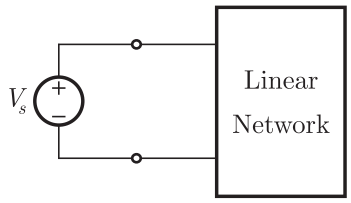
$$\sum_{k=1}^{k=n} V_k I_k = 0$$

One application of Tellegen's theorem is checking the quantities obtained when a circuit is analyzed. If the individual branch power dissipations do not add up to zero, then some of the calculated quantities are incorrect.

****

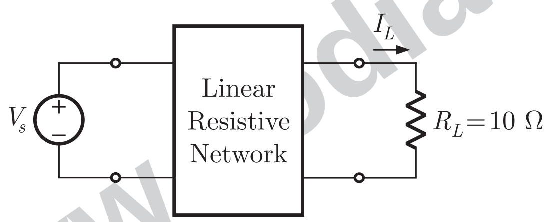
EXERCISE 5.1

MCQ 5.1.1 The linear network in the figure contains resistors and dependent sources only. When $V_s = 10$ V, the power supplied by the voltage source is 40 W. What will be the power supplied by the source if $V_s = 5$ V ?



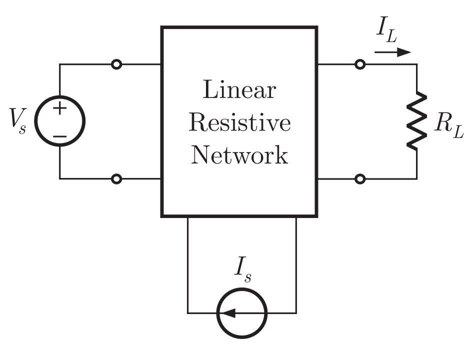
(A) 20 W
(B) 10 W
(C) 40 W
(D) can not be determined

MCQ 5.1.2 In the circuit below, it is given that when $V_s = 20$ V, $I_L = 200$ mA. What values of I_L and V_s will be required such that power absorbed by R_L is 2.5 W ?



(A) 1 A, 2.5 V
(B) 0.5 A, 2 V
(C) 0.5 A, 50 V
(D) 2 A, 1.25 V

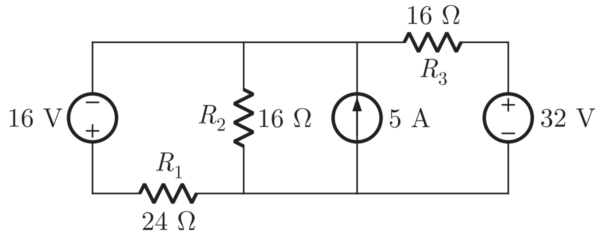
MCQ 5.1.3 For the circuit shown in figure below, some measurements are made and listed in the table.



	V_s	I_s	I_L
1.	14 V	6 A	2 A
2.	18 V	2 A	6 A

Which of the following equation is true for I_L ?
(A) $I_L = 0.6V_s + 0.4I_s$
(B) $I_L = 0.2V_s - 0.3I_s$
(C) $I_L = 0.2V_s + 0.3I_s$
(D) $I_L = 0.4V_s - 0.6I_s$

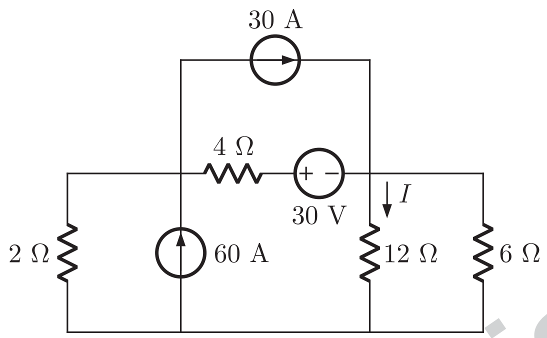
MCQ 5.1.4 In the circuit below, the voltage drop across the resistance R_2 will be equal to



(A) 46 volt (B) 38 volt

(C) 22 volt (D) 14 volt

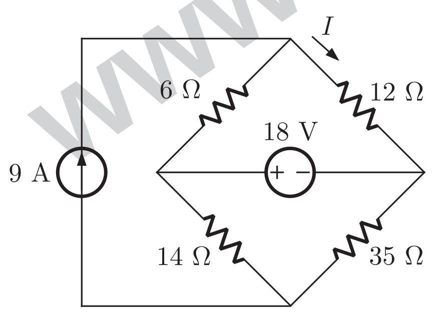
MCQ 5.1.5 In the circuit below, current $I = I_1 + I_2 + I_3$, where I_1, I_2 and I_3 are currents due to 60 A, 30 A and 30 V sources acting alone. The values of I_1, I_2 and I_3 are respectively



(A) 8 A, 8 A, -4 A (B) 12 A, 12 A, -5 A

(C) 4 A, 4 A, -1 A (D) 2 A, 2 A, -4 A

MCQ 5.1.6 In the circuit below, current I is equal to sum of two currents I_1 and I_2. What are the values of I_1 and I_2 ?



(A) 6 A, 1 A (B) 9 A, 6 A

(C) 3 A, 1 A (D) 3 A, 4 A

MCQ 5.1.7 A network consists only of independent current sources and resistors. If the values of all the current sources are doubled, then values of node voltages

(A) remains same

(B) will be doubled

(C) will be halved

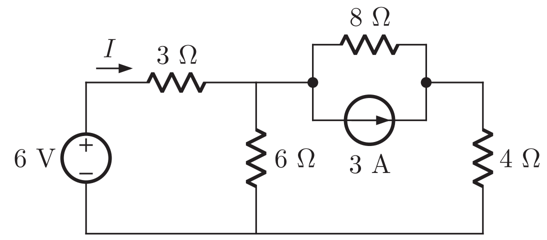
(D) changes in some other way.

MCQ 5.1.8 Consider a network which consists of resistors and voltage sources only. If the values of all the voltage sources are doubled, then the values of mesh current will be

(A) doubled (B) same

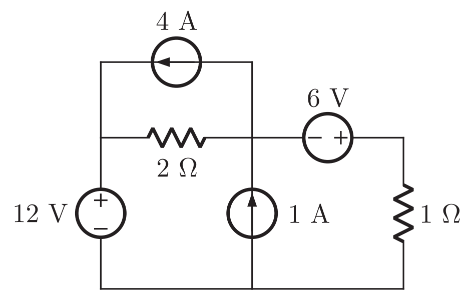
(C) halved (D) none of these

MCQ 5.1.9 The value of current I in the circuit below is equal to



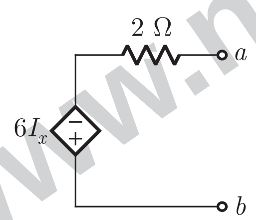
(A) $\frac{2}{7}$ A (B) 1 A
(C) 2 A (D) 4 A

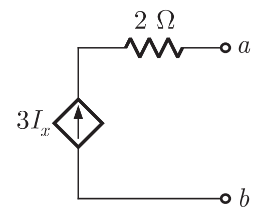
MCQ 5.1.10 In the circuit below, the 12 V source

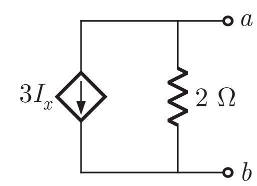


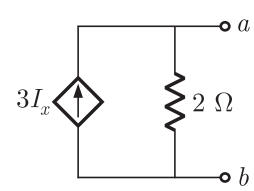
(A) absorbs 36 W
(B) delivers 4 W
(C) absorbs 100 W
(D) delivers 36 W

MCQ 5.1.11 Which of the following circuits is equivalent to the circuit shown below ?



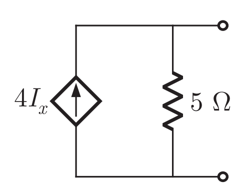
(A)


(B)


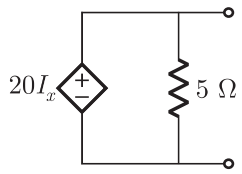
(C)


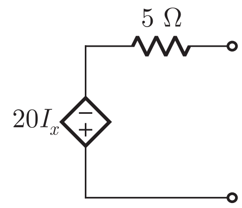
(D) None of these

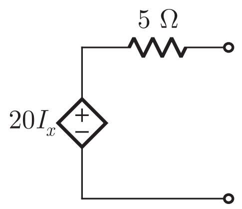
MCQ 5.1.12 Consider a dependent current source shown in figure below.



The source transformation of above is given by

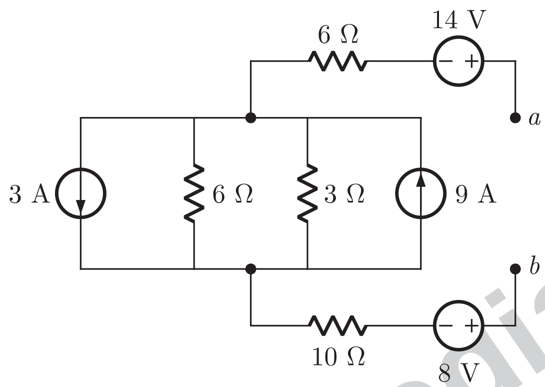
(A) 

(B) 

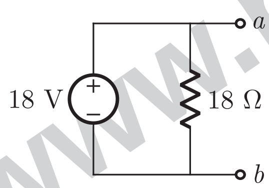
(C) 

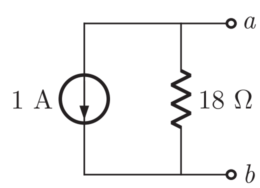
(D) Source transformation does not applicable to dependent sources

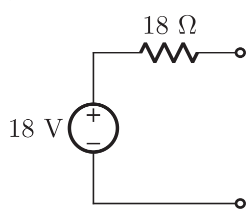
MCQ 5.1.13 Consider a circuit shown in the figure

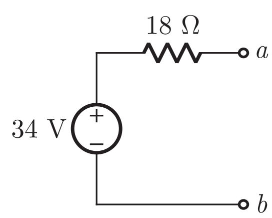


Which of the following circuit is equivalent to the above circuit ?

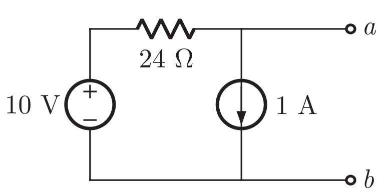
(A) 

(B) 

(C) 

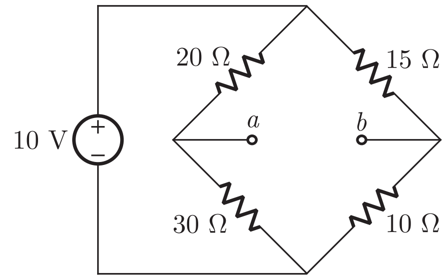
(D) 

MCQ 5.1.14 For the circuit shown in the figure the Thevenin voltage and resistance seen from the terminal a-b are respectively



(A) 34 V, 0 Ω
(B) 20 V, 24 Ω
(C) 14 V, 0 Ω
(D) -14 V, 24 Ω

MCQ 5.1.15 In the following circuit, Thevenin voltage and resistance across terminal a and b respectively are



(A) 10 V, 18 Ω (B) 2 V, 18 Ω
(C) 10 V, 18.67 Ω (D) 2 V, 18.67 Ω

MCQ 5.1.16 The value of R_{Th} and V_{Th} such that the circuit of figure (B) is the Thevenin equivalent circuit of the circuit shown in figure (A), will be equal to

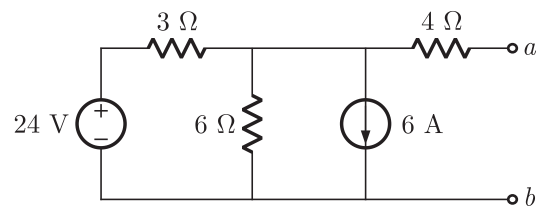


Fig.(A)

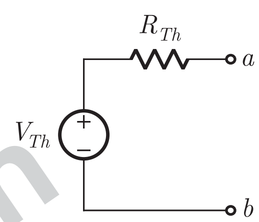


Fig.(B)

(A) $R_{Th} = 6\,\Omega$, $V_{Th} = 4$ V
(B) $R_{Th} = 6\,\Omega$, $V_{Th} = 28$ V
(C) $R_{Th} = 2\,\Omega$, $V_{Th} = 24$ V
(D) $R_{Th} = 10\,\Omega$, $V_{Th} = 14$ V

MCQ 5.1.17 What values of R_{Th} and V_{Th} will cause the circuit of figure (B) to be the equivalent circuit of figure (A) ?

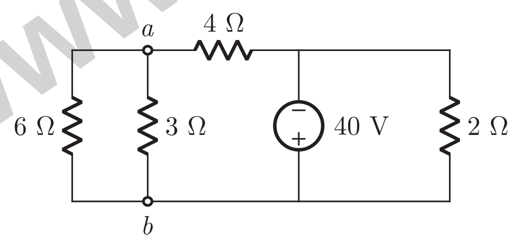


Fig.(A)

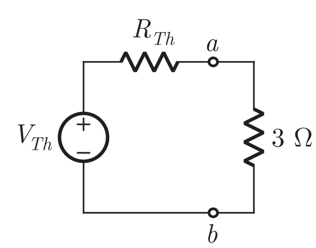


Fig.(B)

(A) 2.4 Ω, −24 V (B) 3 Ω, 16 V
(C) 10 Ω, 24 V (D) 10 Ω, −24 V

Common Data For Q. 18 and 19 :

Consider the two circuits shown in figure (A) and figure (B) below

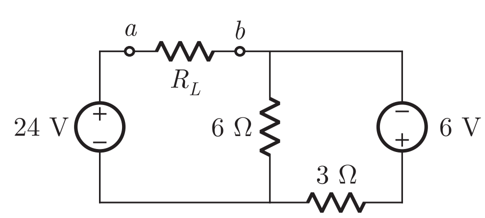


Fig.(A)

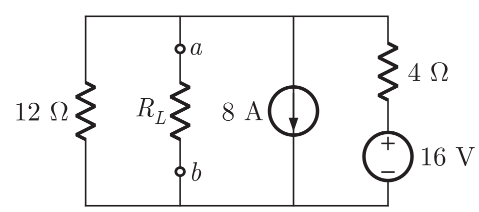


Fig.(B)

MCQ 5.1.18 The value of Thevenin voltage across terminals a-b of figure (A) and figure (B) respectively are

(A) 30 V, 36 V (B) 28 V, −12 V

(C) 18 V, 12 V (D) 30 V, −12 V

MCQ 5.1.19 The value of Thevenin resistance across terminals a-b of figure (A) and figure (B) respectively are

(A) zero, 3 Ω (B) 9 Ω, 16 Ω

(C) 2 Ω, 3 Ω (D) zero, 16 Ω

MCQ 5.1.20 For a network having resistors and independent sources, it is desired to obtain Thevenin equivalent across the load which is in parallel with an ideal current source. Then which of the following statement is true ?

(A) The Thevenin equivalent circuit is simply that of a voltage source.

(B) The Thevenin equivalent circuit consists of a voltage source and a series resistor.

(C) The Thevenin equivalent circuit does not exist but the Norton equivalent does exist.

(D) None of these

MCQ 5.1.21 The Thevenin equivalent circuit of a network consists only of a resistor (Thevenin voltage is zero). Then which of the following elements might be contained in the network ?

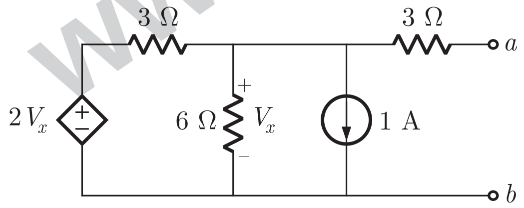
(A) resistor and independent sources

(B) resistor only

(C) resistor and dependent sources

(D) resistor, independent sources and dependent sources.

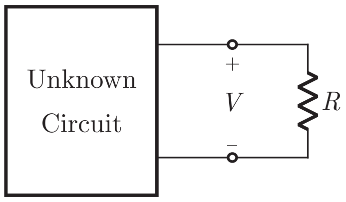
MCQ 5.1.22 For the circuit shown in the figure, the Thevenin's voltage and resistance looking into a-b are



(A) 2 V, 3 Ω (B) 2 V, 2 Ω

(C) 6 V, −9 Ω (D) 6 V, −3 Ω

MCQ 5.1.23 For the following circuit, values of voltage V for different values of R are given in the table.



R	V
3 Ω	6 V
8 Ω	8 V

The Thevenin voltage and resistance of the unknown circuit are respectively.

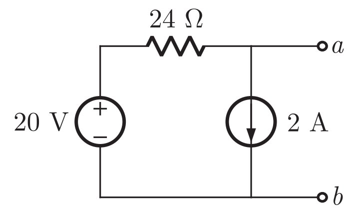
(A) 14 V, 4 Ω

(B) 4 V, 1 Ω

(C) 14 V, 6 Ω

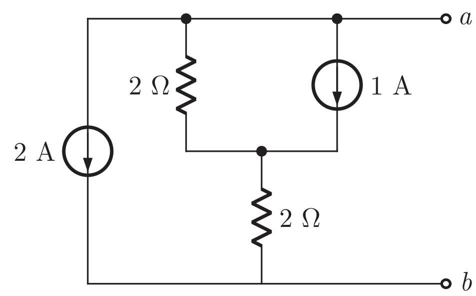
(D) 10 V, 2 Ω

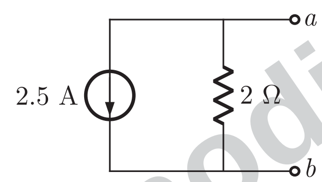
MCQ 5.1.24 In the circuit shown below, the Norton equivalent current and resistance with respect to terminal a-b is

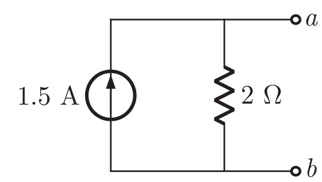


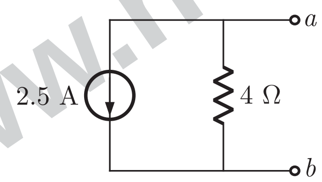
(A) $\frac{17}{6}$ A, 0 Ω
(B) 2 A, 24 Ω
(C) $-\frac{7}{6}$ A, 24 Ω
(D) -2 A, 24 Ω

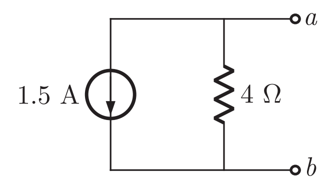
MCQ 5.1.25 The Norton equivalent circuit for the circuit shown in figure is given by



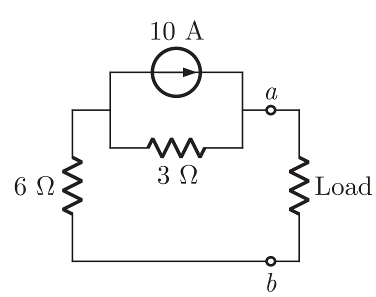
(A) 

(B) 

(C) 

(D) 

MCQ 5.1.26 What are the values of equivalent Norton current source (I_N) and equivalent resistance (R_N) across the load terminal of the circuit shown in figure ?



	I_N	R_N
(A)	10 A	2 Ω
(B)	10 A	9 Ω
(C)	3.33 A	9 Ω
(D)	6.66 A	2 Ω

MCQ 5.1.27 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in parallel with an ideal voltage sources.
Consider the following statements :

1. Thevenin equivalent circuit across this terminal does not exist.
2. The Thevenin equivalent circuit exists and it is simply that of a voltage source.
3. The Norton equivalent circuit for this terminal does not exist.

Which of the above statements is/are true ?

(A) 1 and 3 (B) 1 only

(C) 2 and 3 (D) 3 only

MCQ 5.1.28 For a network consisting of resistors and independent sources only, it is desired to obtain Thevenin's or Norton's equivalent across a load which is in series with an ideal current sources.

Consider the following statements

1. Norton equivalent across this terminal is not feasible.
2. Norton equivalent circuit exists and it is simply that of a current source only.
3. Thevenin's equivalent circuit across this terminal is not feasible.

Which of the above statements is/are correct ?

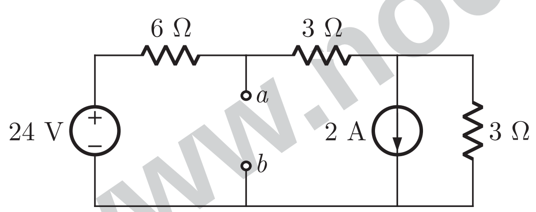
(A) 1 and 3

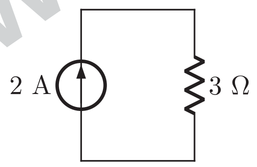
(B) 2 and 3

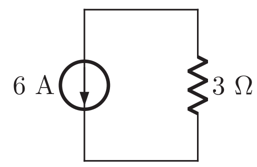
(C) 1 only

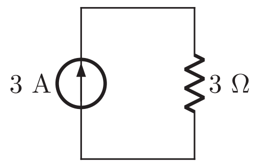
(D) 3 only

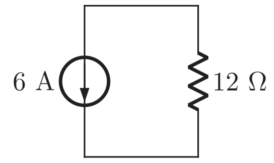
MCQ 5.1.29 The Norton equivalent circuit of the given network with respect to the terminal a-b, is



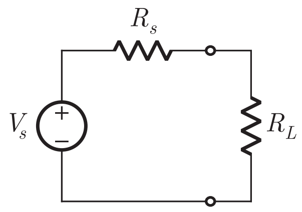
(A)


(B)


(C)


(D)


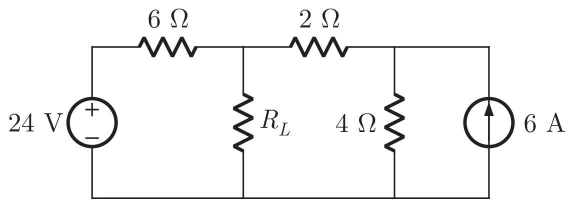
MCQ 5.1.30 In the circuit below, if R_L is fixed and R_s is variable then for what value of R_s power dissipated in R_L will be maximum ?



(A) $R_S = R_L$ (B) $R_S = 0$

(C) $R_S = R_L/2$ (D) $R_S = 2R_L$

MCQ 5.1.31 In the circuit shown below the maximum power transferred to R_L is P_{max}, then



(A) $R_L = 12\,\Omega,\ P_{max} = 12$ W
(B) $R_L = 3\,\Omega,\ P_{max} = 96$ W
(C) $R_L = 3\,\Omega,\ P_{max} = 48$ W
(D) $R_L = 12\,\Omega,\ P_{max} = 24$ W

MCQ 5.1.32 In the circuit shown in figure (A) if current $I_1 = 2$ A, then current I_2 and I_3 in figure (B) and figure (C) respectively are

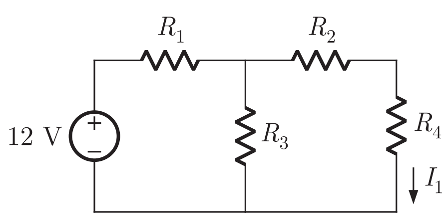


Fig.(A)

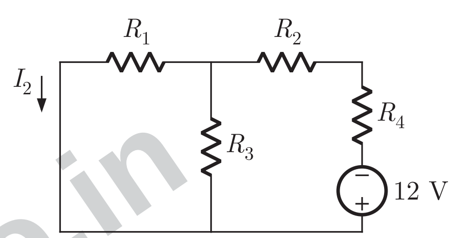


Fig.(B)

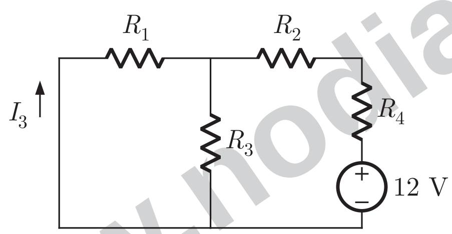


Fig.(C)

(A) 2 A, 2 A
(B) −2 A, 2 A
(C) 2 A, −2 A
(D) −2 A, −2 A

MCQ 5.1.33 In the circuit of figure (A), if $I_1 = 20$ mA, then what is the value of current I_2 in the circuit of figure (B) ?

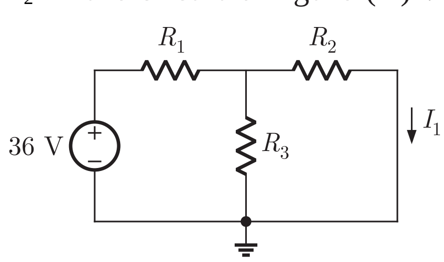


Fig.(A)

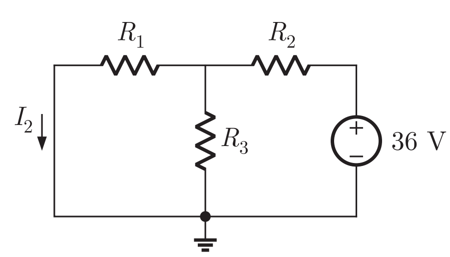


Fig.(B)

(A) 40 mA
(B) −20 mA
(C) 20 mA
(D) R_1, R_2 and R_3 must be known

MCQ 5.1.34 If $V_1 = 2$ V in the circuit of figure (A), then what is the value of V_2 in the circuit of figure (B) ?

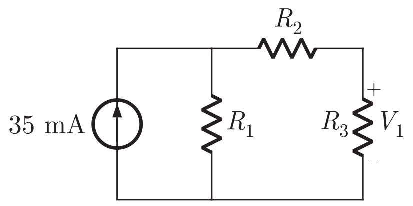


Fig.(A)

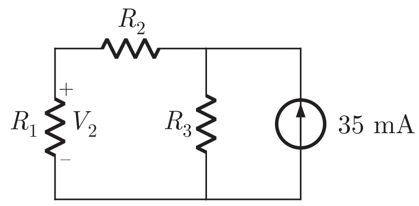
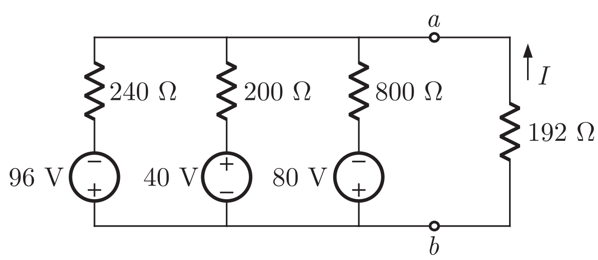


Fig.(B)

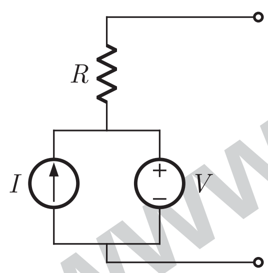
(A) 2 V
(B) -2 V
(C) 4 V
(D) R_1, R_2 and R_3 must be known

MCQ 5.1.35 The value of current I in the circuit below is equal to

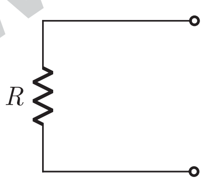


(A)100 mA
(B) 10 mA
(C) 233.34 mA
(D) none of these

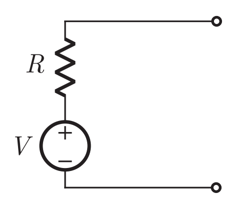
MCQ 5.1.36 A simple equivalent circuit of the two-terminal network shown in figure is



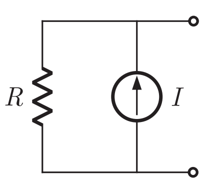
(A)



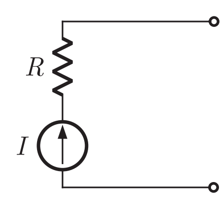
(B)



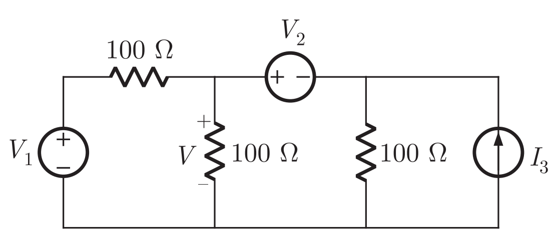
(C)



(D)

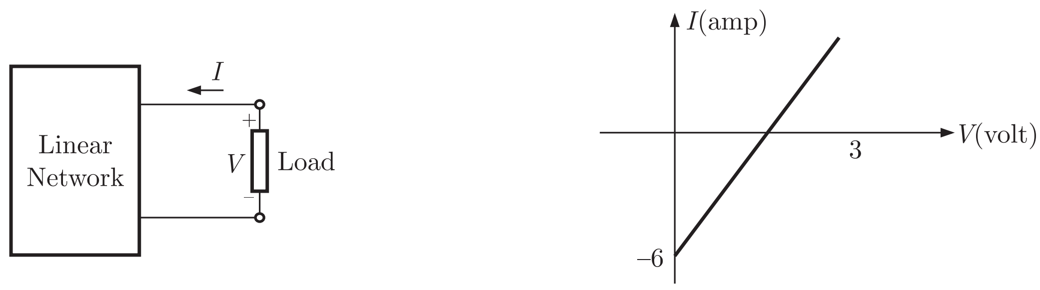


MCQ 5.1.37 If $V = AV_1 + BV_2 + CI_3$ in the following circuit, then values of A, B and C respectively are



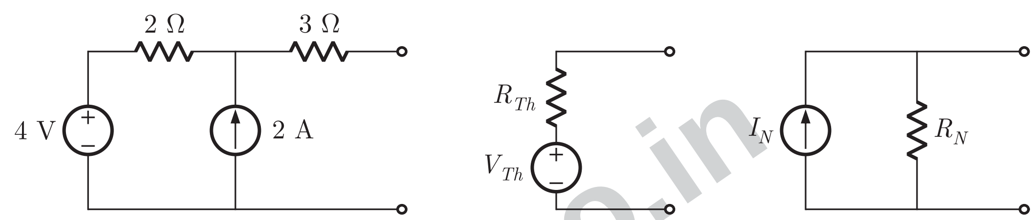
(A) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$
(B) $\frac{1}{3}, \frac{1}{3}, \frac{100}{3}$
(C) $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$
(D) $\frac{1}{3}, \frac{2}{3}, \frac{100}{3}$

MCQ 5.1.38 For the linear network shown below, V-I characteristic is also given in the figure. The value of Norton equivalent current and resistance respectively are



(A) 3 A, 2 Ω (B) 6 Ω, 2 Ω
(C) 6 A, 0.5 Ω (D) 3 A, 0.5 Ω

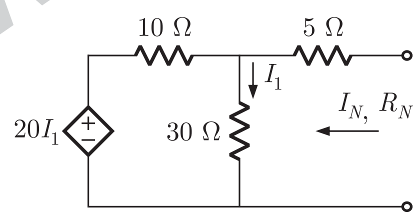
MCQ 5.1.39 In the following circuit a network and its Thevenin and Norton equivalent are given.



The value of the parameter are

	V_{Th}	R_{Th}	I_N	R_N
(A)	4 V	2 Ω	2 A	2 Ω
(B)	4 V	2 Ω	2 A	3 Ω
(C)	8 V	1.2 Ω	$\frac{30}{3}$ A	1.2 Ω
(D)	8 V	5 Ω	$\frac{8}{5}$ A	5 Ω

MCQ 5.1.40 For the following circuit the value of equivalent Norton current I_N and resistance R_N are



(A) 2 A, 20 Ω (B) 2 A, –20 Ω
(C) 0 A, 20 Ω (D) 0 A, –20 Ω

MCQ 5.1.41 Consider the following circuits shown below

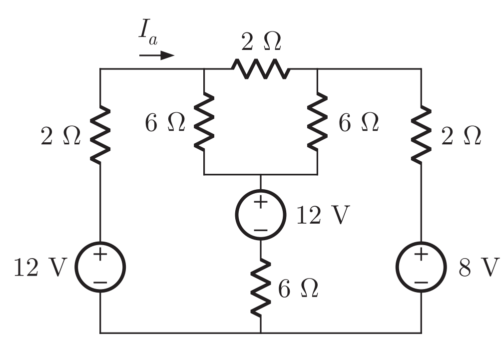


Fig (A)

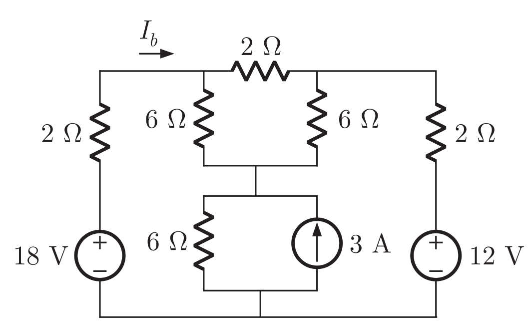


Fig (B)

The relation between I_a and I_b is

(A) $I_b = I_a + 6$

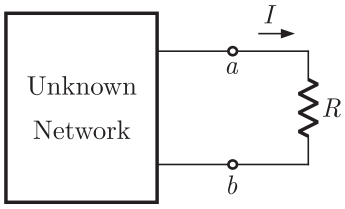
(B) $I_b = I_a + 2$

(C) $I_b = 1.5 I_a$

(D) $I_b = I_a$

Common Data For Q. 42 and 43 :

In the following circuit, some measurements were made at the terminals a, b and given in the table below.



R	I
3 Ω	2 A
5 Ω	1.6 A

MCQ 5.1.42 The Thevenin equivalent of the unknown network across terminal a-b is

(A) 3 Ω, 14 V (B) 5 Ω, 16 V

(C) 16 Ω, 38 V (D) 10 Ω, 26 V

MCQ 5.1.43 The value of R that will cause I to be 1 A, is

(A) 22 Ω (B) 16 Ω

(C) 8 Ω (D) 11 Ω

MCQ 5.1.44 In the circuit shown in fig (A) if current $I_1 = 2.5$ A then current I_2 and I_3 in fig (B) and (C) respectively are

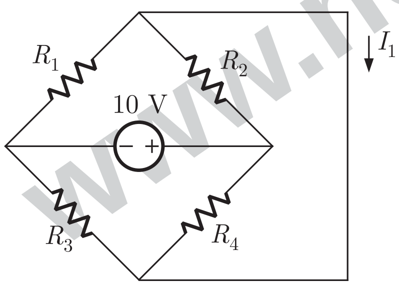


Fig.(A)

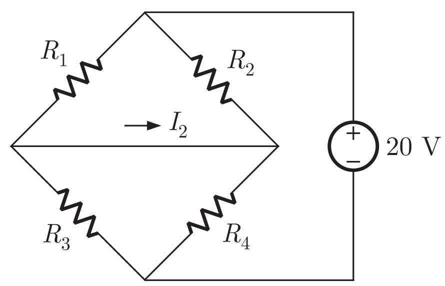


Fig.(B)

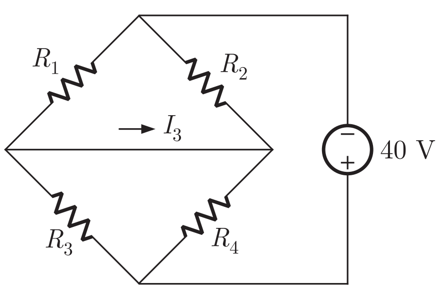
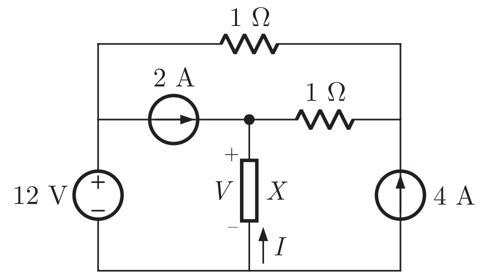


Fig.(C)

(A) 5 A, 10 A (B) −5 A, 10 A

(C) 5 A, −10 A (D) −5 A, −10 A

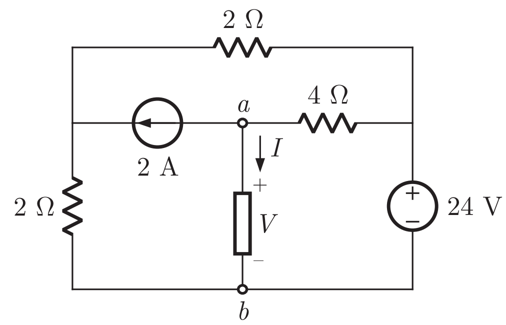
MCQ 5.1.45 The V-I relation of the unknown element X in the given network is $V = AI + B$. The value of A (in ohm) and B (in volt) respectively are

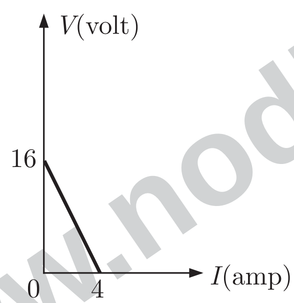


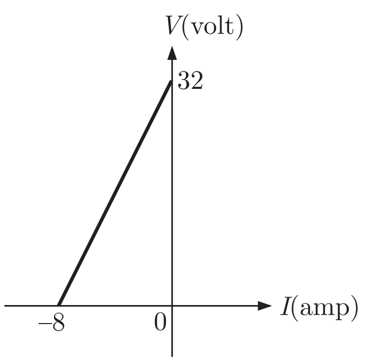
(A) 2, 20 (B) 2, 8

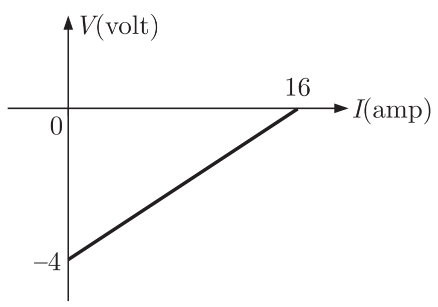
(C) 0.5, 4 (D) 0.5, 16

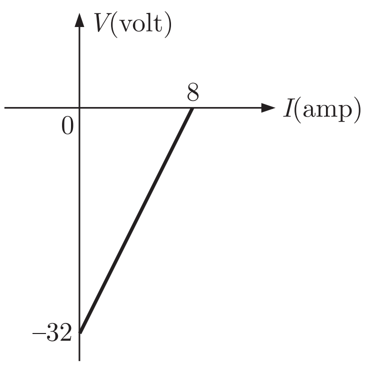
MCQ 5.1.46 For the following network the V-I curve with respect to terminals a-b, is given by



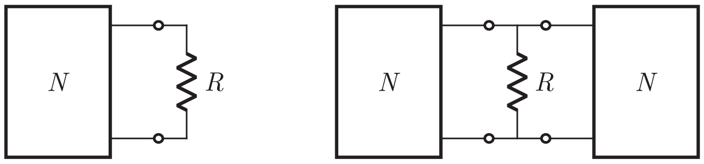
(A)


(B)


(C)


(D)


MCQ 5.1.47 A network N feeds a resistance R as shown in circuit below. Let the power consumed by R be P. If an identical network is added as shown in figure, the power consumed by R will be



(A) equal to P (B) less than P

(C) between P and $4P$ (D) more than $4P$

MCQ 5.1.48 A certain network consists of a large number of ideal linear resistors, one of which is R and two constant ideal source. The power consumed by R is P_1 when only the first source is active, and P_2 when only the second source is active. If both sources are active simultaneously, then the power consumed by R is

(A) $P_1 \pm P_2$
(B) $\sqrt{P_1} \pm \sqrt{P_2}$
(C) $(\sqrt{P_1} \pm \sqrt{P_2})^2$
(D) $(P_1 \pm P_2)^2$

MCQ 5.1.49 If the $60\,\Omega$ resistance in the circuit of figure (A) is to be replaced with a current source I_s and $240\,\Omega$ shunt resistor as shown in figure (B), then magnitude and direction of required current source would be

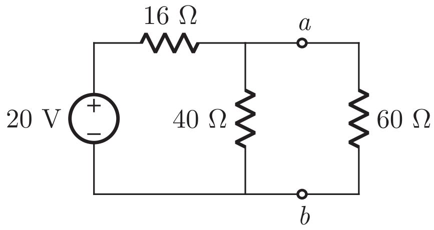


Fig.(A)

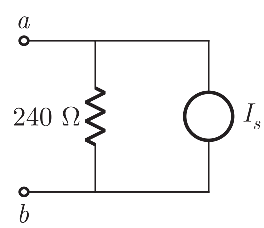
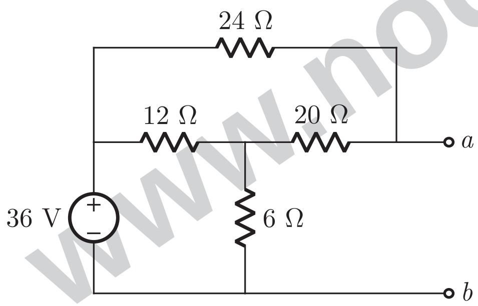


Fig.(B)

(A) 200 mA, upward
(B) 150 mA, downward
(C) 50 mA, downward
(D) 150 mA, upward

MCQ 5.1.50 The Thevenin's equivalent of the circuit shown in the figure is

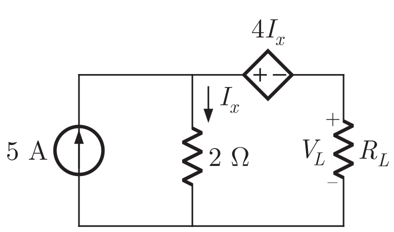


(A) 4 V, $48\,\Omega$
(B) 24 V, $12\,\Omega$
(C) 24 V, $24\,\Omega$
(D) 12 V, $12\,\Omega$

MCQ 5.1.51 The voltage V_L across the load resistance in the figure is given by

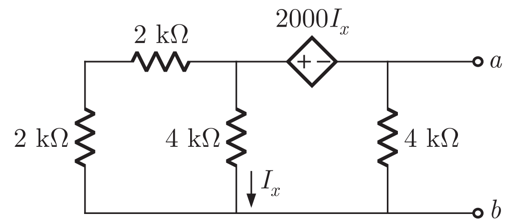
$$V_L = V\left(\frac{R_L}{R + R_L}\right)$$

V and R will be equal to



(A) -10 V, $2\,\Omega$
(B) 10 V, $2\,\Omega$
(C) -10 V, $-2\,\Omega$
(D) none of these

MCQ 5.1.52 In the circuit given below, viewed from a-b, the circuit can be reduced to an equivalent circuit as



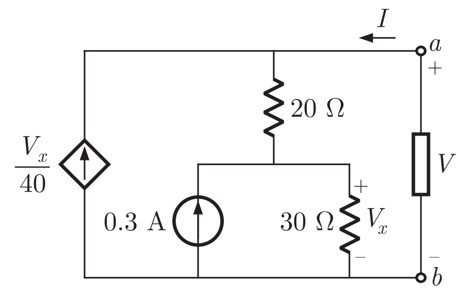
(A) 10 volt source in series with 2 kΩ resistor

(B) 1250 Ω resistor only

(C) 20 V source in series with 1333.34 Ω resistor

(D) 800 Ω resistor only

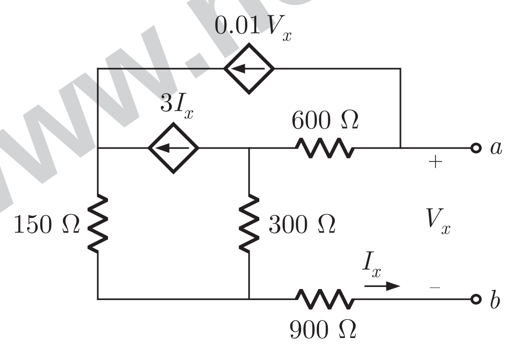
MCQ 5.1.53 The V-I equation for the network shown in figure, is given by



(A) $7V = 200I + 54$ (B) $V = 100I + 36$

(C) $V = 200I + 54$ (D) $V = 50I + 54$

MCQ 5.1.54 In the following circuit the value of open circuit voltage and Thevenin resistance at terminals a,b are



(A) $V_{oc} = 100$ V, $R_{Th} = 1800\ \Omega$

(B) $V_{oc} = 0$ V, $R_{Th} = 270\ \Omega$

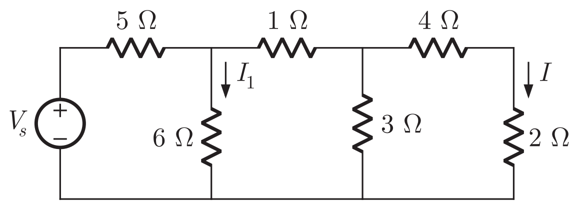
(C) $V_{oc} = 100$ V, $R_{Th} = 90\ \Omega$

(D) $V_{oc} = 0$ V, $R_{Th} = 90\ \Omega$

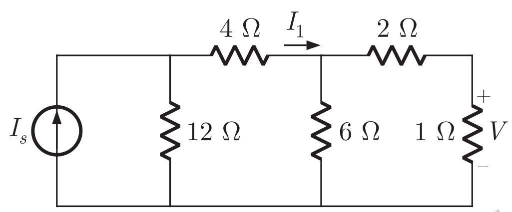
****

EXERCISE 5.2

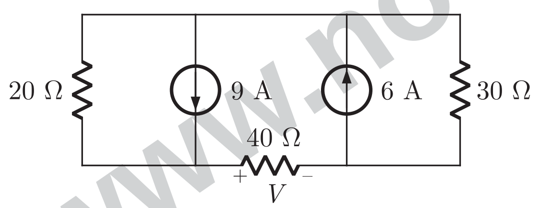
QUES 5.2.1 In the given network, if $V_s = V_0$, $I = 1\text{ A}$. If $V_s = 2V_0$ then what is the value of I_1 (in Amp) ?



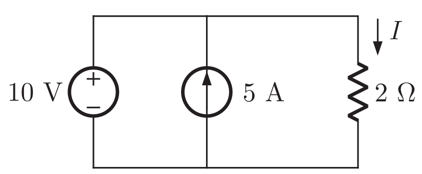
QUES 5.2.2 In the given network, if $I_s = I_0$ then $V = 1$ volt. What is the value of I_1 (in Amp) if $I_s = 2I_0$?



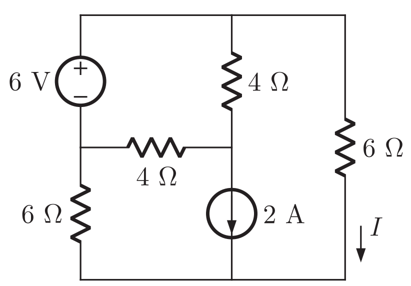
QUES 5.2.3 In the circuit below, the voltage V across the $40\,\Omega$ resistor would be equal to ____ Volts.



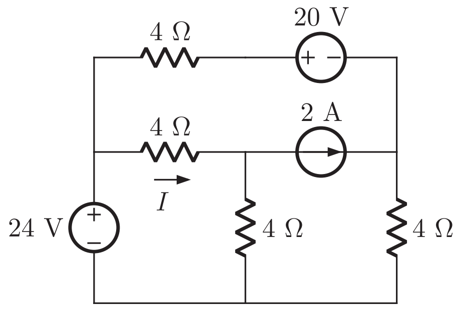
QUES 5.2.4 The value of current I flowing through $2\,\Omega$ resistance in the given circuit, equals to ____ Amp.



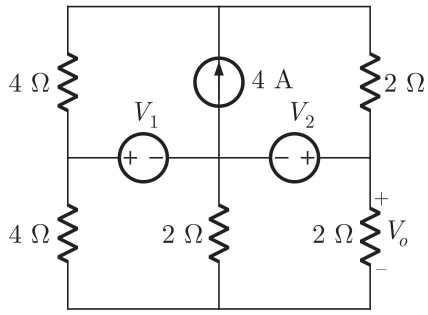
QUES 5.2.5 In the given circuit, the value of current I will be _____Amps.



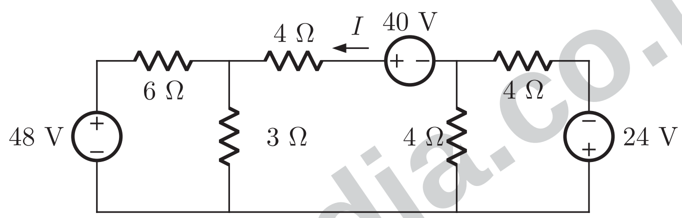
QUES 5.2.6 What is the value of current I in the given network (in Amp) ?



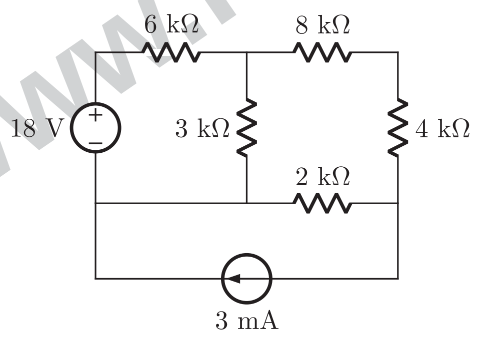
QUES 5.2.7 In the given network if $V_1 = V_2 = 0$, then what is the value of V_o (in volts) ?



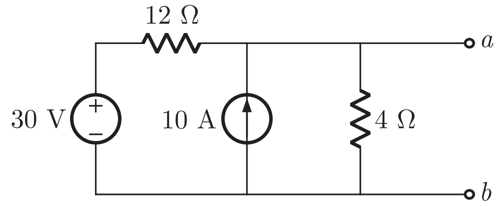
QUES 5.2.8 What is the value of current I in the circuit shown below (in Amp) ?



QUES 5.2.9 How much power is being dissipated by the $4\,\text{k}\Omega$ resistor in the network (in mW) ?

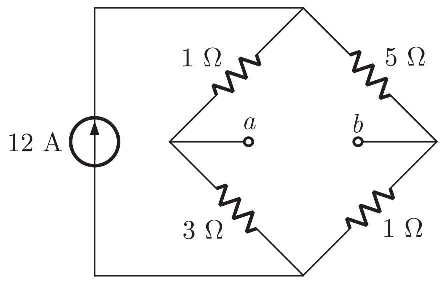


QUES 5.2.10 Thevenin equivalent resistance R_{Th} between the nodes a and b in the following circuit is _ _ _ _ Ω.



Common Data For Q. 11 and 12 :

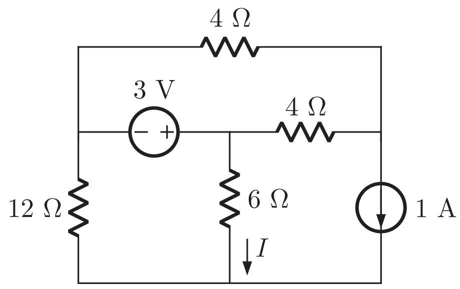
Consider the circuit shown in the figure.



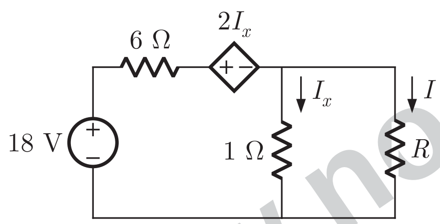
QUES 5.2.11 The equivalent Thevenin voltage across terminal *a*-*b* is____ Volts.

QUES 5.2.12 The Norton equivalent current with respect to terminal *a*-*b* is ____ Amps

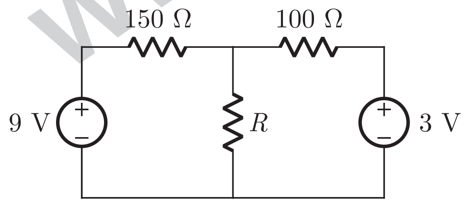
QUES 5.2.13 In the circuit given below, what is the value of current I (in Amp) through $6\,\Omega$ resistor



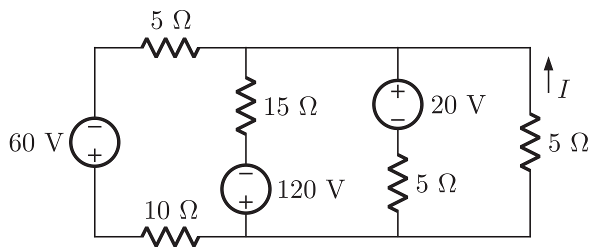
QUES 5.2.14 For the circuit below, what value of R will cause $I = 3$ A (in Ω) ?



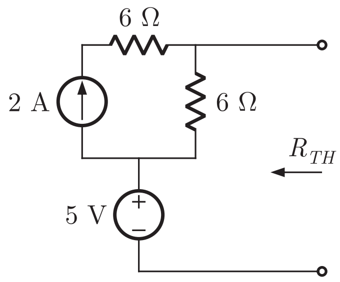
QUES 5.2.15 The maximum power that can be transferred to the resistance R in the circuit is ____ mili watts.



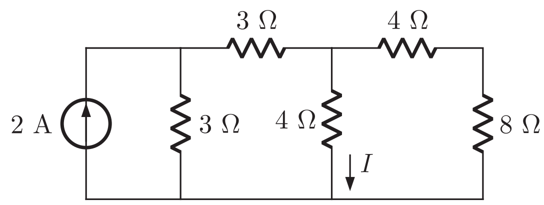
QUES 5.2.16 The value of current I in the following circuit is equal to _____Amp.



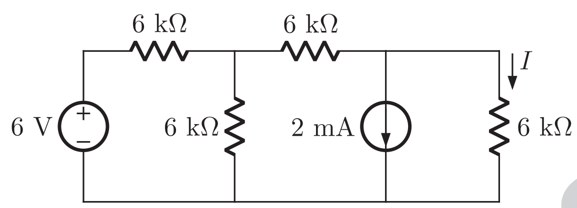
QUES 5.2.17 For the following circuit the value of R_{Th} is____Ω.



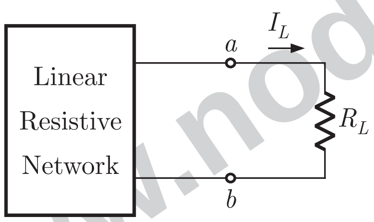
QUES 5.2.18 What is the value of current I in the given network (in Amp) ?



QUES 5.2.19 The value of current I in the figure is _ _ _ _ _ mA.



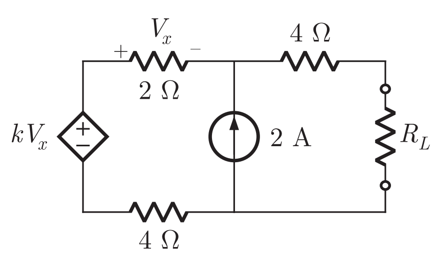
QUES 5.2.20 For the circuit of figure, some measurements were made at the terminals a-b and given in the table below.



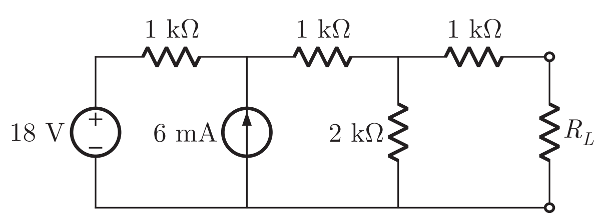
R_L	I_L
2 Ω	10 A
10 Ω	6 A

What is the value of I_L (in Amps) for $R_L = 20\,\Omega$?

QUES 5.2.21 In the circuit below, for what value of k, load $R_L = 2\,\Omega$ absorbs maximum power ?

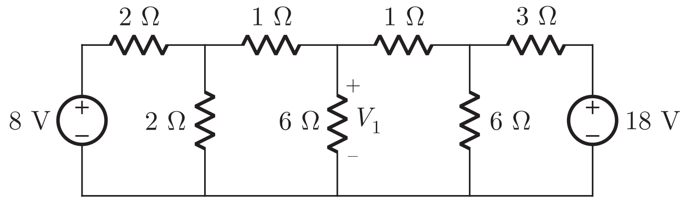


QUES 5.2.22 In the circuit shown below, the maximum power that can be delivered to the load R_L is equal to _ _ _ _ _ mW.

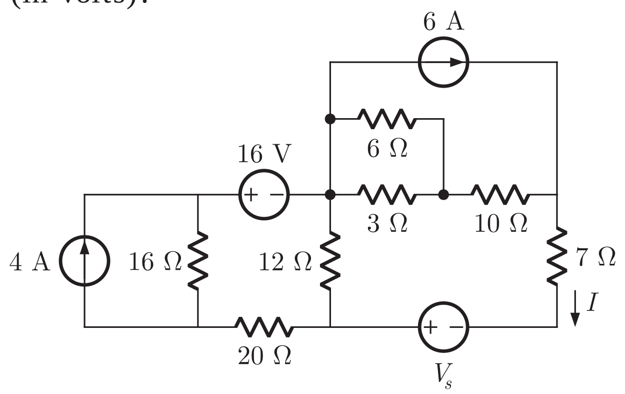


QUES 5.2.23 A practical DC current source provide 20 kW to a 50 Ω load and 20 kW to a 200 Ω load. The maximum power, that can drawn from it, is _ _ _ _ kW.

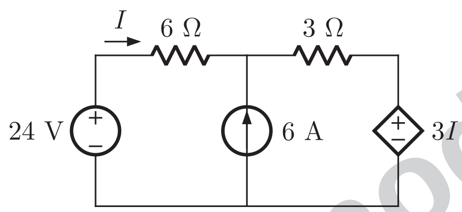
QUES 5.2.24 In the following circuit the value of voltage V_1 is _ _ _ _ Volts.



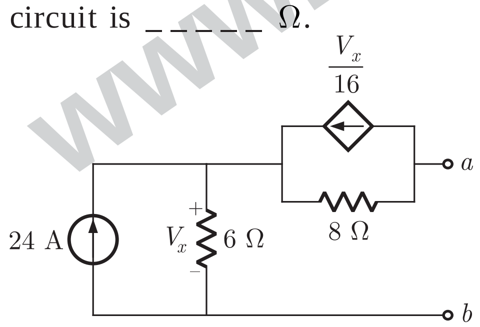
QUES 5.2.25 If $I = 5$ A in the circuit below, then what is the value of voltage source V_s (in volts)?



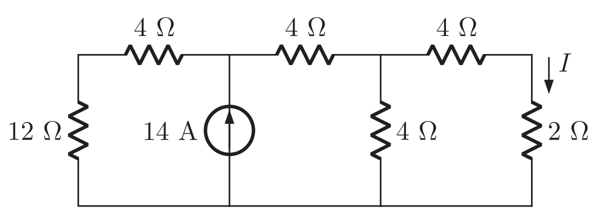
QUES 5.2.26 For the following circuit, what is the value of current I (in Amp) ?



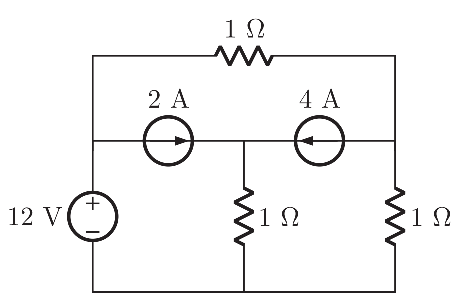
QUES 5.2.27 The Thevenin equivalent resistance between terminal a and b in the following circuit is _ _ _ _ _ Ω.



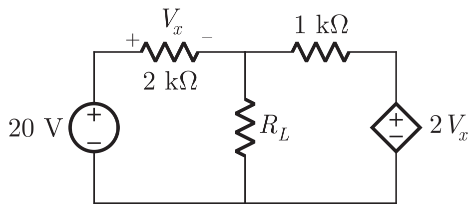
QUES 5.2.28 In the circuit shown below, what is the value of current I (in Amps) ?



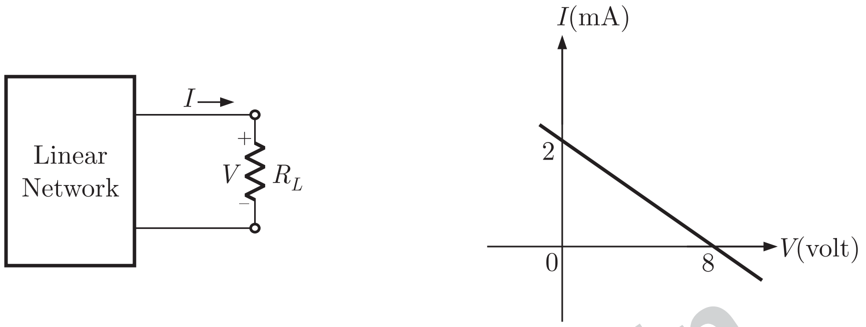
QUES 5.2.29 The power delivered by 12 V source in the given network is _ _ _ _ watts.



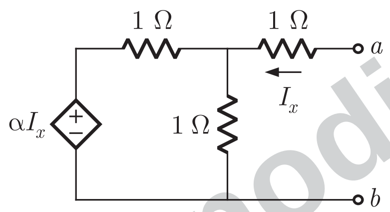
QUES 5.2.30 In the circuit shown, what value of R_L (in Ω) maximizes the power delivered to R_L ?



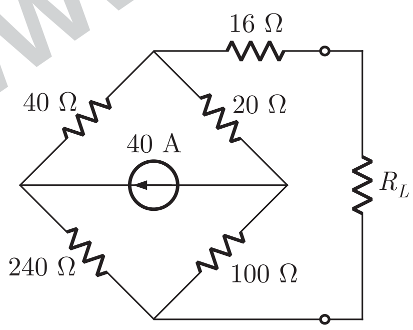
QUES 5.2.31 The V-I relation for the circuit below is plotted in the figure. The maximum power that can be transferred to the load R_L will be ______ mW



QUES 5.2.32 In the following circuit equivalent Thevenin resistance between nodes a and b is $R_{Th} = 3\,\Omega$. The value of α is_____

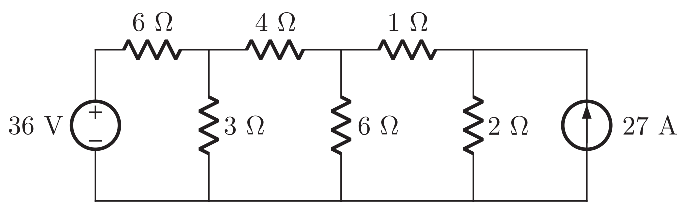


QUES 5.2.33 The maximum power that can be transferred to the load resistor R_L from the current source in the figure is ____ watts.



Common Data For Q. 34 and 35

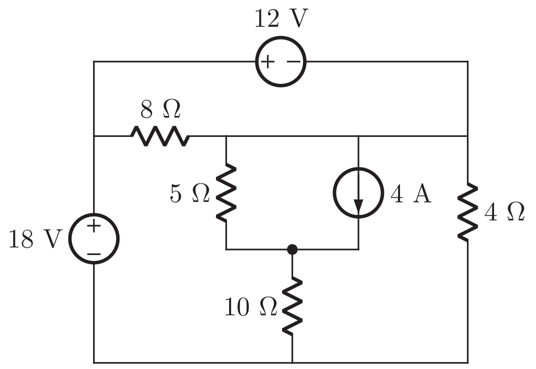
An electric circuit is fed by two independent sources as shown in figure.



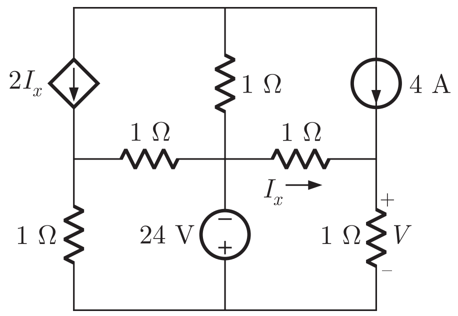
QUES 5.2.34 The power supplied by 36 V source will be _____ watts.

QUES 5.2.35 The power supplied by 27 A source will be _____ watts.

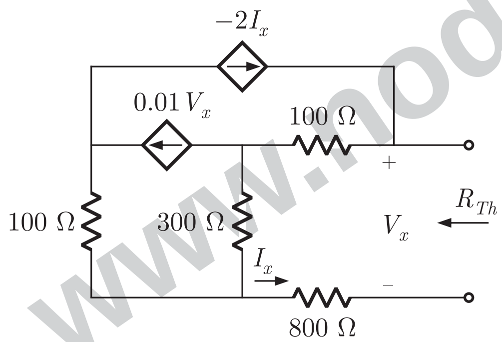
QUES 5.2.36 In the circuit shown in the figure, what is the power dissipated in $4\,\Omega$ resistor (in watts)



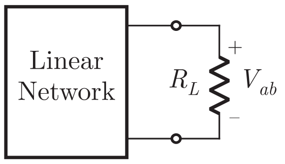
QUES 5.2.37 What is the value of voltage V in the following network (in volts) ?



QUES 5.2.38 For the circuit shown in figure below the value of R_{Th} is ____Ω.



QUES 5.2.39 Consider the network shown below :



The power absorbed by load resistance R_L is shown in table :

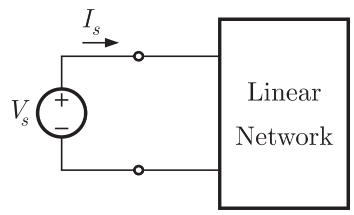
R_L	10 kΩ	30 kΩ
P	3.6 mW	4.8 mW

The value of R_L (in kΩ), that would absorb maximum power, is____

****

SOLUTIONS 5.1

SOL 5.1.1 Option (B) is correct.



For, $V_s = 10\text{ V},\ P = 40\text{ W}$

So, $I_s = \frac{P}{V_s} = \frac{40}{10} = 4\text{ A}$

Now, $V_s' = 5\text{ V}$, so $I_s' = 2\text{ A}$ (From linearity)

New value of the power supplied by source is

$$P_s' = V_s' I_s' = 5 \times 2 = 10\text{ W}$$

Note: Linearity does not apply to power calculations.

SOL 5.1.2 Option (C) is correct.

From linearity, we know that in the circuit $\frac{V_s}{I_L}$ ratio remains constant

$$\frac{V_s}{I_L} = \frac{20}{200 \times 10^{-3}} = 100$$

Let current through load is I_L' when the power absorbed is 2.5 W, so

$$P_L = (I_L')^2 R_L$$

$$2.5 = (I_L')^2 \times 10$$

$$I_L' = 0.5\text{ A}$$

$$\frac{V_s}{I_L} = \frac{V_s'}{I_L'} = 100$$

So, $V_s' = 100 I_L' = 100 \times 0.5 = 50\text{ V}$

Thus required values are

$$I_L' = 0.5\text{ A},\ V_s' = 50\text{ V}$$

SOL 5.1.3 Option (D) is correct.

From linearity,

$$I_L = AV_s + BI_s, \quad A \text{ and } B \text{ are constants}$$

From the table $2 = 14A + 6B$ ...(1)

$6 = 18A + 2B$ ...(2)

Solving equation (1) & (2)

$$A = 0.4,\ B = -0.6$$

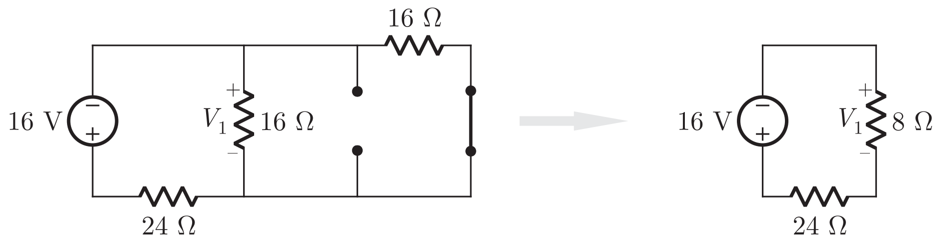
So, $I_L = 0.4V_s - 0.6I_s$

SOL 5.1.4 Option (B) is correct.

The circuit has 3 independent sources, so we apply superposition theorem to obtain the voltage drop.

Due to 16 V source only : (Open circuit 5 A source and Short circuit 32 V source)

Let voltage across R_2 due to 16 V source only is V_1.



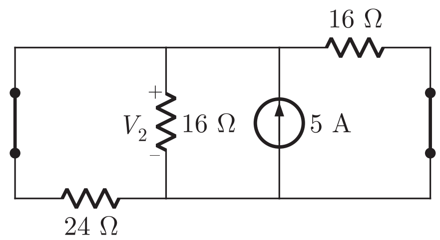
Using voltage division

$$V_1 = -\frac{8}{24+8}(16)$$

$$= -4 \text{ V}$$

Due to 5 A source only : (Short circuit both the 16 V and 32 V sources)

Let voltage across R_2 due to 5 A source only is V_2.

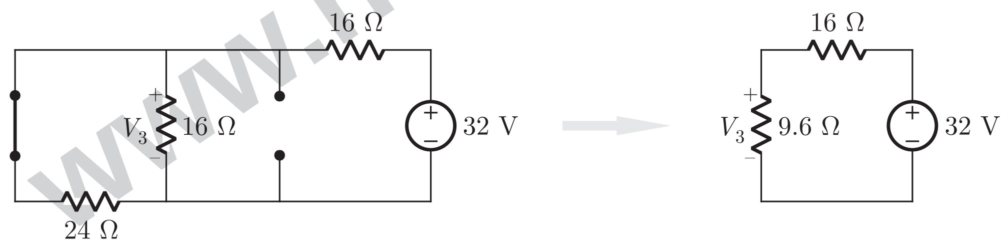


$$V_2 = (24\,\Omega \,||\, 16\,\Omega \,||\, 16\,\Omega) \times 5$$

$$= 6 \times 5 = 30 \text{ volt}$$

Due to 32 V source only : (Short circuit 16 V source and open circuit 5 A source)

Let voltage across R_2 due to 32 V source only is V_3



Using voltage division

$$V_3 = \frac{9.6}{16+9.6}(32) = 12 \text{ V}$$

By superposition, the net voltage across R_2 is

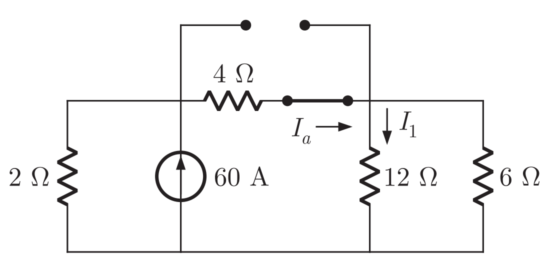
$$V = V_1 + V_2 + V_3 = -4 + 30 + 12 = 38 \text{ volt}$$

ALTERNATIVE METHOD :

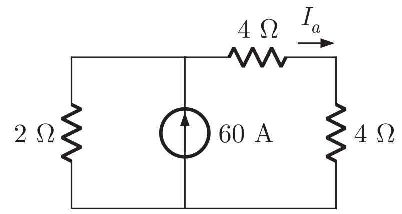
The problem may be solved by applying a node equation at the top node.

SOL 5.1.5 Option (C) is correct

Due to 60 A Source Only : (Open circuit 30 A and short circuit 30 V sources)



$$12\,\Omega \,||\, 6\,\Omega = 4\,\Omega$$



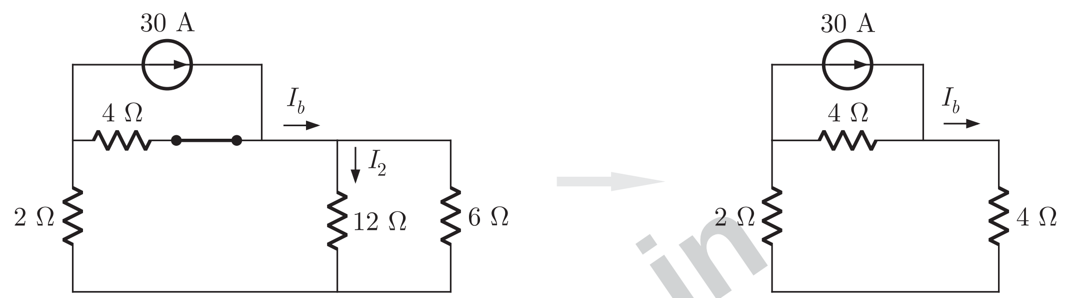
Using current division

$$I_a = \frac{2}{2+8}(60) = 12 \text{ A}$$

Again, I_a will be distributed between parallel combination of $12\,\Omega$ and $6\,\Omega$

$$I_1 = \frac{6}{12+6}(12) = 4 \text{ A}$$

Due to 30 A source only : (Open circuit 60 A and short circuit 30 V sources)



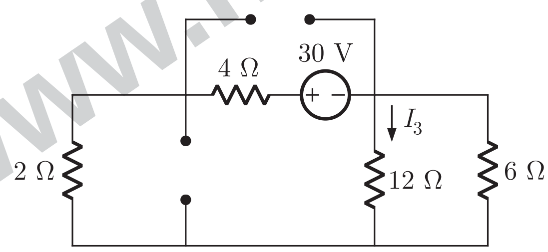
Using current division

$$I_b = \frac{4}{4+6}(30) = 12 \text{ A}$$

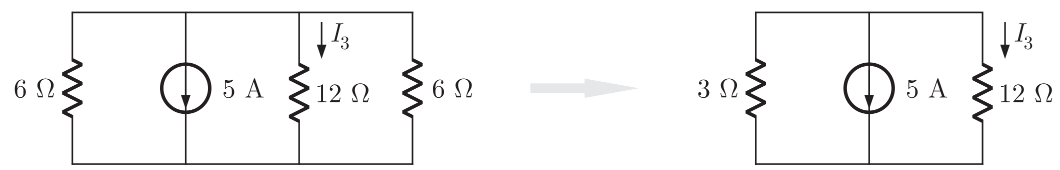
I_b will be distributed between parallel combination of $12\,\Omega$ and $6\,\Omega$

$$I_2 = \frac{6}{12+6}(12) = 4 \text{ A}$$

Due to 30 V Source Only : (Open circuit 60 A and 30 A sources)



Using source transformation



Using current division

$$I_3 = -\frac{3}{12+3}(5) = -1 \text{ A}$$

SOL 5.1.6 Option (C) is correct.

Using superposition, $I = I_1 + I_2$

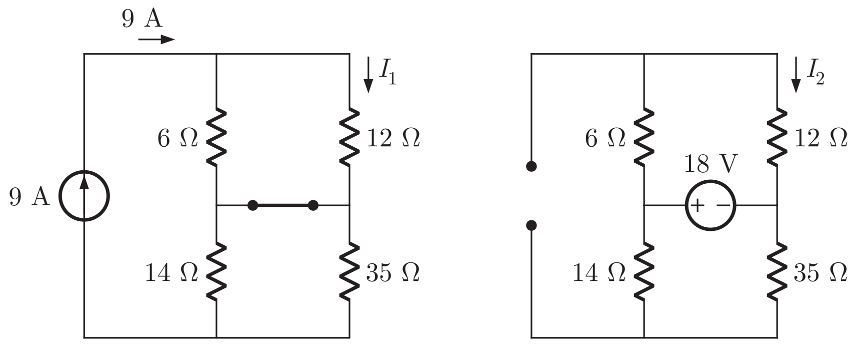
Let I_1 is the current due to 9 A source only. (i.e. short 18 V source)

$$I_1 = \frac{6}{6+12}(9) = 3 \text{ A} \qquad \text{(current division)}$$

Let I_2 is the current due to 18 V source only (i.e. open 9 A source)

$$I_2 = \frac{18}{6+12} = 1 \text{ A}$$

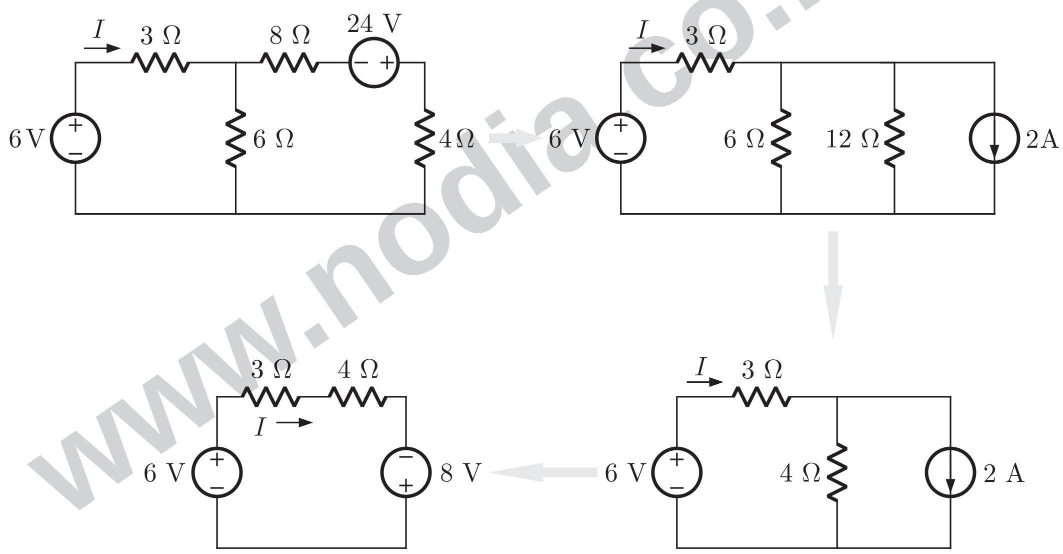
So, $I_1 = 3\text{ A},\ I_2 = 1\text{ A}$



SOL 5.1.7 Option (B) is correct.
From superposition theorem, it is known that if all source values are doubled, then node voltages also be doubled.

SOL 5.1.8 Option (A) is correct.
From the principal of superposition, doubling the values of voltage source doubles the mesh currents.

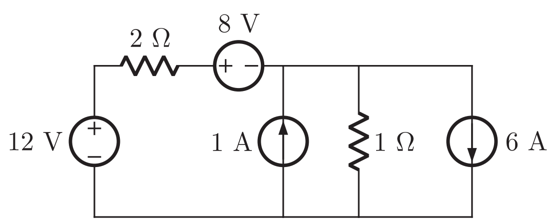
SOL 5.1.9 Option (C) is correct.
Using source transformation, we can obtain I in following steps.



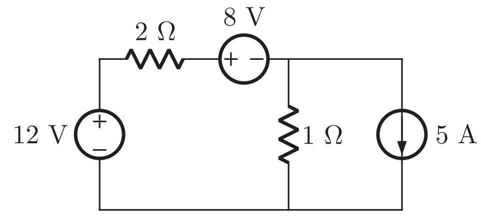
$$I = \frac{6+8}{3+4} = \frac{14}{7} = 2\text{ A}$$

ALTERNATIVE METHOD :
Try to solve the problem by obtaining Thevenin equivalent for right half of the circuit.

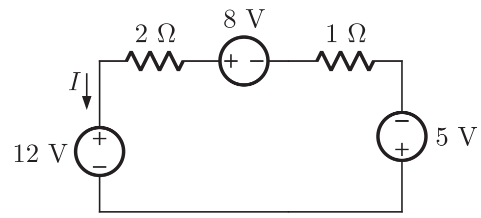
SOL 5.1.10 Option (D) is correct.
Using source transformation of 4 A and 6 V source.



Adding parallel current sources



Source transformation of 5 A source



Applying KVL around the anticlockwise direction

$$-5 - I + 8 - 2I - 12 = 0$$
$$-9 - 3I = 0$$
$$I = -3\text{ A}$$

Power absorbed by 12 V source

$$P_{12\text{ V}} = 12 \times I \qquad \text{(Passive sign convention)}$$
$$= 12 \times -3 = -36\text{ W}$$

or, 12 V source supplies 36 W power.

SOL 5.1.11 Option (B) is correct.

We know that source transformation also exists for dependent source, so

Current source values

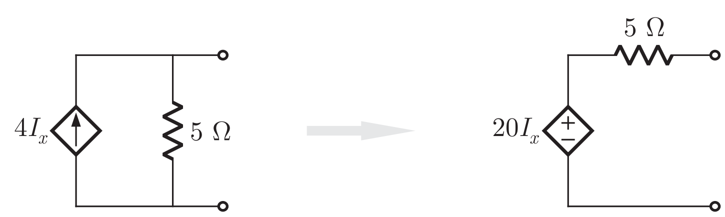
$$I_s = \frac{6I_x}{2} = 3I_x \text{ (downward)}$$
$$R_s = 2\,\Omega$$

SOL 5.1.12 Option (C) is correct.

We know that source transformation is applicable to dependent source also.

Values of equivalent voltage source

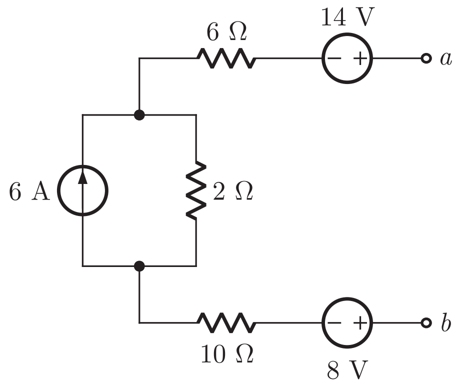
$$V_s = (4I_x)(5) = 20I_x$$
$$R_s = 5\,\Omega$$



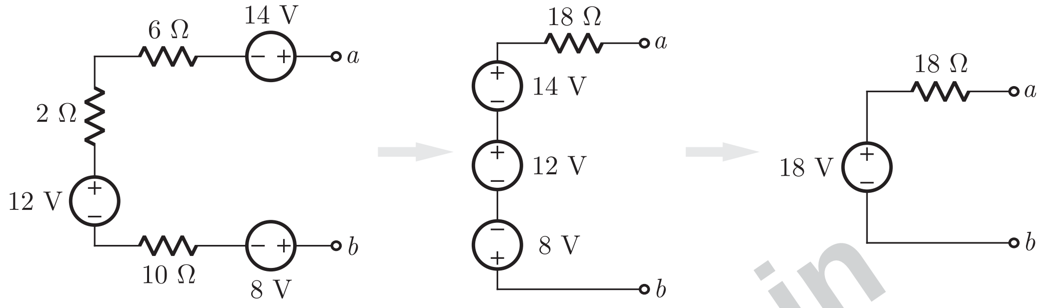
SOL 5.1.13 Option (C) is correct.

Combining the parallel resistance and adding the parallel connected current sources.

$$9\text{ A} - 3\text{ A} = 6\text{ A (upward)}$$
$$3\,\Omega\,||\,6\,\Omega = 2\,\Omega$$



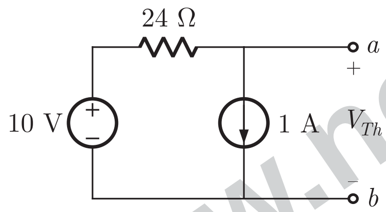
Source transformation of 6 A source



SOL 5.1.14 Option (D) is correct.

Thevenin Voltage : (Open Circuit Voltage)

The open circuit voltage between a-b can be obtained as



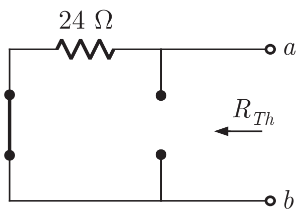
Writing KCL at node a

$$\frac{V_{Th}-10}{24}+1=0$$

$$V_{Th}-10+24=0 \text{ or } V_{Th}=-14 \text{ volt}$$

Thevenin Resistance :

To obtain Thevenin's resistance, we set all independent sources to zero i.e., short circuit all the voltage sources and open circuit all the current sources.



$$R_{Th}=24\,\Omega$$

SOL 5.1.15 Option (B) is correct.

Thevenin Voltage :

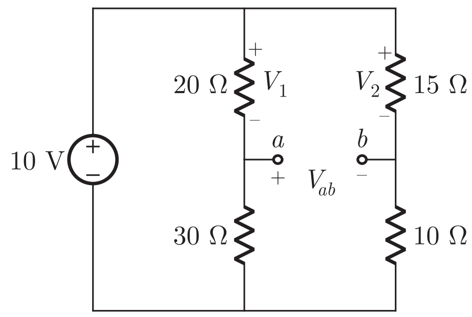
Using voltage division $V_1=\frac{20}{20+30}(10)=4$ volt

and, $V_2=\frac{15}{15+10}(10)=6$ volt

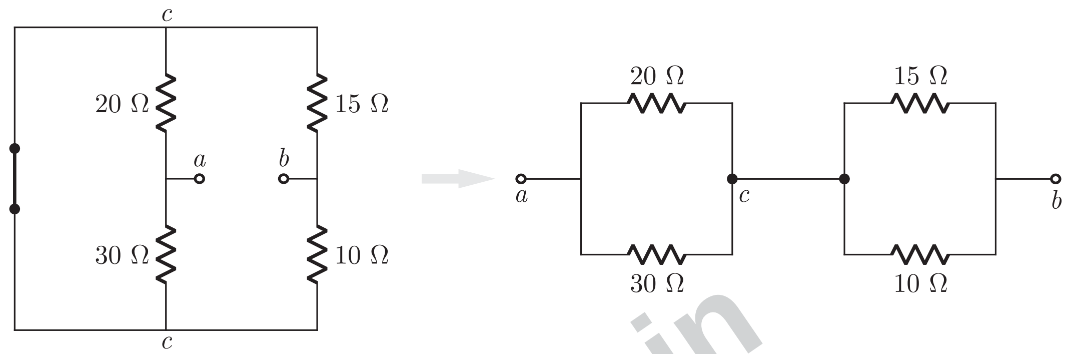
Applying KVL, $V_1-V_2+V_{ab}=0$

$$4-6+V_{ab}=0$$

$$V_{Th} = V_{ab} = -2 \text{ volt}$$



Thevenin Resistance :

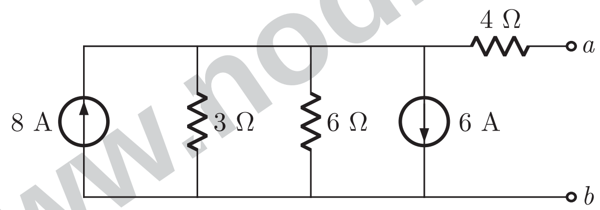


$$R_{ab} = [20\,\Omega\,||\,30\,\Omega] + [15\,\Omega\,||\,10\,\Omega] = 12\,\Omega + 6\,\Omega = 18\,\Omega$$

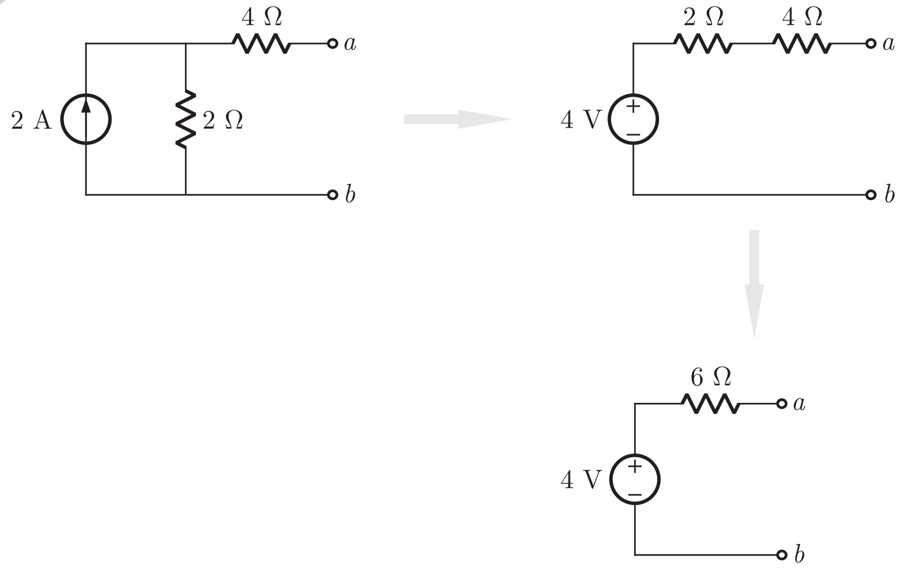
$$R_{Th} = R_{ab} = 18\,\Omega$$

SOL 5.1.16 Option (A) is a correct.

Using source transformation of 24 V source



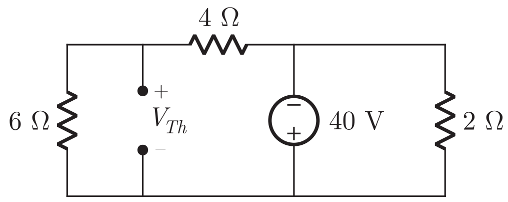
Adding parallel connected sources



So, $V_{Th} = 4\text{ V},\ R_{Th} = 6\,\Omega$

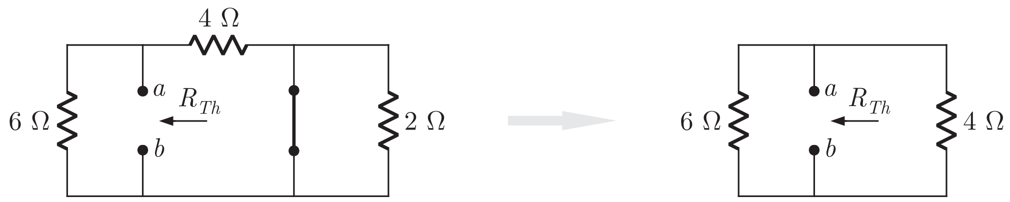
SOL 5.1.17 Option (A) is correct.

Thevenin Voltage: (Open Circuit Voltage)



$$V_{Th} = \frac{6}{6+4}(-40) = -24 \text{ volt} \qquad \text{(using voltage division)}$$

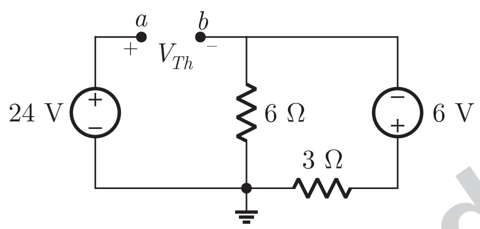
Thevenin Resistance :



$$R_{Th} = 6\,\Omega \,||\, 4\,\Omega = \frac{6\times 4}{6+4} = 2.4\,\Omega$$

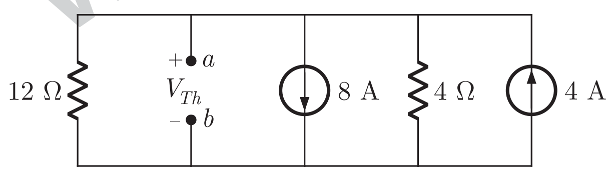
SOL 5.1.18 Option (B) is correct.

For the circuit of figure (A)



$$V_{Th} = V_a - V_b$$
$$V_a = 24 \text{ V}$$
$$V_b = \frac{6}{6+3}(-6) = -4 \text{ V} \qquad \text{(Voltage division)}$$
$$V_{Th} = 24 - (-4) = 28 \text{ V}$$

For the circuit of figure (B), using source transformation

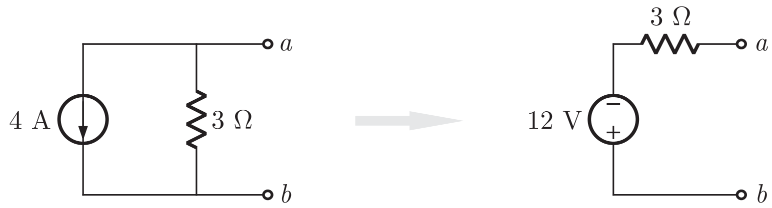


Combining parallel resistances,

$$12\,\Omega \,||\, 4\,\Omega = 3\,\Omega$$

Adding parallel current sources,

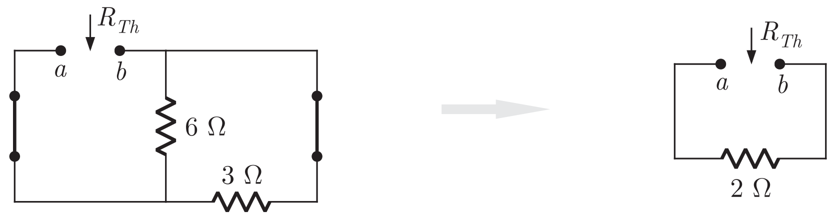
$$8 - 4 = 4 \text{ A (downward)}$$



$$V_{Th} = -12 \text{ V}$$

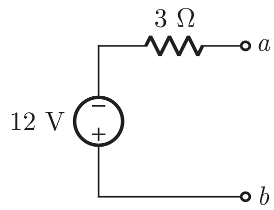
SOL 5.1.19 Option (C) is correct.

For the circuit for fig (A)



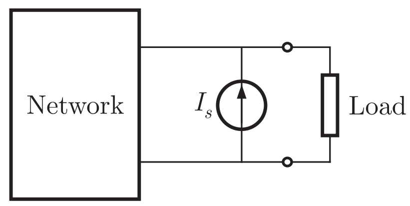
$$R_{Th} = R_{ab} = 6\,\Omega\,||\,3\,\Omega = 2\,\Omega$$

For the circuit of fig (B), as obtained in previous solution.



$$R_{Th} = 3\,\Omega$$

SOL 5.1.20 Option (B) is correct.



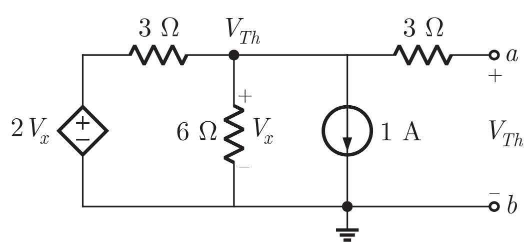
The current source connected in parallel with load does not affect Thevenin equivalent circuit. Thus, Thevenin equivalent circuit will contain its usual form of a voltage source in series with a resistor.

SOL 5.1.21 Option (C) is correct.

The network consists of resistor and dependent sources because if it has independent source then there will be an open circuit Thevenin voltage present.

SOL 5.1.22 Option (D) is correct.

Thevenin Voltage (Open Circuit Voltage) :



Applying KCL at top middle node

$$\frac{V_{Th} - 2V_x}{3} + \frac{V_{Th}}{6} + 1 = 0$$

$$\frac{V_{Th} - 2V_{Th}}{3} + \frac{V_{Th}}{6} + 1 = 0 \qquad (V_{Th} = V_x)$$

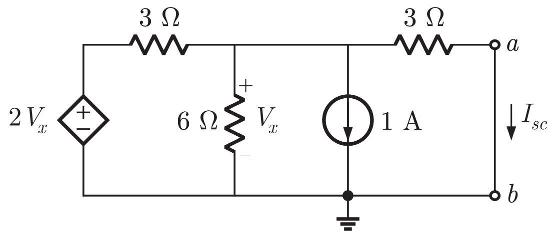
$$-2V_{Th} + V_{Th} + 6 = 0$$

$$V_{Th} = 6 \text{ volt}$$

Thevenin Resistance :

$$R_{Th} = \frac{\text{Open circuit voltage}}{\text{Short circuit current}} = \frac{V_{Th}}{I_{sc}}$$

To obtain Thevenin resistance, first we find short circuit current through a-b



Writing KCL at top middle node

$$\frac{V_x - 2V_x}{3} + \frac{V_x}{6} + 1 + \frac{V_x - 0}{3} = 0$$

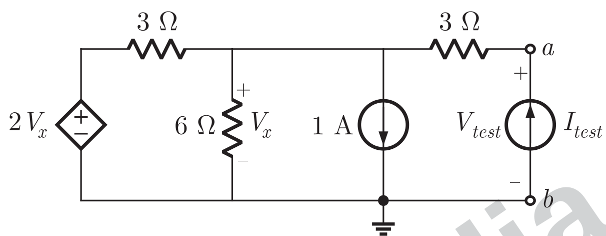
$$-2V_x + V_x + 6 + 2V_x = 0 \text{ or } V_x = -6 \text{ volt}$$

$$I_{sc} = \frac{V_x - 0}{3} = -\frac{6}{3} = -2 \text{ A}$$

Thevenin's resistance, $R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{6}{2} = -3\,\Omega$

ALTERNATIVE METHOD :

Since dependent source is present in the circuit, we put a test source across a-b to obtain Thevenin's equivalent.



By applying KCL at top middle node

$$\frac{V_x - 2V_x}{3} + \frac{V_x}{6} + 1 + \frac{V_x - V_{test}}{3} = 0$$

$$-2V_x + V_x + 6 + 2V_x - 2V_{test} = 0$$

$$2V_{test} - V_x = 6 \qquad ...(1)$$

We have $I_{test} = \frac{V_{test} - V_x}{3}$

$$3I_{test} = V_{test} - V_x$$

$$V_x = V_{test} - 3I_{test}$$

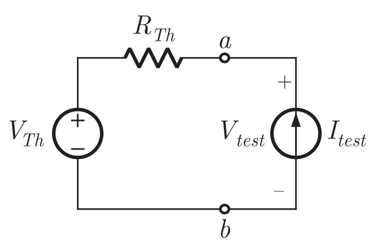
Put V_x into equation (1)

$$2V_{test} - (V_{test} - 3I_{test}) = 6$$

$$2V_{test} - V_{test} + 3I_{test} = 6$$

$$V_{test} = 6 - 3I_{test} \qquad ...(2)$$

For Thevenin's equivalent circuit



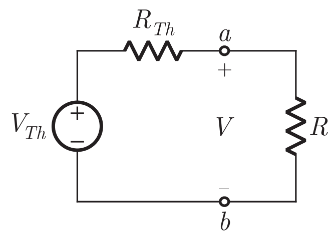
$$\frac{V_{test} - V_{Th}}{R_{Th}} = I_{test}$$

$$V_{test} = V_{Th} + R_{Th} I_{test} \qquad ...(3)$$

Comparing equation (2) and (3)

$$V_{Th} = 6 \text{ V}, \; R_{Th} = -3\,\Omega$$

SOL 5.1.23 Option (D) is correct.



Using voltage division

$$V = V_{Th}\left(\frac{R}{R+R_{Th}}\right)$$

From the table,

$$6 = V_{Th}\left(\frac{3}{3+R_{Th}}\right) \quad \text{...(1)}$$

$$8 = V_{Th}\left(\frac{8}{8+R_{Th}}\right) \quad \text{...(2)}$$

Dividing equation (1) and (2), we get

$$\frac{6}{8} = \frac{3(8+R_{Th})}{8(3+R_{Th})}$$

$$6+2R_{Th} = 8+R_{Th}$$

$$R_{Th} = 2\,\Omega$$

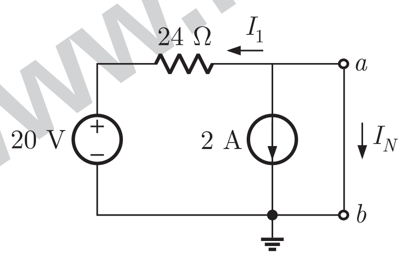
Substituting R_{Th} into equation (1)

$$6 = V_{Th}\left(\frac{3}{3+2}\right) \text{ or } V_{Th} = 10 \text{ V}$$

SOL 5.1.24 Option (C) is correct.

Norton Current : (Short Circuit Current)

The Norton equivalent current is equal to the short-circuit current that would flow when the load replaced by a short circuit as shown below



Applying KCL at node a

$$I_N + I_1 + 2 = 0$$

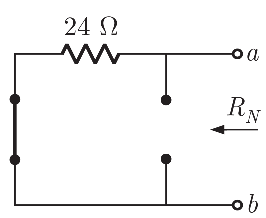
Since $$I_1 = \frac{0-20}{24} = -\frac{5}{6} \text{ A}$$

So, $$I_N - \frac{5}{6} + 2 = 0$$

$$I_N = -\frac{7}{6} \text{ A}$$

Norton Resistance :

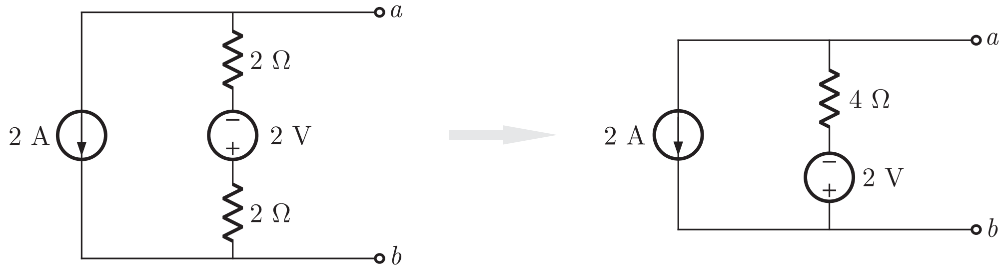
Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) to obtain Norton's equivalent resistance R_N.



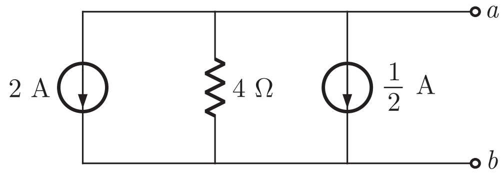
$$R_N = 24\,\Omega$$

SOL 5.1.25 Option (C) is correct.

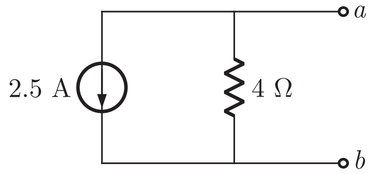
Using source transformation of 1 A source



Again, source transformation of 2 V source



Adding parallel current sources

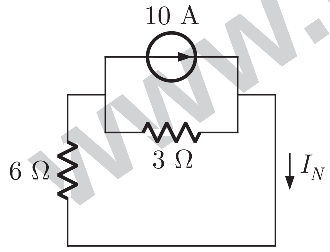


ALTERNATIVE METHOD :

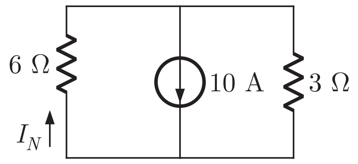
Try to solve the problem using superposition method.

SOL 5.1.26 Option (C) is correct.

Short circuit current across terminal a-b is



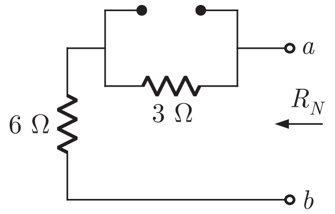
For simplicity circuit can be redrawn as



$$I_N = \frac{3}{3+6}(10) \qquad \text{(Current division)}$$

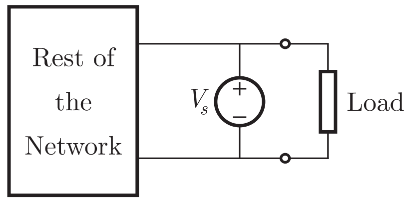
$$= 3.33 \text{ A}$$

Norton's equivalent resistance



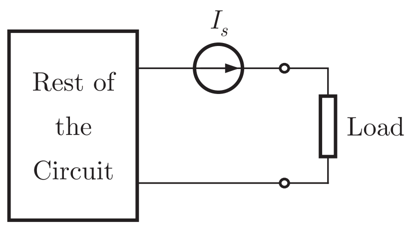
$$R_N = 6 + 3 = 9\,\Omega$$

SOL 5.1.27 Option (C) is correct.



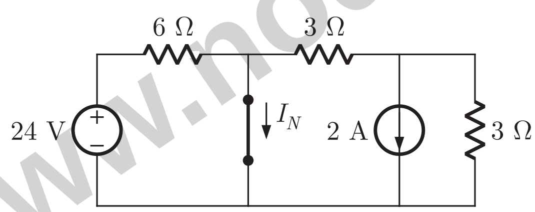
The voltage across load terminal is simply V_s and it is independent of any other current or voltage. So, Thevenin equivalent is $V_{Th} = V_s$ and $R_{Th} = 0$ (Voltage source is ideal).
Norton equivalent does not exist because of parallel connected voltage source.

SOL 5.1.28 Option (B) is correct.

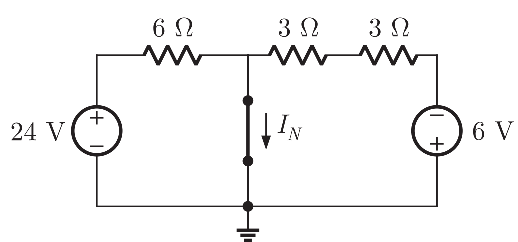


The output current from the network is equal to the series connected current source only, so $I_N = I_s$. Thus, effect of all other component in the network does not change I_N.
In this case Thevenin's equivalent is not feasible because of the series connected current source.

SOL 5.1.29 Option (C) is correct.
Norton Current : (Short Circuit Current)



Using source transformation



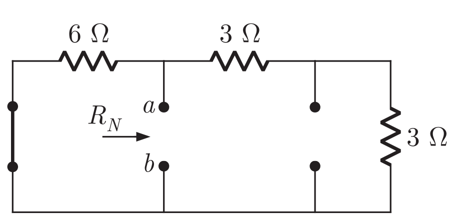
Nodal equation at top center node

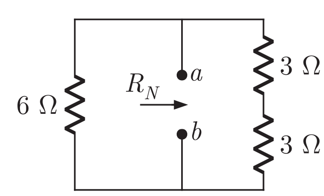
$$\frac{0-24}{6} + \frac{0-(-6)}{3+3} + I_N = 0$$

$$-4 + 1 + I_N = 0$$

$$I_N = 3\text{ A}$$

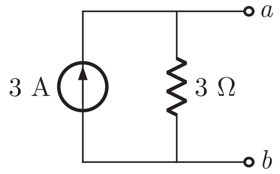
Norton Resistance :



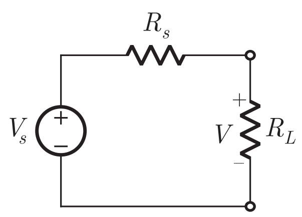


$$R_N = R_{ab} = 6\,||\,(3+3) = 6\,||\,6 = 3\,\Omega$$

So, Norton equivalent will be



SOL 5.1.30 Option (B) is correct.



$$V = V_s\left(\frac{R_L}{R_s+R_L}\right)$$

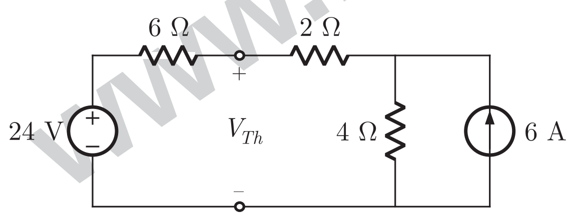
Power absorbed by R_L

$$P_L = \frac{(V)^2}{R_L} = \frac{V_s^2 R_L}{(R_s+R_L)^2}$$

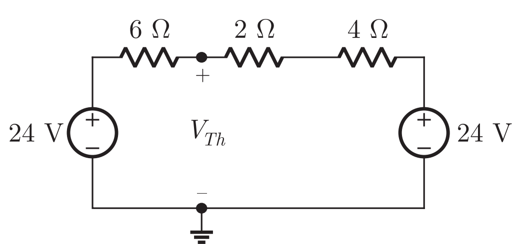
From above expression, it is known that power is maximum when $R_s = 0$

NOTE :
Do not get confused with maximum power transfer theorem. According to maximum power transfer theorem if R_L is variable and R_s is fixed then power dissipated by R_L is maximum when $R_L = R_s$.

SOL 5.1.31 Option (C) is correct.
We solve this problem using maximum power transfer theorem. First, obtain Thevenin equivalent across R_L.
Thevenin Voltage : (Open circuit voltage)



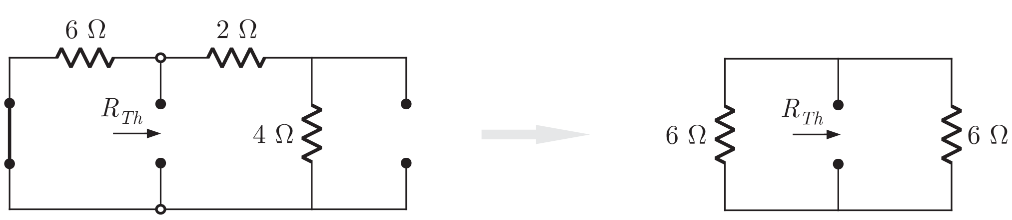
Using source transformation



Using nodal analysis $\dfrac{V_{Th}-24}{6} + \dfrac{V_{Th}-24}{2+4} = 0$

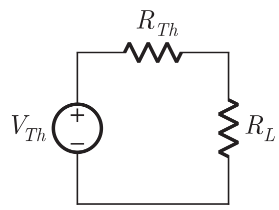
$$2V_{Th} - 48 = 0 \Rightarrow V_{Th} = 24\text{ V}$$

Thevenin Resistance :



$$R_{Th} = 6\,\Omega\,||\,6\,\Omega = 3\,\Omega$$

Circuit becomes as



For maximum power transfer

$$R_L = R_{Th} = 3\,\Omega$$

Value of maximum power

$$P_{\max} = \frac{(V_{Th})^2}{4R_L} = \frac{(24)^2}{4\times 3} = 48\text{ W}$$

SOL 5.1.32 Option (D) is correct.

This can be solved by reciprocity theorem. But we have to take care that the polarity of voltage source have the same correspondence with branch current in each of the circuit.

In figure (B) and figure (C), polarity of voltage source is reversed with respect to direction of branch current so

$$\frac{V_1}{I_1} = -\frac{V_2}{I_2} = -\frac{V_3}{I_3}$$

$$I_2 = I_3 = -2\text{ A}$$

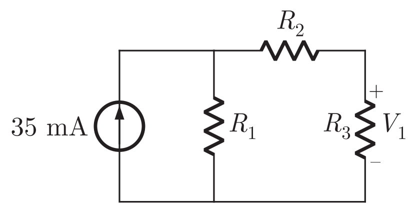
SOL 5.1.33 Option (C) is correct.

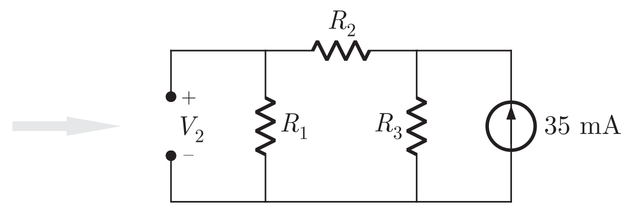
According to reciprocity theorem in any linear bilateral network when a single voltage source V_a in branch a produces a current I_b in branches b, then if the voltage source V_a is removed(i.e. branch a is short circuited) and inserted in branch b, then it will produce a current I_b in branch a.

So, $I_2 = I_1 = 20\text{ mA}$

SOL 5.1.34 Option (A) is correct.

According to reciprocity theorem in any linear bilateral network when a single current source I_a in branch a produces a voltage V_b in branches b, then if the current source I_a is removed(i.e. branch a is open circuited) and inserted in branch b, then it will produce a voltage V_b in branch a.





So, $V_2 = 2\text{ volt}$

SOL 5.1.35 Option (A) is correct.

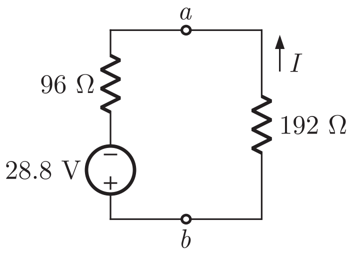
We use Millman's theorem to obtain equivalent resistance and voltage across a-b.

$$V_{ab} = \frac{-\frac{96}{240} + \frac{40}{200} + \frac{-80}{800}}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}} = -\frac{144}{5} = -28.8\text{ V}$$

The equivalent resistance

$$R_{ab} = \frac{1}{\frac{1}{240} + \frac{1}{200} + \frac{1}{800}} = 96\,\Omega$$

Now, the circuit is reduced as



$$I = \frac{28.8}{96+192} = 100 \text{ mA}$$

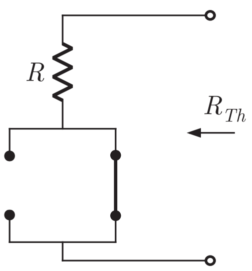
SOL 5.1.36 Option (B) is correct.

Thevenin Voltage: (Open circuit voltage):

The open circuit voltage will be equal to V, i.e. $V_{Th} = V$

Thevenin Resistance:

Set all independent sources to zero i.e. open circuit the current source and short circuit the voltage source as shown in figure

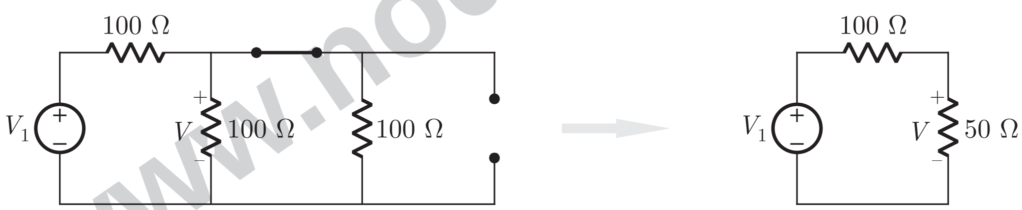


Open circuit voltage $= V_1$

SOL 5.1.37 Option (B) is correct.

V is obtained using super position.

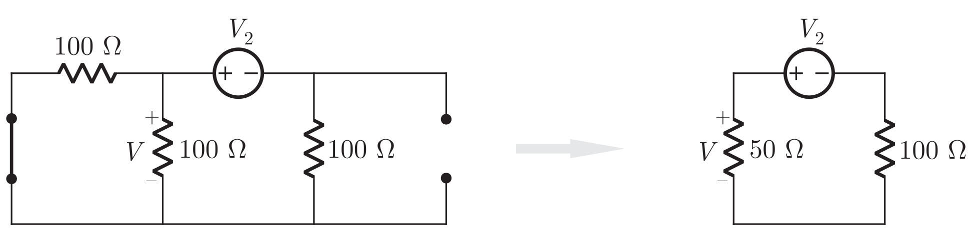
Due to source V_1 only : (Open circuit source I_3 and short circuit source V_2)



$$V = \frac{50}{100+50}(V_1) = \frac{1}{3}V_1 \qquad \text{(using voltage division)}$$

so, $$A = \frac{1}{3}$$

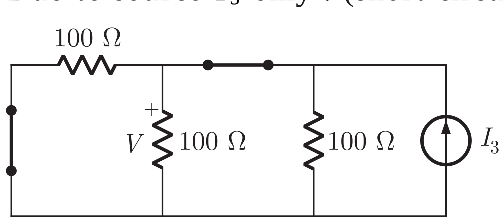
Due to source V_2 only : (Open circuit source I_3 and short circuit source V_1)



$$V = \frac{50}{100+50}(V_2) = \frac{1}{3}V_2 \qquad \text{(Using voltage division)}$$

So, $$B = \frac{1}{3}$$

Due to source I_3 only : (short circuit sources V_1 and V_2)



$$V = I_3[100 \,||\, 100 \,||\, 100] = I_3\left(\frac{100}{3}\right)$$

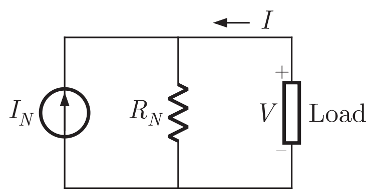
So, $$C = \frac{100}{3}$$

ALTERNATIVE METHOD :

Try to solve by nodal method, taking a supernode corresponding to voltage source V_2.

SOL 5.1.38 Option (C) is correct.

The circuit with Norton equivalent



So, $$I_N + I = \frac{V}{R_N}$$

$$I = \frac{V}{R_N} - I_N \qquad \text{(General form)}$$

From the given graph, the equation of line

$$I = 2V - 6$$

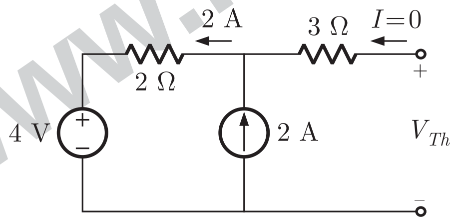
Comparing with general form

$$\frac{1}{R_N} = 2 \text{ or } R_N = 0.5\,\Omega$$

$$I_N = 6\text{ A}$$

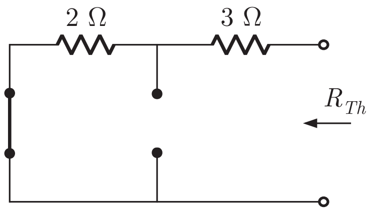
SOL 5.1.39 Option (D) is correct.

Thevenin voltage: (Open circuit voltage)



$$V_{Th} = 4 + (2 \times 2) = 4 + 4 = 8\text{ V}$$

Thevenin Resistance:



$$R_{Th} = 2 + 3 = 5\,\Omega = R_N$$

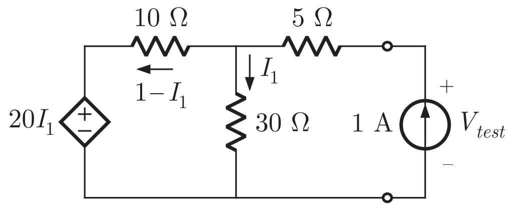
Norton Current:

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{8}{5}\text{ A}$$

SOL 5.1.40 Option (C) is correct.

Norton current, $I_N = 0$ because there is no independent source present in the circuit.

To obtain Norton resistance we put a 1 A test source across the load terminal as shown in figure.



Norton or Thevenin resistance

$$R_N = \frac{V_{test}}{1}$$

Writing KVL in the left mesh

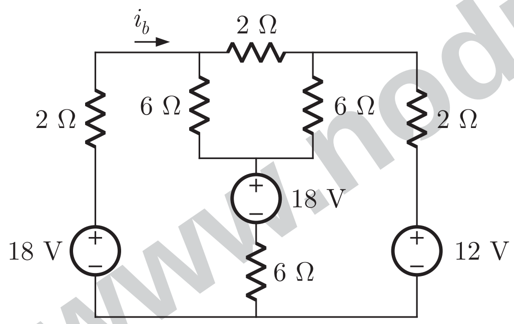
$$20I_1 + 10(1 - I_1) - 30I_1 = 0$$
$$20I_1 - 10I_1 - 30I_1 + 10 = 0$$
$$I_1 = 0.5\text{ A}$$

Writing KVL in the right mesh

$$V_{test} - 5(1) - 30I_1 = 0$$
$$V_{test} - 5 - 30(0.5) = 0$$
$$V_{test} - 5 - 15 = 0$$
$$R_N = \frac{V_{test}}{1} = 20\,\Omega$$

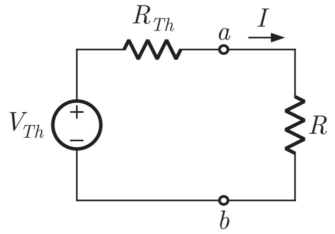
SOL 5.1.41 Option (C) is correct.

In circuit (b) transforming the 3 A source in to 18 V source all source are 1.5 times of that in circuit (a) as shown in figure.



Using principal of linearity, $I_b = 1.5I_a$

SOL 5.1.42 Option (B) is correct.



$$I = \frac{V_{Th}}{R + R_{Th}}$$

From the table, $$2 = \frac{V_{Th}}{3 + R_{Th}} \quad ...(1)$$

$$1.6 = \frac{V_{Th}}{5 + R_{Th}} \quad ...(2)$$

Dividing equation (1) and (2), we get

$$\frac{2}{1.6} = \frac{5 + R_{Th}}{3 + R_{Th}}$$
$$6 + 2R_{Th} = 8 + 1.6R_{Th}$$
$$0.4R_{Th} = 2$$

$$R_{Th} = 5\,\Omega$$

Substituting R_{Th} into equation (1)

$$2 = \frac{V_{Th}}{3+5}$$

$$V_{Th} = 2(8) = 16 \text{ V}$$

SOL 5.1.43 Option (D) is correct.

We have, $$I = \frac{V_{Th}}{R_{Th}+R}$$

$$V_{Th} = 16 \text{ V},\; R_{Th} = 5\,\Omega$$

$$I = \frac{16}{5+R} = 1$$

$$16 = 5 + R$$

$$R = 11\,\Omega$$

SOL 5.1.44 Option (B) is correct.

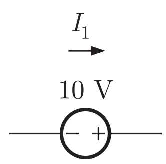


Fig.(A)

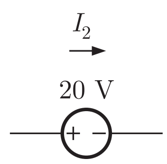


Fig.(B)

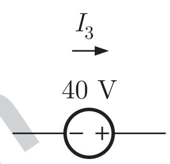


Fig.(C)

It can be solved by reciprocity theorem. Polarity of voltage source should have same correspondence with branch current in each of the circuit. Polarity of voltage source and current direction are shown below

So, $$\frac{V_1}{I_1} = -\frac{V_2}{I_2} = \frac{V_3}{I_3}$$

$$\frac{10}{2.5} = -\frac{20}{I_2} = \frac{40}{I_3}$$

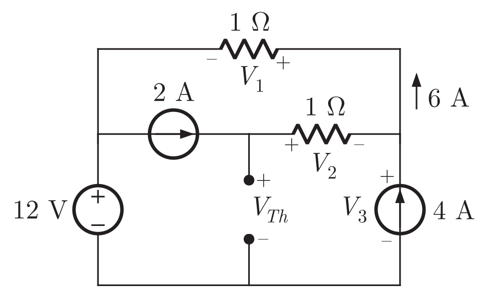
$$I_2 = -5 \text{ A}$$

$$I_3 = 10 \text{ A}$$

SOL 5.1.45 Option (A) is correct.

To obtain V-I equation we find the Thevenin equivalent across the terminal at which X is connected.

Thevenin Voltage : (Open Circuit Voltage)



$$V_1 = 6 \times 1 = 6 \text{ V}$$

$$12 + V_1 - V_3 = 0 \qquad \text{(KVL in outer mesh)}$$

$$V_3 = 12 + 6 = 18 \text{ V}$$

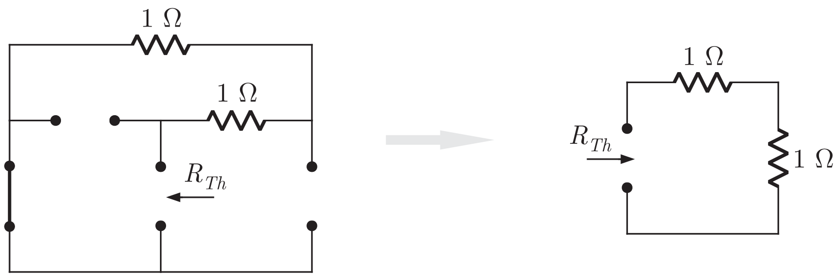
$$V_{Th} - V_2 - V_3 = 0 \qquad \text{(KVL in Bottom right mesh)}$$

$$V_{Th} = V_2 + V_3$$

$(V_2 = 2 \times 1 = 2 \text{ V})$

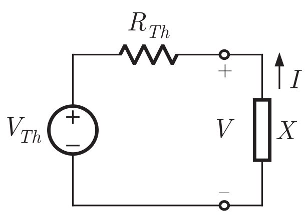
$$V_{Th} = 2 + 18 = 20 \text{ V}$$

Thevenin Resistance :



$$R_{Th} = 1 + 1 = 2\,\Omega$$

Now, the circuit becomes as



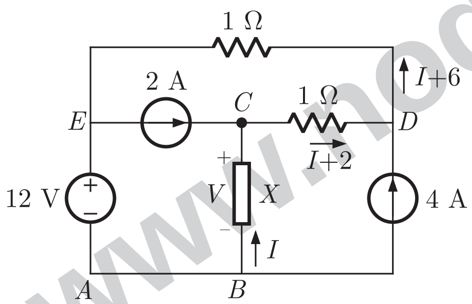
$$I = \frac{V - V_{Th}}{R_{Th}}$$

$$V = R_{Th} I + V_{Th}$$

so $$A = R_{Th} = 2\,\Omega$$

$$B = V_{Th} = 20 \text{ V}$$

ALTERNATIVE METHOD :



In the mesh $ABCDEA$, we have KVL equation as

$$V - 1(I + 2) - 1(I + 6) - 12 = 0$$

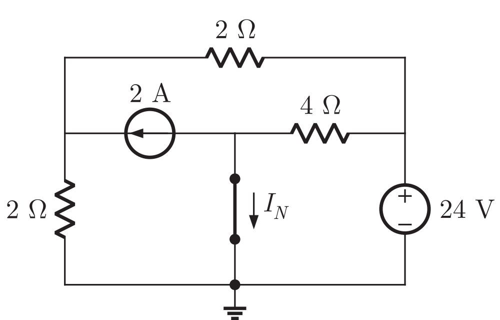
$$V = 2I + 20$$

So, $$A = 2, \quad B = 2$$

SOL 5.1.46 Option (A) is correct.

To obtain V-I relation, we obtain either Norton equivalent or Thevenin equivalent across terminal a-b.

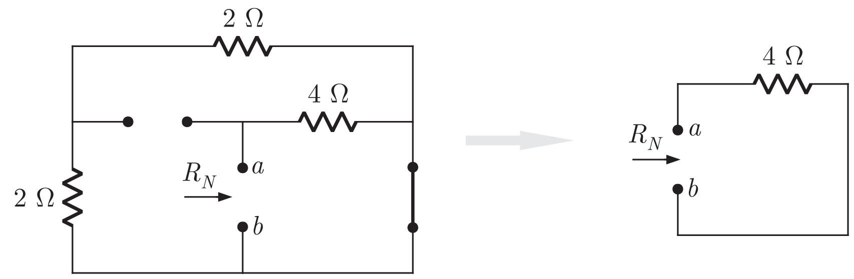
Norton Current (short circuit current) :



Applying nodal analysis at center node

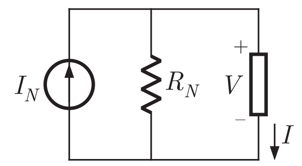
$$I_N + 2 = \frac{24}{4} \text{ or } I_N = 6 - 2 = 4 \text{ A}$$

Norton Resistance :



$$R_N = 4\,\Omega \qquad \text{(Both } 2\,\Omega \text{ resistor are short circuited)}$$

Now, the circuit becomes as

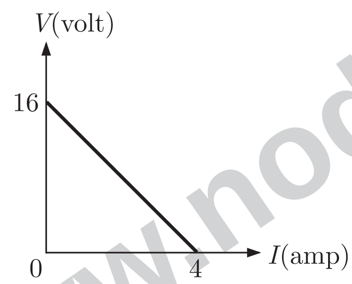


$$I_N = \frac{V}{R_N} + I$$

$$4 = \frac{V}{4} + I$$

$$16 = V + 4I$$

or $$V = -4I + 16$$

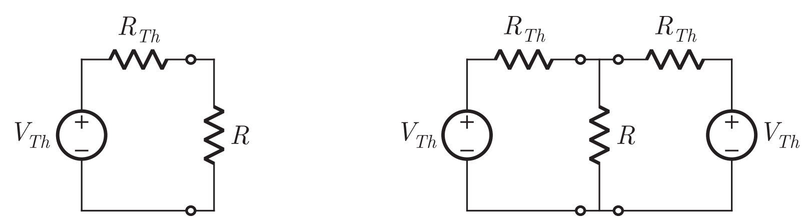


ALTERNATIVE METHOD :

Solve by writing nodal equation at the center node.

SOL 5.1.47 Option (C) is correct.

Let Thevenin equivalent of both networks are as shown below.



$$P = \left(\frac{V_{Th}}{R_{Th} + R}\right)^2 R \qquad \text{(Single network } N\text{)}$$

$$P' = \left(\frac{V_{Th}}{R + \frac{R_{Th}}{2}}\right)^2 R = 4\left(\frac{V_{Th}}{2R + R_{Th}}\right)^2 R \qquad \text{(Two } N \text{ are added)}$$

Thus $P < P' < 4P$

SOL 5.1.48 Option (C) is correct.

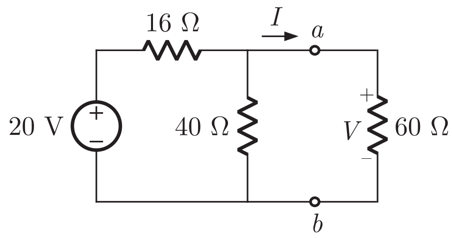
$$I_1 = \sqrt{\frac{P_1}{R}} \text{ and } I_2 = \sqrt{\frac{P_2}{R}}$$

Using superposition $$I = I_1 \pm I_2 = \sqrt{\frac{P_1}{R}} \pm \sqrt{\frac{P_2}{R}}$$

$$I^2 R = (\sqrt{P_1} \pm \sqrt{P_2})^2$$

SOL 5.1.49 Option (B) is correct.

From the substitution theorem we know that any branch within a circuit can be replaced by an equivalent branch provided that replacement branch has the same current through it and voltage across it as the original branch. The voltage across the branch in the original circuit



$$V = \frac{40\,||\,60}{(40\,||\,60)+16}(20) = \frac{24}{40}\times 20 = 12\text{ V}$$

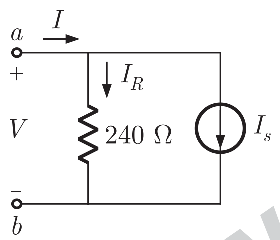
Current entering terminal a-b is

$$I = \frac{V}{R} = \frac{12}{60} = 200\text{ mA}$$

In fig(B), to maintain same voltage $V = 12$ V current through 240 Ω resistor must be

$$I_R = \frac{12}{240} = 50\text{ mA}$$

Using KCL at terminal a, as shown

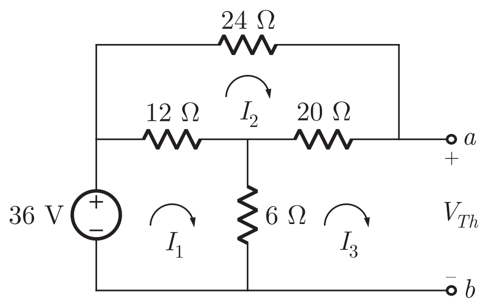


$$I = I_R + I_S$$
$$200 = 50 + I_s$$
$$I_s = 150\text{ mA}, \quad \text{down wards}$$

SOL 5.1.50 Option (B) is correct.

Thevenin voltage : (Open Circuit Voltage)

In the given problem, we use mesh analysis method to obtain Thevenin voltage



$$I_3 = 0 \qquad (a\text{-}b \text{ is open circuit})$$

Writing mesh equations

Mesh 1: $36 - 12(I_1 - I_2) - 6(I_1 - I_3) = 0$

$$36 - 12I_1 + 12I_2 - 6I_1 = 0 \qquad (I_3 = 0)$$

$$3I_1 - 2I_2 = 6 \qquad ...(1)$$

Mesh 2: $-24I_2 - 20(I_2 - I_3) - 12(I_2 - I_1) = 0$

$$-24I_2 - 20I_2 - 12I_2 + 12I_1 = 0 \qquad (I_3 = 0)$$

$$14I_2 = 3I_1 \qquad ...(2)$$

From equation (1) and (2)

$$I_1 = \frac{7}{3}\text{ A},\quad I_2 = \frac{1}{2}\text{ A}$$

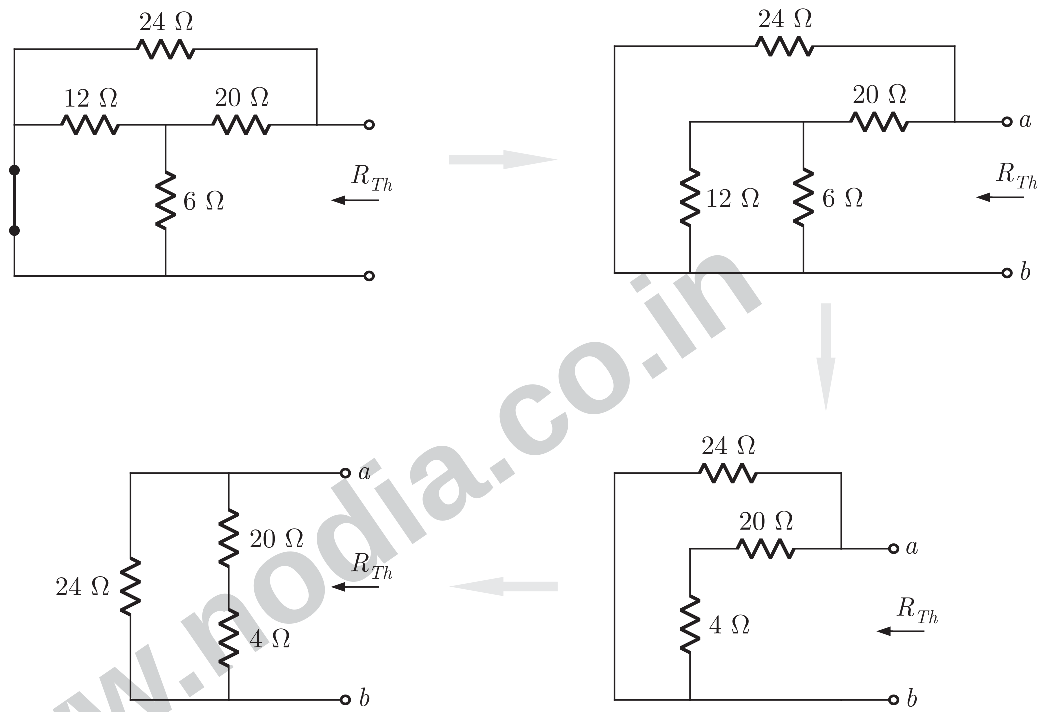
Mesh 3: $\quad -6(I_3 - I_1) - 20(I_3 - I_2) - V_{Th} = 0$

$$-6\left[0 - \frac{7}{3}\right] - 20\left[0 - \frac{1}{2}\right] - V_{Th} = 0$$

$$14 + 10 = V_{Th}$$

$$V_{Th} = 24 \text{ volt}$$

Thevenin Resistance :



$$R_{Th} = (20 + 4)\,||\,24\,\Omega = 24\,\Omega\,||\,24\,\Omega = 12\,\Omega$$

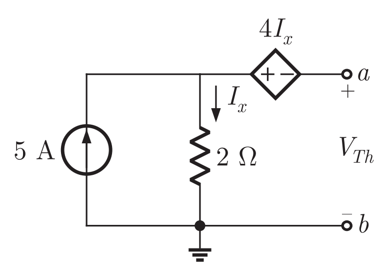
ALTERNATIVE METHOD :

V_{Th} can be obtained by writing nodal equation at node a and at center node.

SOL 5.1.51 Option (C) is correct.

We obtain Thevenin's equivalent across load terminal.

Thevenin Voltage : (Open Circuit Voltage)



Using KCL at top left node

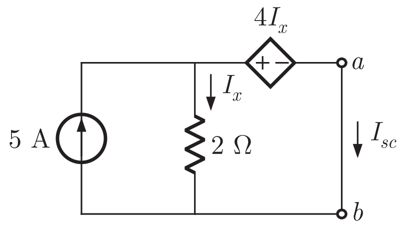
$$5 = I_x + 0 \text{ or } I_x = 5\text{ A}$$

Using KVL $\quad 2I_x - 4I_x - V_{Th} = 0$

$$2(5) - 4(5) = V_{Th} \text{ or } V_{Th} = -10 \text{ volt}$$

Thevenin Resistance :

First we find short circuit current through a-b



Using KCL at top left node

$$5 = I_x + I_{sc}$$
$$I_x = 5 - I_{sc}$$

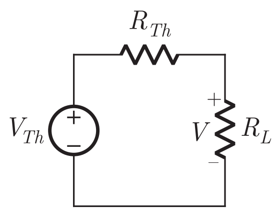
Applying KVL in the right mesh

$$2I_x - 4I_x + 0 = 0 \text{ or } I_x = 0$$

So, $$5 - I_{sc} = 0 \text{ or } I_{sc} = 5\text{ A}$$

Thevenin resistance, $$R_{Th} = \frac{V_{Th}}{I_{sc}} = -\frac{10}{5} = -2\,\Omega$$

Now, the circuit becomes as



$$V = V_{Th}\left(\frac{R}{R+R_L}\right) \qquad \text{(Using voltage division)}$$

So, $$V = V_{Th} = -10\text{ volt}$$
$$R = R_{Th} = -2\,\Omega$$

SOL 5.1.52 Option (D) is correct.

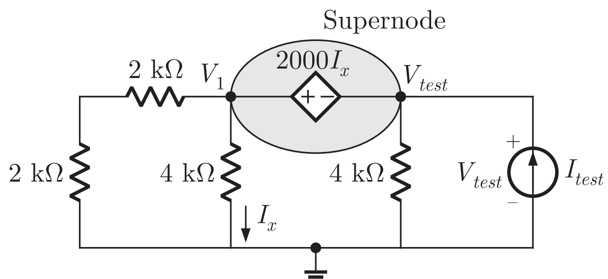
We obtain Thevenin equivalent across terminal a-b.

Thevenin Voltage :

Since there is no independent source present in the network, Thevenin voltage is simply zero i.e. $V_{Th} = 0$

Thevenin Resistance :

Put a test source across terminal a-b



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

For the super node

$$V_1 - V_{test} = 2000I_x$$
$$V_1 - V_{test} = 2000\left(\frac{V_1}{4000}\right) \qquad (I_x = V_1/4000)$$
$$\frac{V_1}{2} = V_{test} \text{ or } V_1 = 2V_{test}$$

Applying KCL to the super node

$$\frac{V_1 - 0}{4k} + \frac{V_1}{4k} + \frac{V_{test}}{4k} = I_{test}$$

$$2V_1 + V_{test} = 4 \times 10^3 I_{test}$$

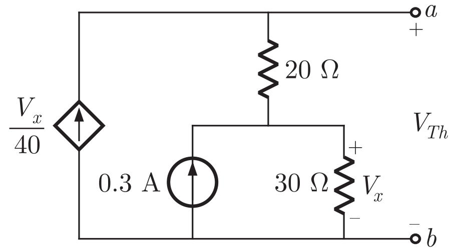
$$2(2V_{test}) + V_{test} = 4 \times 10^3 I_{test} \qquad (V_1 = 2V_{test})$$

$$\frac{V_{test}}{I_{test}} = \frac{4 \times 10^3}{5} = 800\,\Omega$$

SOL 5.1.53 Option (C) is correct.

Equation for V-I can be obtained with Thevenin equivalent across a-b terminals.

Thevenin Voltage: (Open circuit voltage)



Writing KCL at the top node

$$\frac{V_x}{40} = \frac{V_{Th} - V_x}{20}$$

$$V_x = 2V_{Th} - 2V_x$$

$$3V_x = 2V_{Th} \Rightarrow V_x = \frac{2}{3}V_{Th}$$

KCL at the center node

$$\frac{V_x - V_{Th}}{20} + \frac{V_x}{30} = 0.3$$

$$3V_x - 3V_{Th} + 2V_x = 18$$

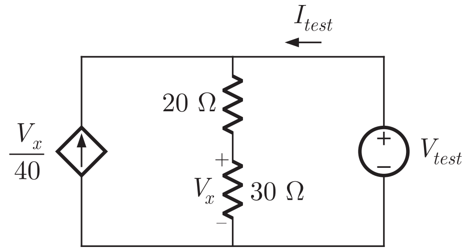
$$5V_x - 3V_{Th} = 18$$

$$5\left(\frac{2}{3}\right)V_{Th} - 3V_{Th} = 18 \qquad \left(V_x = \frac{2}{3}V_{Th}\right)$$

$$10V_{Th} - 9V_{Th} = 54 \text{ or } V_{Th} = 54 \text{ volt}$$

Thevenin Resistance :

When a dependent source is present in the circuit the best way to obtain Thevenin resistance is to remove all independent sources and put a test source across a-b terminals as shown in figure.



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

KCL at the top node

$$\frac{V_x}{40} + I_{test} = \frac{V_{test}}{20 + 30}$$

$$\frac{V_x}{40} + I_{test} = \frac{V_{test}}{50} \qquad ...(1)$$

$$V_x = \frac{30}{30 + 20}(V_{test}) = \frac{3}{5}V_{test} \qquad \text{(using voltage division)}$$

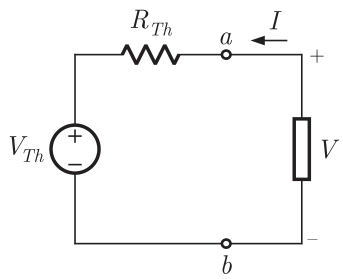
Substituting V_x into equation (1), we get

$$\frac{3V_{test}}{5(40)} + I_{test} = \frac{V_{test}}{50}$$

$$I_{test} = V_{test}\left(\frac{1}{50} - \frac{3}{200}\right) = \frac{V_{test}}{200}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = 200\,\Omega$$

The circuit now reduced as

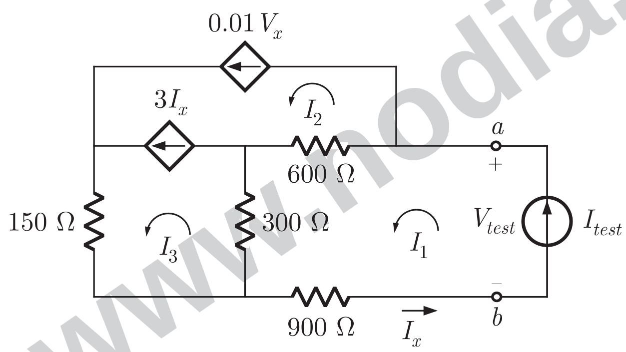


$$I = \frac{V - V_{Th}}{R_{Th}} = \frac{V - 54}{200}$$

$$V = 200I + 54$$

SOL 5.1.54 Option (D) is correct.

To obtain Thevenin resistance put a test source across the terminal a, b as shown.



$$V_{test} = V_x,\; I_{test} = I_x$$

Writing loop equation for the circuit

$$V_{test} = 600(I_1 - I_2) + 300(I_1 - I_3) + 900(I_1)$$

$$V_{test} = (600 + 300 + 900)I_1 - 600I_2 - 300I_3$$

$$V_{test} = 1800I_1 - 600I_2 - 300I_3 \qquad \text{...(1)}$$

The loop current are given as,

$$I_1 = I_{test},\quad I_2 = 0.3V_s,\quad \text{and}\quad I_3 = 3I_{test} + 0.2V_s$$

Substituting theses values into equation (1),

$$V_{test} = 1800I_{test} - 600(0.01V_s) - 300(3I_{test} + 0.01V_s)$$

$$V_{test} = 1800I_{test} - 6V_s - 900I_{test} - 3V_s$$

$$10V_{test} = 900I_{test} \text{ or } V_{test} = 90I_{test}$$

Thevenin resistance

$$R_{Th} = \frac{V_{test}}{I_{test}} = 90\,\Omega$$

Thevenin voltage or open circuit voltage will be zero because there is no independent source present in the network, i.e. $V_{oc} = 0$ V

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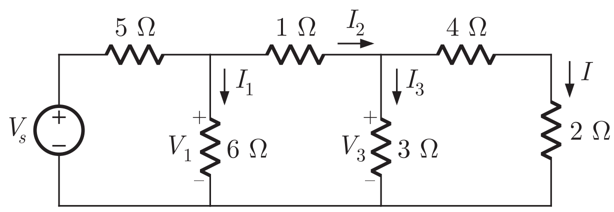
SOLUTIONS 5.2

SOL 5.2.1 Correct answer is 3.

We solve this problem using principal of linearity.



In the left, $4\,\Omega$ and $2\,\Omega$ are in series and has same current $I = 1\,\text{A}$.

$$V_3 = 4I + 2I \qquad \text{(using KVL)}$$
$$= 6I = 6\text{ V}$$
$$I_3 = \frac{V_3}{3} = \frac{6}{3} = 2\text{ A} \qquad \text{(using ohm's law)}$$
$$I_2 = I_3 + I \qquad \text{(using KCL)}$$
$$= 2 + 1 = 3\text{ A}$$
$$V_1 = (1)I_2 + V_3 \qquad \text{(using KVL)}$$
$$= 3 + 6 = 9\text{ V}$$
$$I_1 = \frac{V_1}{6} = \frac{9}{6} = \frac{3}{2}\text{ A} \qquad \text{(using ohm's law)}$$

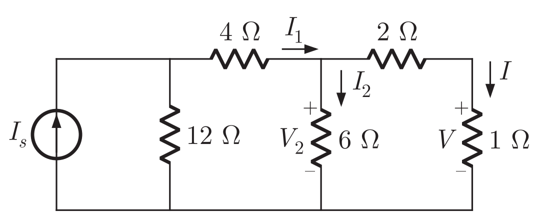
Applying principal of linearity

For $V_s = V_0$, $\quad I_1 = \frac{3}{2}\text{ A}$

So for $V_s = 2V_0$, $\quad I_1 = \frac{3}{2} \times 2 = 3\text{ A}$

SOL 5.2.2 Correct answer is 3.

We solve this problem using principal of linearity.



$$I = \frac{V}{1} = \frac{1}{1} = 1\text{ A} \qquad \text{(using ohm's law)}$$
$$V_2 = 2I + (1)I = 3\text{ V} \qquad \text{(using KVL)}$$
$$I_2 = \frac{V_2}{6} = \frac{3}{6} = \frac{1}{2}\text{ A} \qquad \text{(using ohm's law)}$$
$$I_1 = I_2 + I \qquad \text{(using KCL)}$$
$$= \frac{1}{2} + 1 = \frac{3}{2}\text{ A}$$

Applying principal of superposition

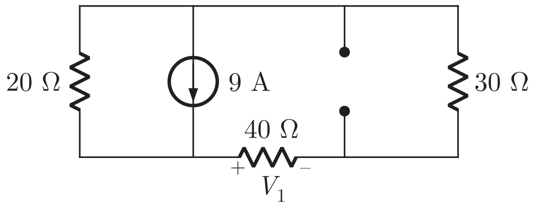
When $I_s = I_0$, and $V = 1\text{ V}$, $\quad I_1 = \frac{3}{2}\text{ A}$

So, if $I_s = 2I_0$, $$I_1 = \frac{3}{2} \times 2 = 3 \text{ A}$$

SOL 5.2.3 Correct answer is 160.

We solve this problem using superposition.

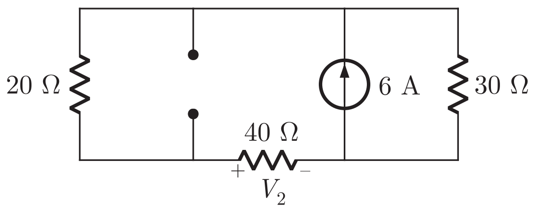
Due to 9 A source only : (Open circuit 6 A source)



Using current division

$$\frac{V_1}{40} = \frac{20}{20 + (40 + 30)}(9) \Rightarrow V_1 = 80 \text{ volt}$$

Due to 6 A source only : (Open circuit 9 A source)



Using current division,

$$\frac{V_2}{40} = \frac{30}{30 + (40 + 20)}(6) \Rightarrow V_2 = 80 \text{ volt}$$

From superposition,

$$V = V_1 + V_2 = 80 + 80 = 160 \text{ volt}$$

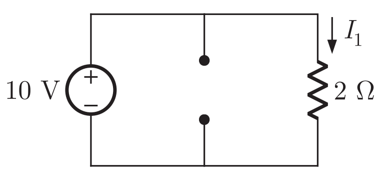
ALTERNATIVE METHOD :

The problem may be solved by transforming both the current sources into equivalent voltage sources and then applying voltage division.

SOL 5.2.4 Correct answer is 5.

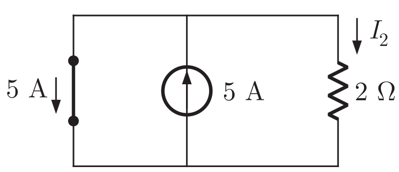
Using super position, we obtain I.

Due to 10 V source only : (Open circuit 5 A source)



$$I_1 = \frac{10}{2} = 5 \text{ A}$$

Due to 5 A source only : (Short circuit 10 V source)



$$I_2 = 0$$

$$I = I_1 + I_2 = 5 + 0 = 5 \text{ A}$$

ALTERNATIVE METHOD :

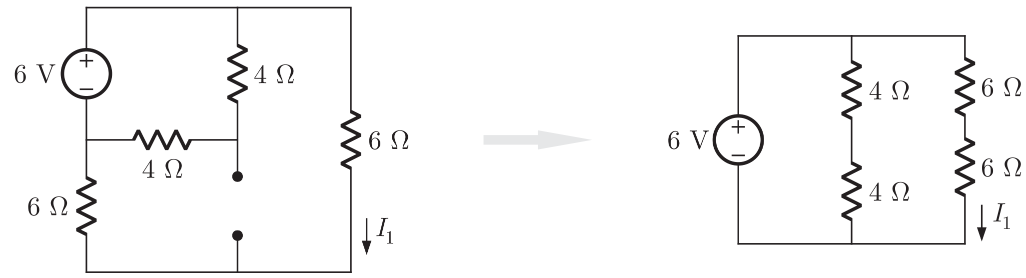
We can see that voltage source is in parallel with resistor and current source so voltage across parallel branches will be 10 V and $I = 10/2 = 5$ A

SOL 5.2.5

Correct answer is -0.5 .

Applying superposition,

Due to 6 V source only : (Open circuit 2 A current source)



$$I_1 = \frac{6}{6+6} = 0.5 \text{ A}$$

Due to 2 A source only : (Short circuit 6 V source)

$$I_2 = \frac{6}{6+6}(-2) \quad \text{(using current division)}$$

$$= -1 \text{ A}$$

$$I = I_1 + I_2 = 0.5 - 1 = -0.5 \text{ A}$$

ALTERNATIVE METHOD :

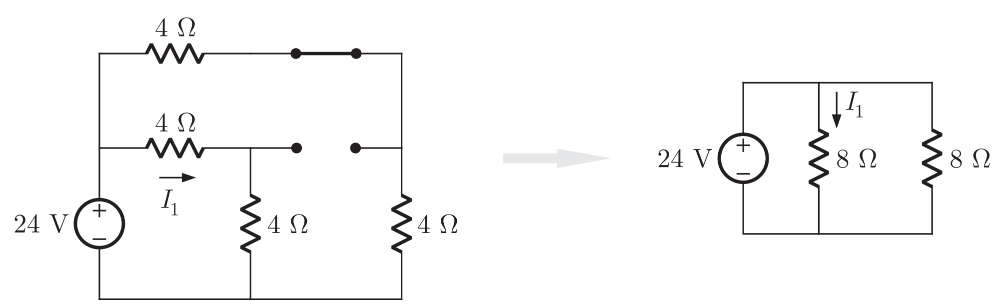
This problem may be solved by using a single KVL equation around the outer loop.

SOL 5.2.6

Correct answer is 4.

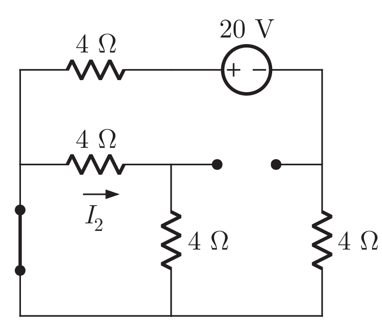
Applying superposition,

Due to 24 V Source Only : (Open circuit 2 A and short circuit 20 V source)



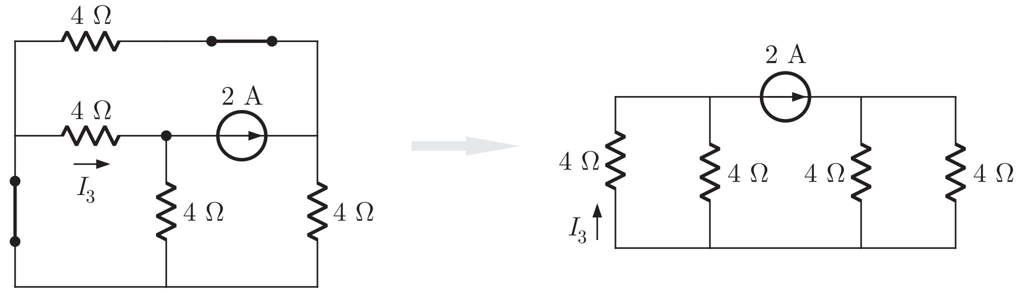
$$I_1 = \frac{24}{8} = 3 \text{ A}$$

Due to 20 V source only : (Short circuit 24 V and open circuit 2 A source)



So $\qquad I_2 = 0 \qquad$ (Due to short circuit)

Due to 2 A source only : (Short circuit 24 V and 20 V sources)



$$I_3 = \frac{4}{4+4}(2) \qquad \text{(using current division)}$$

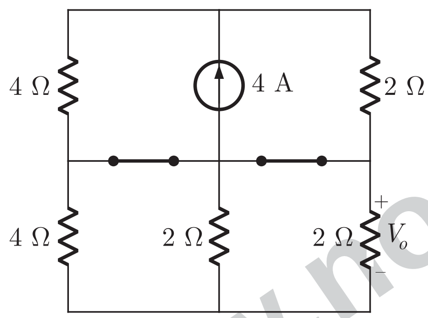
$$= 1\text{ A}$$

So $\qquad I = I_1 + I_2 + I_3 = 3 + 0 + 1 = 4\text{ A}$

Alternate Method: We can see that current in the middle $4\,\Omega$ resistor is $I - 2$, therefore I can be obtained by applying KVL in the bottom left mesh.

SOL 5.2.7 Correct answer is 0.

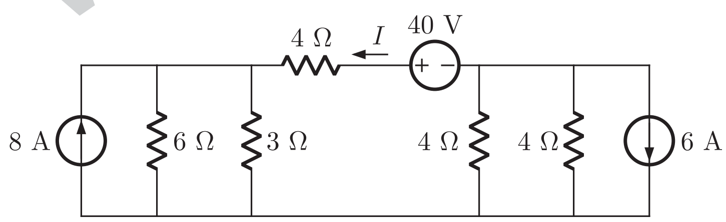
$$V_1 = V_2 = 0 \qquad \text{(short circuit both sources)}$$



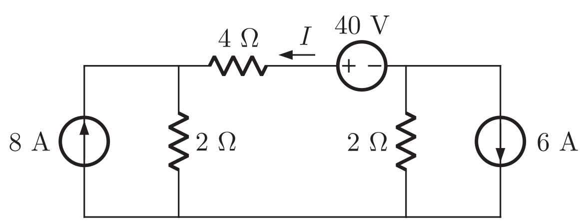
$$V_o = 0$$

SOL 5.2.8 Correct answer is 1.5 .

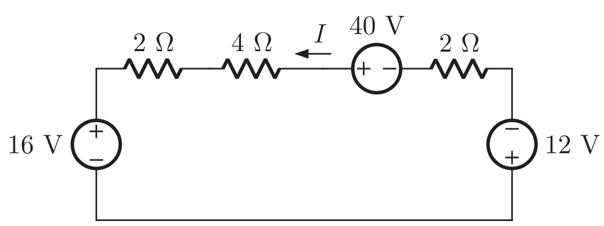
Using source transformation of 48 V source and the 24 V source



using parallel resistances combination



Source transformation of 8 A and 6 A sources



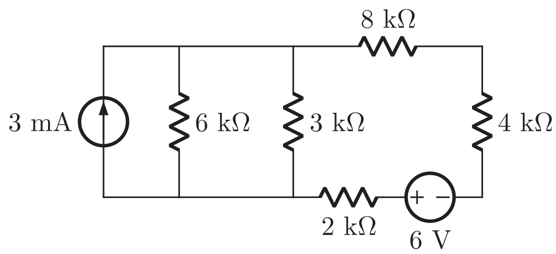
Writing KVL around anticlock wise direction

$$-12 - 2I + 40 - 4I - 2I - 16 = 0$$
$$12 - 8I = 0$$
$$I = \frac{12}{8} = 1.5 \text{ A}$$

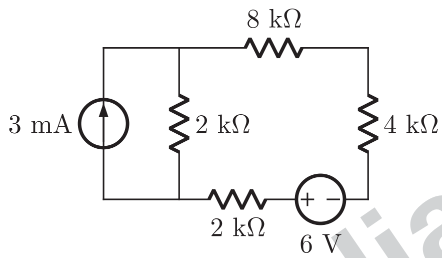
SOL 5.2.9 Correct answer is 2.25 .

We apply source transformation as follows.

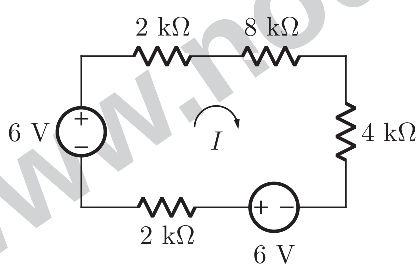
Transforming 3 mA source into equivalent voltage source and 18 V source into equivalent current source.



$6\,\text{k}\Omega$ and $3\,\text{k}\Omega$ resistors are in parallel and equivalent to $2\,\Omega$.



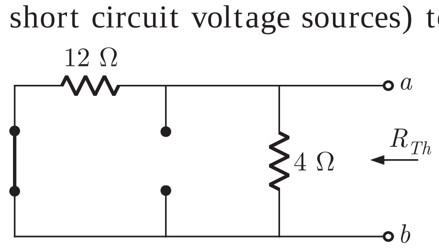
Again transforming $3\,\text{mA}$ source



$$I = \frac{6+6}{2+8+4+2} = \frac{3}{4} \text{ mA}$$
$$P_{4\,\text{k}\Omega} = I^2(4 \times 10^3) = \left(\frac{3}{4}\right)^2 \times 4 = 2.25 \text{ mW}$$

SOL 5.2.10 Correct answer is 3.

Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) to obtain R_{Th}



$$R_{Th} = 12\,\Omega \,||\, 4\,\Omega = 3\,\Omega$$

SOL 5.2.11 Correct answer is 16.8 .

Using current division

$$I_1 = \frac{(5+1)}{(5+1)+(3+1)}(12) = \frac{6}{6+4}(12) = 7.2 \text{ A}$$

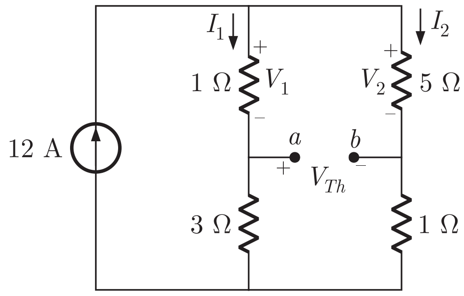
$$V_1 = I_1 \times 1 = 7.2 \text{ V}$$

$$I_2 = \frac{(3+1)}{(3+1)+(5+1)}(12) = 4.8 \text{ A}$$

$$V_2 = 5I_2 = 5 \times 4.8 = 24 \text{ V}$$

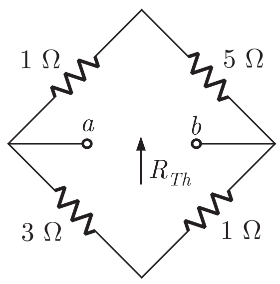
$$V_{Th} + V_1 - V_2 = 0 \qquad \text{(KVL)}$$

$$V_{Th} = V_2 - V_1 = 24 - 7.2 = 16.8 \text{ V}$$



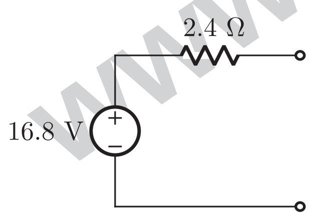
SOL 5.2.12 Correct answer is 7.

We obtain Thevenin's resistance across a-b and then use source transformation of Thevenin's circuit to obtain equivalent Norton circuit.

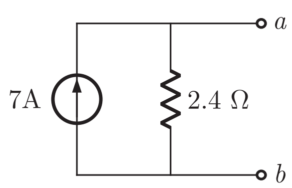


$$R_{Th} = (5+1)\,||\,(3+1) = 6\,||\,4 = 2.4\,\Omega$$

Thevenin's equivalent is



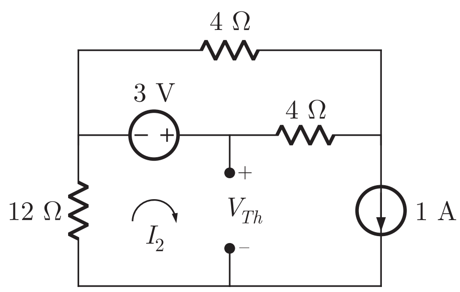
Norton equivalent



SOL 5.2.13 Correct answer is -0.5 .

Current I can be easily calculated by Thevenin's equivalent across $6\,\Omega$.

Thevenin Voltage : (Open Circuit Voltage)

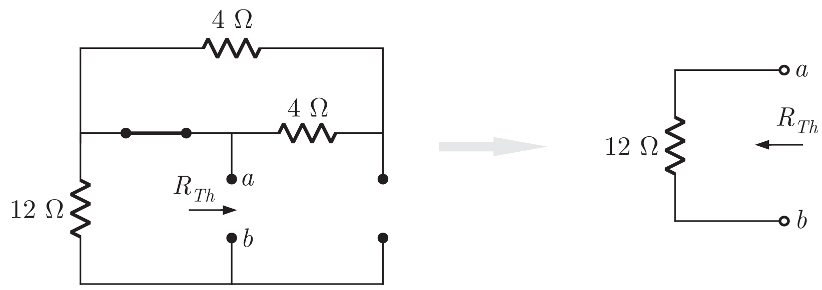


In the bottom mesh $I_2 = 1\text{ A}$

In the bottom left mesh $-V_{Th} - 12I_2 + 3 = 0$

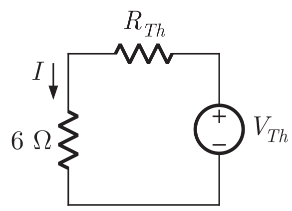
$$V_{Th} = 3 - (12)(1) = -9\text{ V}$$

Thevenin Resistance :



$R_{Th} = 12\,\Omega$ (both $4\,\Omega$ resistors are short circuit)

so, circuit becomes as



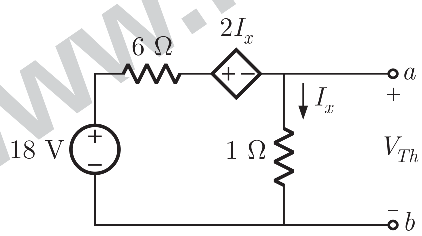
$$I = \frac{V_{Th}}{R_{Th} + 6} = \frac{-9}{12+6} = -\frac{9}{18} = -0.5\text{ A}$$

Note: The problem can be solved easily by a single node equation. Take the nodes connecting the top $4\,\Omega$, 3 V and $4\,\Omega$ as supernode and apply KCL.

SOL 5.2.14 Correct answer is 0.

We obtain Thevenin's equivalent across R.

Thevenin Voltage : (Open circuit voltage)



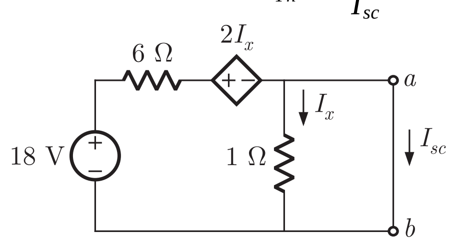
Applying KVL $18 - 6I_x - 2I_x - (1)I_x = 0$

$$I_x = \frac{18}{9} = 2\text{ A}$$

$$V_{Th} = (1)I_x = (1)(2) = 2\text{ V}$$

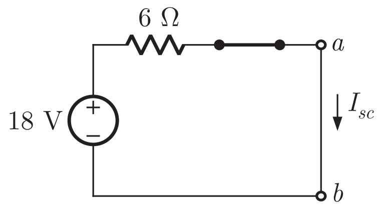
Thevenin Resistance :

$R_{Th} = \frac{V_{Th}}{I_{sc}}$ $I_{sc} \rightarrow$ Short circuit current



$I_x = 0$ (Due to short circuit)

So dependent source also becomes zero.

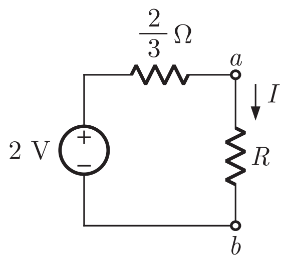


$$I_{sc} = \frac{18}{6} = 3\text{ A}$$

Thevenin resistance,

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{2}{3}\,\Omega$$

Now, the circuit becomes as



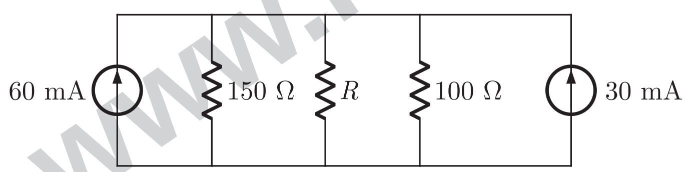
$$I = \frac{2}{\frac{2}{3} + R} = 3$$

$$2 = 2 + 3R$$

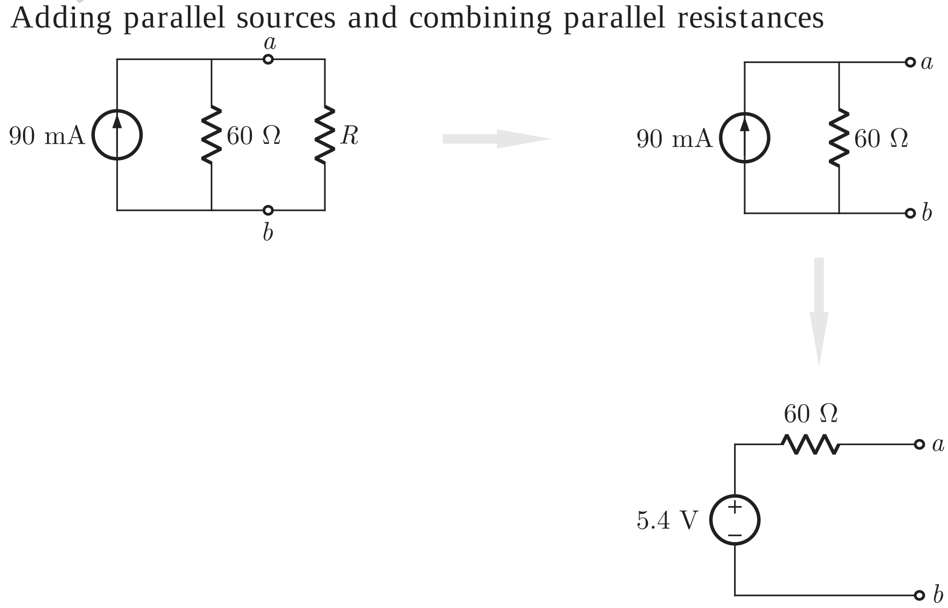
$$R = 0$$

SOL 5.2.15 Correct answer is 121.5 .

We obtain Thevenin's equivalent across R. By source transformation of both voltage sources



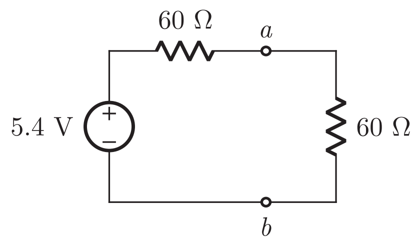
Adding parallel sources and combining parallel resistances



Here, $V_{Th} = 5.4\text{ V}, \quad R_{Th} = 60\,\Omega$

For maximum power transfer

$$R = R_{Th} = 60\,\Omega$$



Maximum Power absorbed by R

$$P = \frac{(V_{Th})^2}{4R} = \frac{(5.4)^2}{4 \times 60} = 121.5 \text{ mW}$$

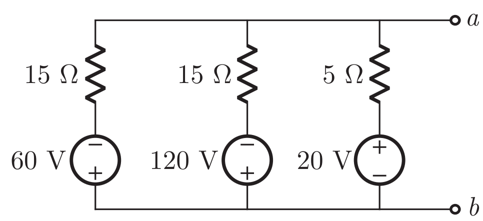
ALTERNATIVE METHOD :

Thevenin voltage (open circuit voltage) may be obtained using node voltage method also.

SOL 5.2.16

Correct answer is 3.

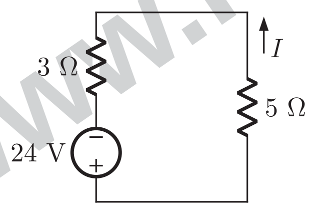
First we obtain equivalent voltage and resistance across terminal a-b using Millman's theorem.



$$V_{ab} = \frac{-\frac{60}{15} + (-\frac{120}{15}) + \frac{20}{5}}{\frac{1}{15} + \frac{1}{15} + \frac{1}{5}} = -24 \text{ V}$$

$$R_{ab} = \frac{1}{\frac{1}{15} + \frac{1}{15} + \frac{1}{5}} = 3\,\Omega$$

So, the circuit is reduced as

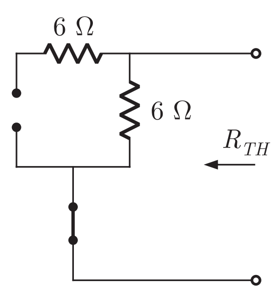


$$I = \frac{24}{3+5} = 3 \text{ A}$$

SOL 5.2.17

Correct answer is 6.

Set all independent sources to zero as shown,

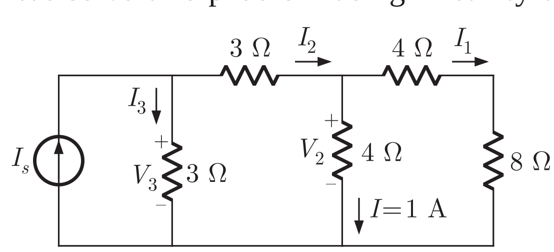


$$R_{Th} = 6\,\Omega$$

SOL 5.2.18

Correct answer is 0.5 .

We solve this problem using linearity and taking assumption that $I = 1$ A.



In the circuit,

$$V_2 = 4I = 4\text{ V} \qquad \text{(Using Ohm's law)}$$

$$I_2 = I + I_1 \qquad \text{(Using KCL)}$$

$$= 1 + \frac{V_2}{4+8} = 1 + \frac{4}{12} = \frac{4}{3}\text{ A}$$

$$V_3 = 3I_2 + V_2 \qquad \text{(Using KVL)}$$

$$= 3 \times \frac{4}{3} + 4 = 8\text{ V}$$

$$I_s = I_3 + I_2 \qquad \text{(Using KCL)}$$

$$= \frac{V_3}{3} + I_2 = \frac{8}{3} + \frac{4}{3} = 4\text{ A}$$

Applying superposition

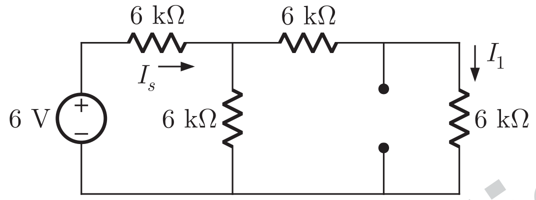
When $I_s = 4\text{ A}, \; I = 1\text{ A}$

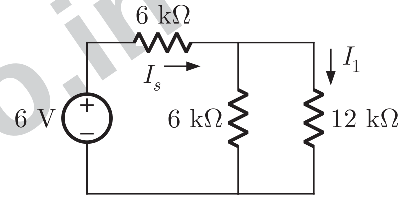
But actually $I_s = 2\text{ A}$, So $I = \frac{1}{4} \times 2 = 0.5\text{ A}$

SOL 5.2.19 Correct answer is -1.

Solving with superposition,

Due to 6 V Source Only : (Open Circuit 2 mA source)

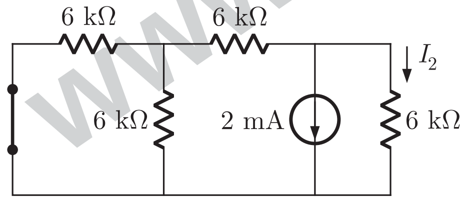




$$I_s = \frac{6}{6 + 6\,||\,12} = \frac{6}{6+4} = 0.6\text{ mA}$$

$$I_1 = \frac{6}{6+12}(I_s) = \frac{6}{18} \times 0.6 = 0.2\text{ mA} \qquad \text{(Using current division)}$$

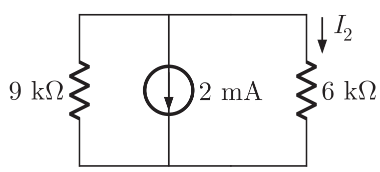
Due to 2 mA source only : (Short circuit 6 V source) :



Combining resistances,

$$6\text{ k}\Omega\,||\,6\text{ k}\Omega = 3\text{ k}\Omega$$

$$3\text{ k}\Omega + 6\text{ k}\Omega = 9\text{ k}\Omega$$



$$I_2 = \frac{9}{9+6}(-2) = -1.2\text{ mA} \qquad \text{(Current division)}$$

$$I = I_1 + I_2 \qquad \text{(Using superposition)}$$

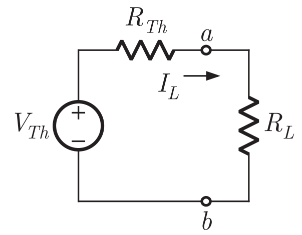
$$= 0.2 - 1.2 = -1\text{ mA}$$

ALTERNATIVE METHOD :

Try to solve the problem using source conversion.

SOL 5.2.20 Correct answer is 4.

We find Thevenin equivalent across a-b.



$$I_L = \frac{V_{Th}}{R_{Th}+R_L}$$

From the data given in table

$$10 = \frac{V_{Th}}{R_{Th}+2} \quad ...(1)$$

$$6 = \frac{V_{Th}}{R_{Th}+10} \quad ...(2)$$

Dividing equation (1) and (2), we get

$$\frac{10}{6} = \frac{R_{Th}+10}{R_{Th}+2}$$

$$10R_{Th}+20 = 6R_{Th}+60$$

$$4R_{Th} = 40 \Rightarrow R_{Th} = 10\,\Omega$$

Substituting R_{Th} into equation (1)

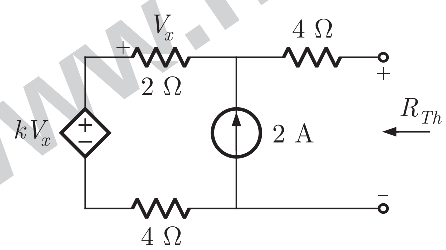
$$10 = \frac{V_{Th}}{10+2}$$

$$V_{Th} = 10(12) = 120 \text{ V}$$

For $R_L = 20\,\Omega$, $\quad I_L = \dfrac{V_{Th}}{R_{Th}+R_L}$

$$= \frac{120}{10+20} = 4 \text{ A}$$

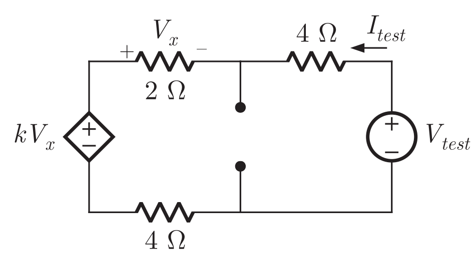
SOL 5.2.21 Correct answer is 4.



For maximum power transfer

$$R_{Th} = R_L = 2\,\Omega$$

To obtain R_{Th} set all independent sources to zero and put a test source across the load terminals.



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Using KVL,

$$V_{test} - 4I_{test} - 2I_{test} - kV_x - 4I_{test} = 0$$

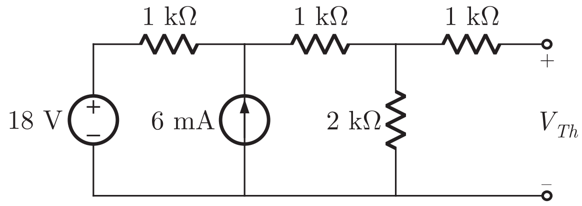
$$V_{test} - 10I_{test} - k(-2I_{test}) = 0 \quad (V_x = -2I_{test})$$

$$V_{test} = (10 - 2k) I_{test}$$
$$R_{Th} = \frac{V_{test}}{I_{test}} = 10 - 2k = 2$$
$$8 = 2k \text{ or } k = 4$$

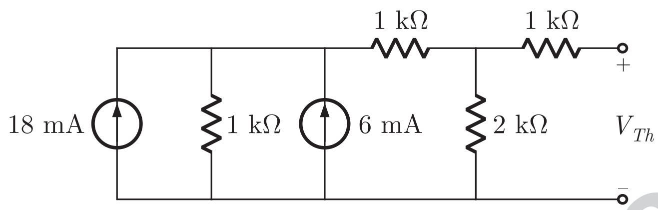
SOL 5.2.22 Correct answer is 18.

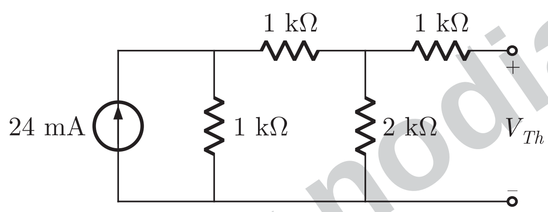
To calculate maximum power transfer, first we will find Thevenin equivalent across load terminals.

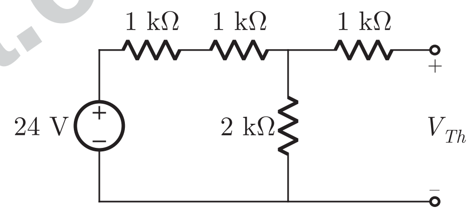
Thevenin Voltage: (Open Circuit Voltage)



Using source transformation

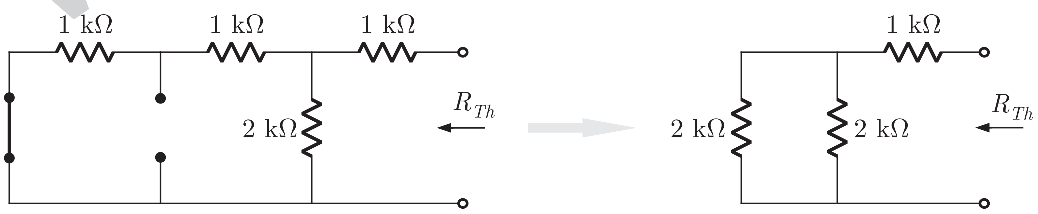






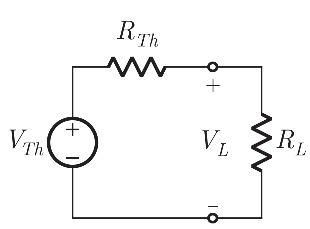
$$V_{Th} = \frac{2}{2+2}(24) \qquad \text{(Using voltage division)}$$
$$= 12 \text{ V}$$

Thevenin Resistance :



$$R_{Th} = 1 + 2\,||\,2 = 1 + 1 = 2 \text{ k}\Omega$$

Circuit becomes as



$$V_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

For maximum power transfer $R_L = R_{Th}$

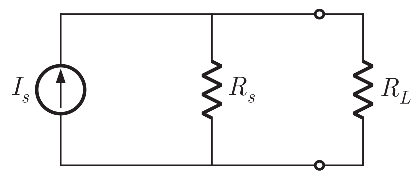
$$V_L = \frac{V_{Th}}{2R_{Th}} \times R_{Th} = \frac{V_{Th}}{2}$$

So maximum power absorbed by R_L

$$P_{max} = \frac{V_L^2}{R_L} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(12)^2}{4 \times 2} = 18 \text{ mW}$$

SOL 5.2.23 Correct answer is 22.5 .

The circuit is as shown below



When $R_L = 50\,\Omega$, power absorbed in load will be

$$\left(\frac{R_s}{R_s + 50} I_s\right)^2 50 = 20 \text{ kW} \qquad ...(1)$$

When $R_L = 200\,\Omega$, power absorbed in load will be

$$\left(\frac{R_s}{R_s + 200} I_s\right)^2 200 = 20 \text{ kW} \qquad ...(2)$$

Dividing equation (1) and (2), we have

$$(R_s + 200)^2 = 4(R_s + 50)^2$$

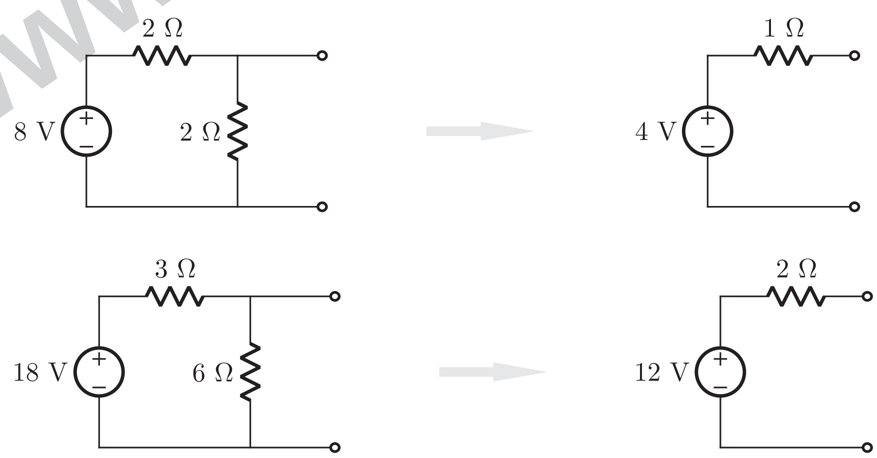
$$R_s = 100\,\Omega \text{ and } I_s = 30 \text{ A}$$

From maximum power transfer, the power supplied by source current I_s will be maximum when load resistance is equal to source resistance i.e. $R_L = R_s$. Maximum power is given as

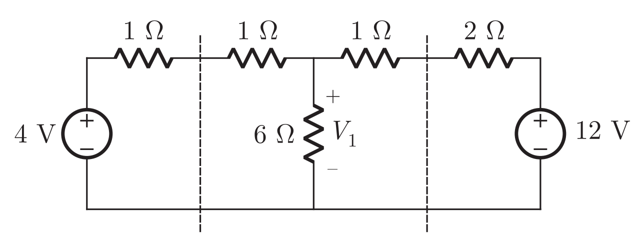
$$P_{max} = \frac{I_s^2 R_s}{4} = \frac{(30)^2 \times 100}{4} = 22.5 \text{ kW}$$

SOL 5.2.24 Correct answer is 6.

If we solve this circuit directly by nodal analysis, then we have to deal with three variables. We can replace the left most and write most circuit by their Thevenin equivalent as shown below.



Now the circuit becomes as shown



Writing node equation at the top center node

$$\frac{V_1 - 4}{1 + 1} + \frac{V_1}{6} + \frac{V_1 - 12}{1 + 2} = 0$$

$$\frac{V_1+4}{2}+\frac{V_1}{6}+\frac{V_1-12}{3} = 0$$

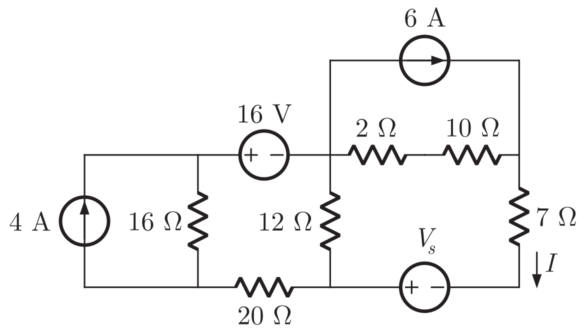
$$3V_1-12+V_1+2V_1-24 = 0$$

$$6V_1 = 36$$

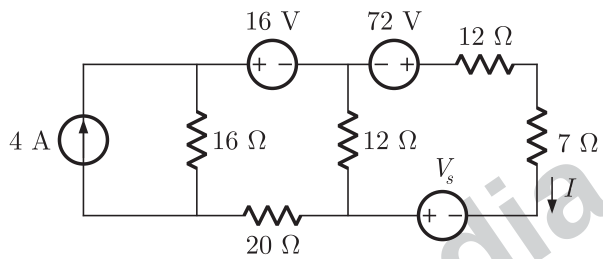
$$V_1 = 6\text{ V}$$

SOL 5.2.25 Correct answer is 56.

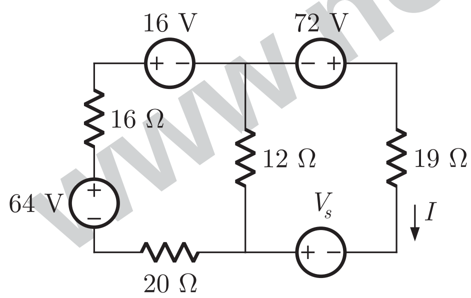
$6\,\Omega$ and $3\,\Omega$ resistors are in parallel, which is equivalent to $2\,\Omega$.



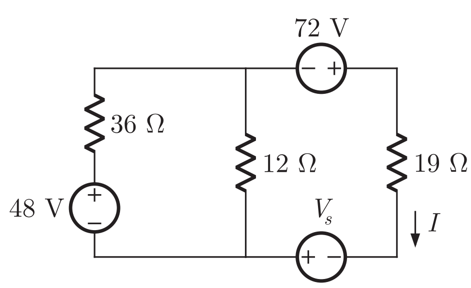
Using source transformation of 6 A source



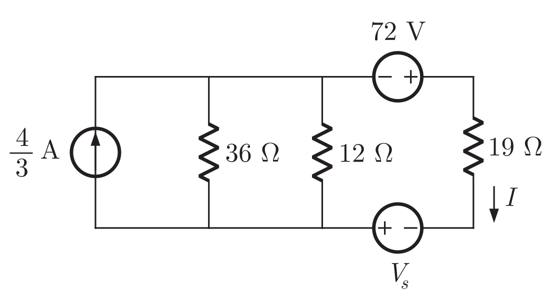
Source transform of 4 A source

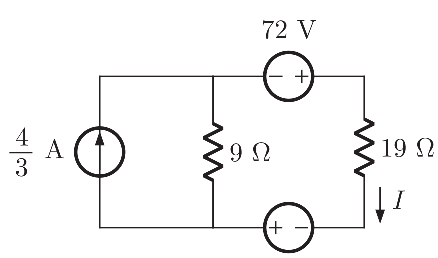


Adding series resistors and sources on the left

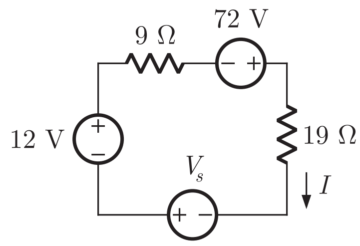


Source transformation of 48 V source





Source transformation of $\frac{4}{3}$ A source.



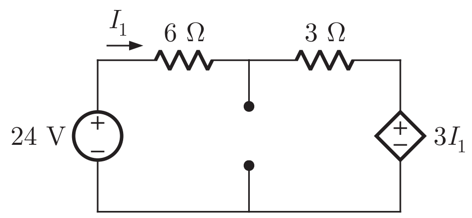
$$I = \frac{12 + 72 + V_s}{19 + 9}$$

$$V_s = (28 \times I) - 12 - 72 = (28 \times 5) - 12 - 72 = 56 \text{ V}$$

SOL 5.2.26 Correct answer is 0.5 .

We obtain I using superposition.

Due to 24 V source only : (Open circuit 6 A)

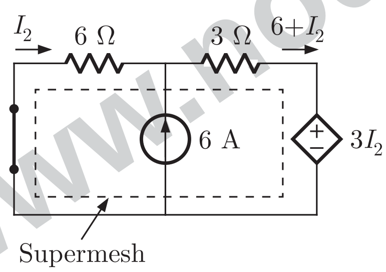


Applying KVL

$$24 - 6I_1 - 3I_1 - 3I_1 = 0$$

$$I_1 = \frac{24}{12} = 2 \text{ A}$$

Due to 6 A source only : (Short circuit 24 V source)



Applying KVL to supermesh

$$-6I_2 - 3(6 + I_2) - 3I_2 = 0$$

$$6I_2 + 18 + 3I_2 + 3I_2 = 0$$

$$I_2 = -\frac{18}{12} = -\frac{3}{2} \text{ A}$$

From superposition, $I = I_1 + I_2$

$$= 2 - \frac{3}{2} = \frac{1}{2} = 0.5 \text{ A}$$

ALTERNATIVE METHOD :

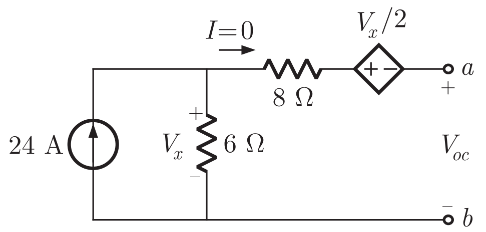
Note that current in $3\,\Omega$ resistor is $(I + 6)$ A, so by applying KVL around the outer loop, we can find current I.

SOL 5.2.27 Correct answer is 11.

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{\text{Open circuit voltage}}{\text{short circuit}}$$

Thevenin Voltage: (Open Circuit Voltage V_{oc})

Using source transformation of the dependent source



Applying KCL at top left node

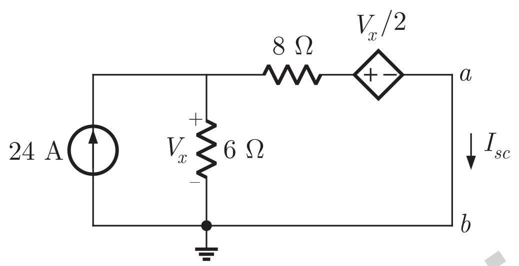
$$24 = \frac{V_x}{6} \Rightarrow V_x = 144 \text{ V}$$

Using KVL, $$V_x - 8I - \frac{V_x}{2} - V_{oc} = 0$$

$$144 - 0 - \frac{144}{2} = V_{oc}$$

$$V_{oc} = 72 \text{ V}$$

Short circuit current (I_{sc}):



Applying KVL in the right mesh

$$V_x - 8I_{sc} - \frac{V_x}{2} = 0$$

$$\frac{V_x}{2} = 8I_{sc}$$

$$V_x = 16I_{sc}$$

KCL at the top left node

$$24 = \frac{V_x}{6} + \frac{V_x - V_x/2}{8}$$

$$24 = \frac{V_x}{6} + \frac{V_x}{16}$$

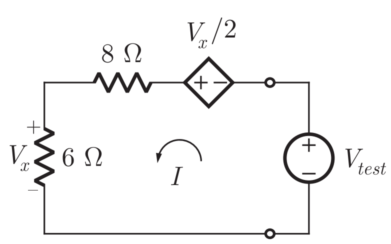
$$V_x = \frac{1152}{11} \text{ V}$$

$$I_{sc} = \frac{V_x}{16} = \frac{1152}{11 \times 16} = \frac{72}{11} \text{ A}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{72}{\frac{72}{11}} = 11\,\Omega$$

ALTERNATIVE METHOD :

We can obtain Thevenin equivalent resistance without calculating the Thevenin voltage (open circuit voltage). Set all independent sources to zero (i.e. open circuit current sources and short circuit voltage sources) and put a test source V_{test} between terminal a-b as shown



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

$$6I + 8I - \frac{V_x}{2} - V_{test} = 0 \qquad \text{(KVL)}$$

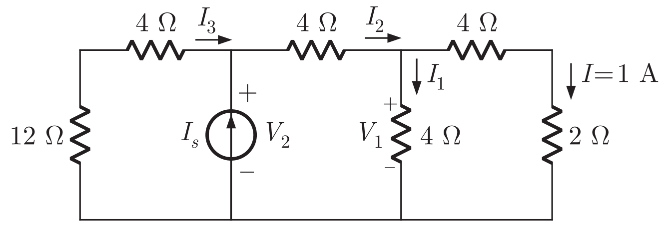
$$14I - \frac{6I}{2} - V_{test} = 0 \qquad V_x = 6I_{test} \text{ (Using Ohm's law)}$$

$$11I = V_{test}$$

So $$R_{Th} = \frac{V_{test}}{I_{test}} = 11\,\Omega$$

SOL 5.2.28 Correct answer is 4.

We solve this problem using linearity and assumption that $I = 1$ A.



$$V_1 = 4I + 2I \qquad \text{(Using KVL)}$$
$$= 6\text{ V}$$
$$I_2 = I_1 + I \qquad \text{(Using KCL)}$$
$$= \frac{V_1}{4} + I = \frac{6}{4} + 1 = 2.5\text{ A}$$
$$V_2 = 4I_2 + V_1 \qquad \text{(Using KVL)}$$
$$= 4(2.5) + 6 = 16\text{ V}$$
$$I_s + I_3 = I_2 \qquad \text{(Using KCL)}$$
$$I_s - \frac{V_2}{4+12} = I_2$$
$$I_s = \frac{16}{16} + 2.5 = 3.5\text{ A}$$

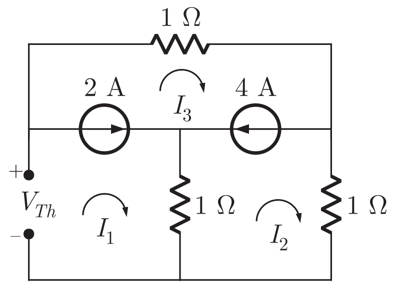
When $I_s = 3.5$ A, $I = 1$ A

But $I_s = 14$ A, so $I = \frac{.1}{3.5} \times 14 = 4$ A

SOL 5.2.29 Correct answer is 120.

This problem will easy to solve if we obtain Thevenin equivalent across the 12 V source.

Thevenin Voltage : (Open Circuit Voltage)



Mesh currents are

Mesh 1: $I_1 = 0$ (due to open circuit)

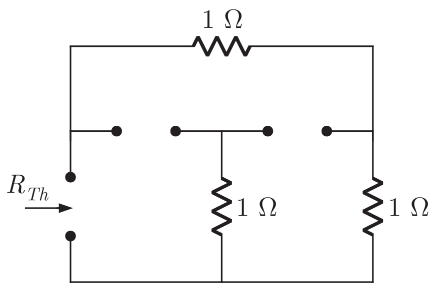
Mesh 2: $I_1 - I_3 = 2$ or $I_3 = -2$ A

Mesh 3: $I_3 - I_2 = 4$ or $I_2 = -6$ A

Mesh equation for outer loop

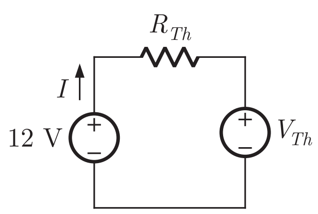
$$V_{Th} - 1 \times I_3 - 1 \times I_2 = 0$$
$$V_{Th} - (-2) - (-6) = 0$$
$$V_{Th} + 2 + 6 = 0$$
$$V_{Th} = -8 \text{ V}$$

Thevenin Resistance :



$$R_{Th} = 1 + 1 = 2\,\Omega$$

circuit becomes as

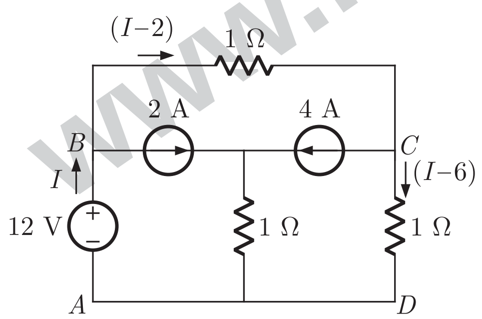


$$I = \frac{12 - V_{Th}}{R_{Th}} = \frac{12 - (-8)}{2} = 10 \text{ A}$$

Power supplied by 12 V source

$$P_{12\text{ V}} = 10 \times 12 = 120 \text{ W}$$

ALTERNATIVE METHOD :



KVL in the loop $ABCDA$

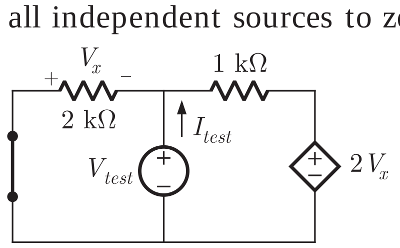
$$12 - 1(I - 2) - 1(I - 6) = 0$$
$$2I = 20$$
$$I = 10 \text{ A}$$

Power supplied by 12 V source

$$P_{12\text{ V}} = 10 \times 12 = 120 \text{ W}$$

SOL 5.2.30 Correct answer is 286.

For maximum power transfer $R_L = R_{Th}$. To obtain Thevenin resistance set all independent sources to zero and put a test source across load terminals.



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Writing KCL at the top center node

$$\frac{V_{test}}{2k} + \frac{V_{test} - 2V_x}{1k} = I_{test} \quad ...(1)$$

Also, $V_{test} + V_x = 0$ (KVL in left mesh)

so $V_x = - V_{test}$

Substituting $V_x = - V_{test}$ into equation (1)

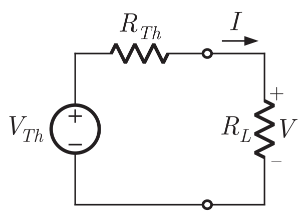
$$\frac{V_{test}}{2k} + \frac{V_{test} - 2(- V_{test})}{1k} = I_{test}$$

$$V_{test} + 6V_{test} = 2I_{test}$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{2}{7}\,\text{k}\Omega \simeq 286\,\Omega$$

SOL 5.2.31 Correct answer is 4.

Redrawing the circuit in Thevenin equivalent form



$$I = \frac{V_{Th} - V}{R_{Th}}$$

or, $V = - R_{Th} I + V_{Th}$ (General form)

From the given graph

$$V = - 4I + 8$$

So, by comparing $R_{Th} = 4\,\text{k}\Omega, \quad V_{Th} = 8\,\text{V}$

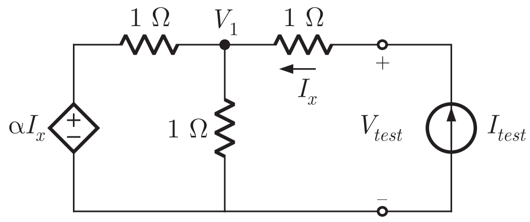
For maximum power transfer $R_L = R_{Th}$

Maximum power absorbed by R_L

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8)^2}{4 \times 4} = 4\,\text{mW}$$

SOL 5.2.32 Correct answer is 3.

To fine out Thevenin equivalent of the circuit put a test source between node a and b,



$$R_{Th} = \frac{V_{test}}{I_{test}}$$

Writing node equation at V_1

$$\frac{V_1 - \alpha I_x}{1} + \frac{V_1}{1} = I_x$$

$$2V_1 = (1 + \alpha) I_x \quad ...(1)$$

I_x is the branch current in $1\,\Omega$ resistor given as

$$I_x = \frac{V_{test} - V_1}{1}$$

$$V_1 = V_{test} - I_x$$

Substituting V_1 into equation (1)

$$2(V_{test} - I_x) = (1+\alpha)I_x$$

$$2V_{test} = (3+\alpha)I_x$$

$$2V_{test} = (3+\alpha)I_{test} \qquad (I_x = I_{test})$$

$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{3+\alpha}{2} = 3$$

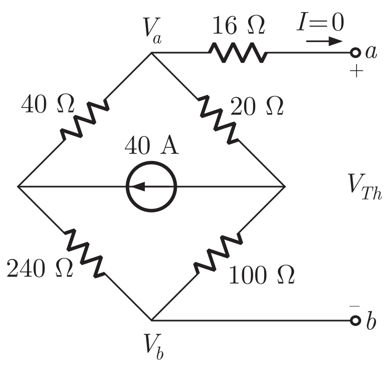
$$3+\alpha = 6$$

$$\alpha = 3\,\Omega$$

SOL 5.2.33 Correct answer is 16.

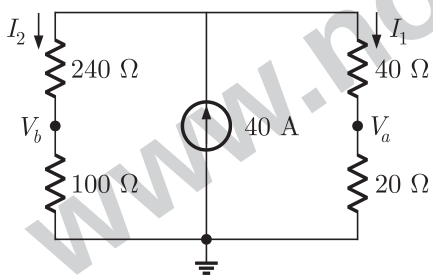
We obtain Thevenin equivalent across the load terminals

Thevenin Voltage : (Open circuit voltage)



$$V_{Th} = V_a - V_b$$

Rotating the circuit, makes it simple



$$I_1 = \frac{340}{340+60}(40) = 34\text{ A} \qquad \text{(Current division)}$$

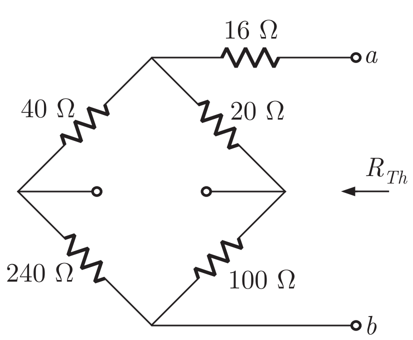
$$V_a = 20I_1 = 20 \times 34 = 680\text{ V} \qquad \text{(Ohm's Law)}$$

Similarly, $$I_2 = \frac{60}{60+340}(40) = 6\text{ A} \qquad \text{(Current division)}$$

$$V_b = 100I_2 = 100 \times 6 = 600\text{ V} \qquad \text{(Ohm's Law)}$$

Thevenin voltage $V_{Th} = 680 - 600 = 80$ V

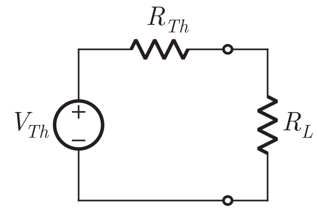
Thevenin Resistance :



$$R_{Th} = 16 + (240+40)\,||\,(20+100)$$

$$= 16 + (280 \,||\, 120) = 16 + 84 = 100\,\Omega$$

Now, circuit reduced as



For maximum power transfer

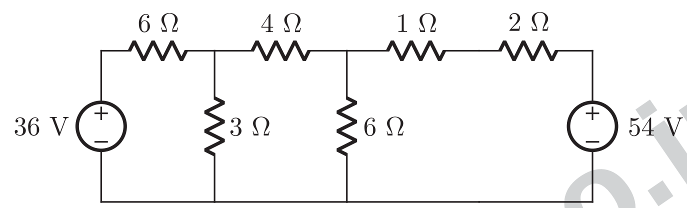
$$R_L = R_{Th} = 100\,\Omega$$

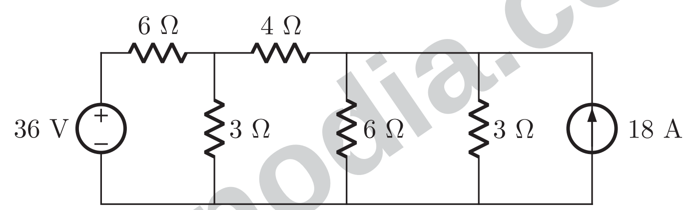
Maximum power transferred to R_L

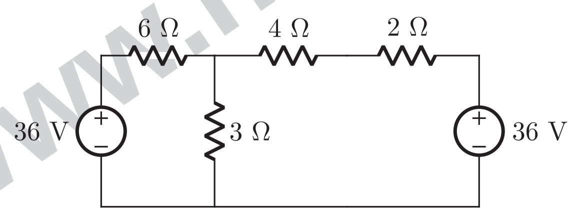
$$P_{\max} = \frac{(V_{Th})^2}{4R_L} = \frac{(80)^2}{4 \times 100} = 16 \text{ W}$$

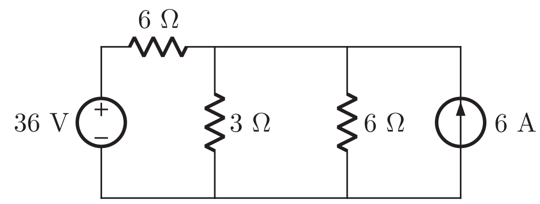
SOL 5.2.34 Correct answer is 108.

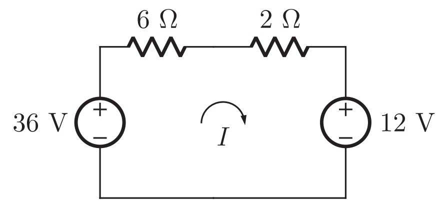
We use source transformation as follows











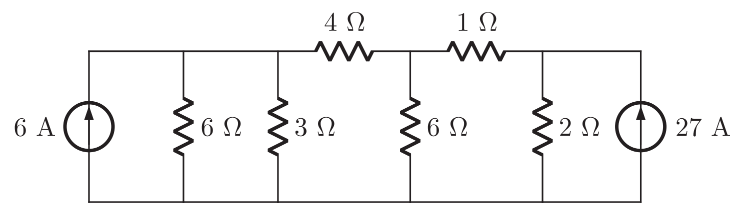
$$I = \frac{36 - 12}{6 + 2} = 3 \text{ A}$$

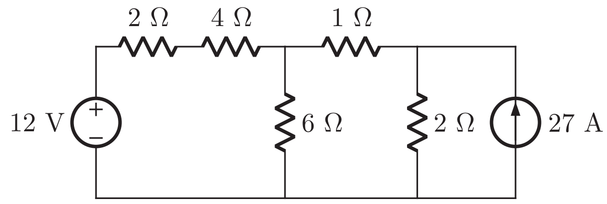
Power supplied by 36 V source

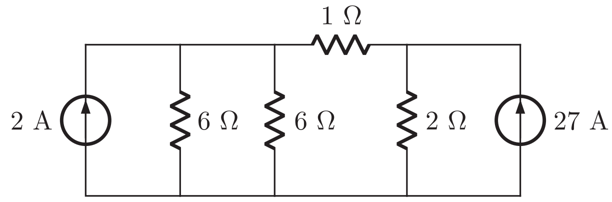
$$P_{36\text{ V}} = 3 \times 36 = 108 \text{ W}$$

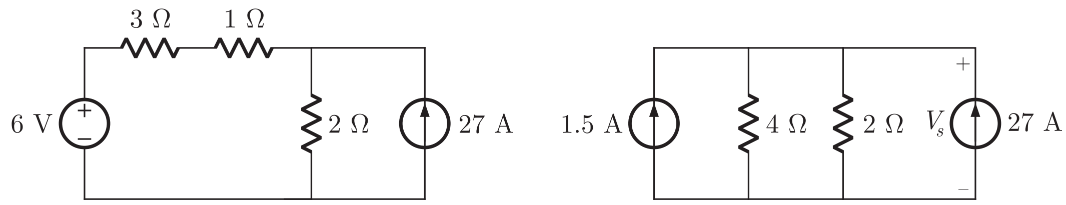
SOL 5.2.35 Correct answer is 1026.

Now, we do source transformation from left to right as shown









$$V_s = (27+1.5)(4\,\Omega\,||\,2\,\Omega)$$
$$= 28.5 \times \frac{4}{3}$$
$$= 38 \text{ V}$$

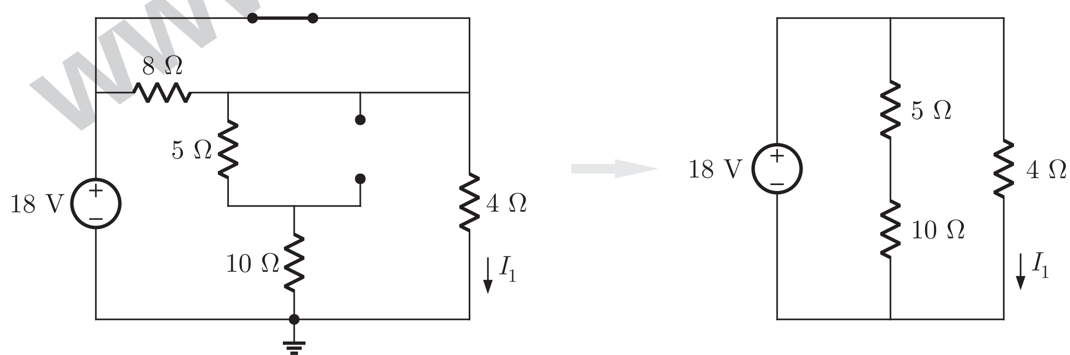
Power supplied by 27 A source

$$P_{27\,\text{A}} = V_s \times 27 = 38 \times 27$$
$$= 1026 \text{ W}$$

SOL 5.2.36 Correct answer is 9.

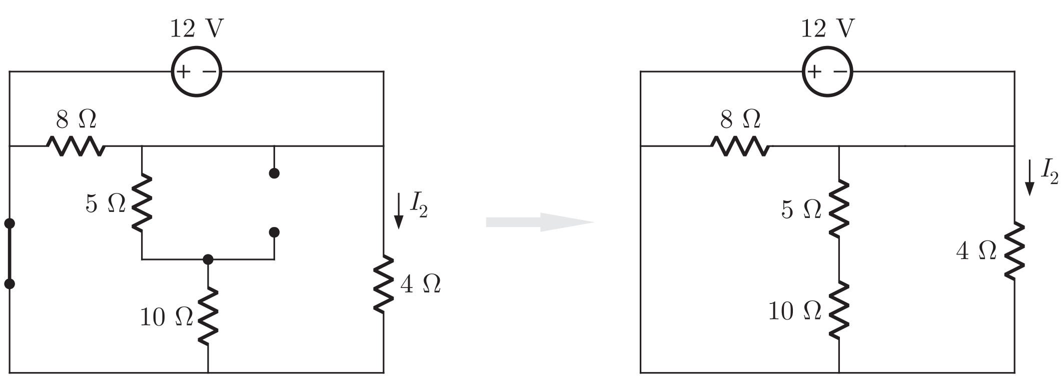
First, we find current I in the $4\,\Omega$ resistors using superposition.

Due to 18 V source only : (Open circuit 4 A and short circuit 12 V source)



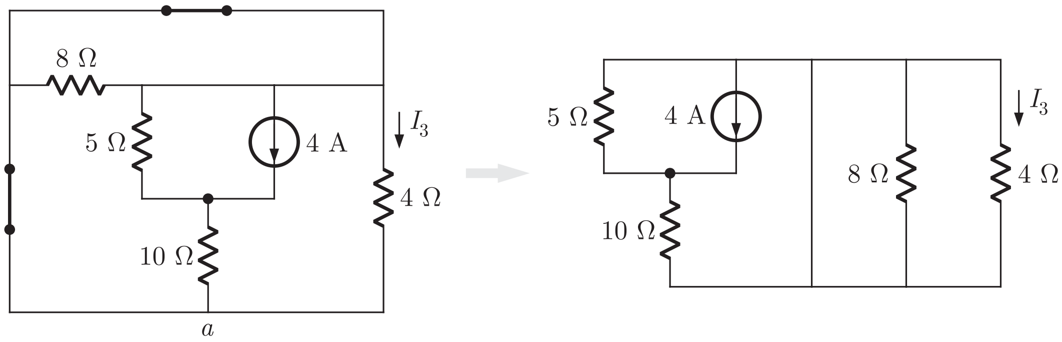
$$I_1 = \frac{18}{4} = 4.5 \text{ A}$$

Due to 12 V source only : (Open circuit 4 A and short circuit 18 V source)



$$I_2 = -\frac{12}{4} = -3\text{ A}$$

Due to 4 A source only : (Short circuit 12 V and 18 V sources)



$$I_3 = 0 \quad \text{(Due to short circuit)}$$

So,
$$I = I_1 + I_2 + I_3$$
$$= 4.5 - 3 + 0$$
$$= 1.5\text{ A}$$

Power dissipated in 4 Ω resistor

$$P_{4\,\Omega} = I^2(4) = (1.5)^2 \times 4 = 9\text{ W}$$

Alternate Method: Let current in 4 Ω resistor is I, then by applying KVL around the outer loop

$$18 - 12 - 4I = 0$$
$$I = \frac{6}{4} = 1.5\text{ A}$$

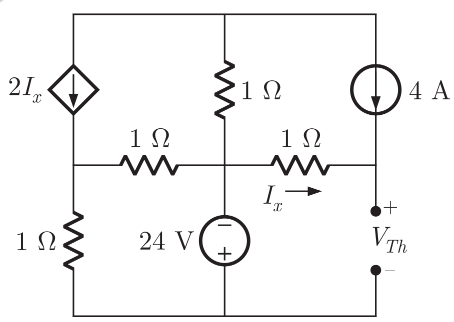
So, power dissipated in 4 Ω resistor

$$P_{4\,\Omega} = I^2(4) = (1.5)^2 \times 4$$
$$= 9\text{ W}$$

SOL 5.2.37 Correct answer is -10.

Using, Thevenin equivalent circuit

Thevenin Voltage : (Open Circuit Voltage)



$$I_x = -4\text{ A} \quad \text{(due to open circuit)}$$

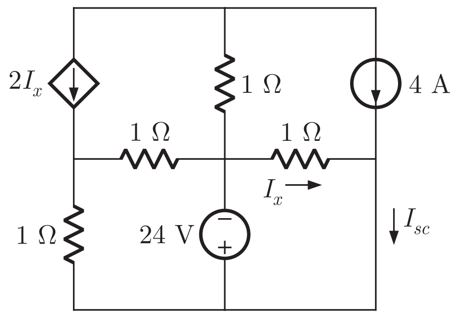
Writing KVL in bottom right mesh

$$-24 - (1)I_x - V_{Th} = 0$$
$$V_{Th} = -24 + 4 = -20\text{ V}$$

Thevenin Resistance :

$$R_{Th} = \frac{\text{open circuit voltage}}{\text{short circuit current}} = \frac{V_{oc}}{I_{sc}}$$
$$V_{oc} = V_{Th} = -20\text{ V}$$

I_{sc} is obtained as follows



$$I_x = -\frac{24}{1} = -24\text{ A}$$

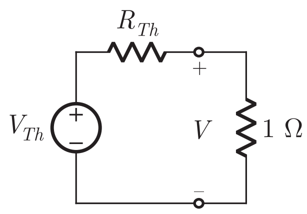
$$I_x + 4 = I_{sc} \qquad \text{(using KCL)}$$

$$-24 + 4 = I_{sc}$$

$$I_{sc} = -20\text{ A}$$

$$R_{Th} = \frac{-20}{-20} = 1\,\Omega$$

The circuit is as shown below



$$V = \frac{1}{1+R_{Th}}(V_{Th}) = \frac{1}{1+1}(-20) = -10\text{ volt} \quad \text{(Using voltage division)}$$

ALTERNATIVE METHOD :

Note that current in bottom right most $1\,\Omega$ resistor is $(I_x + 4)$, so applying KVL around the bottom right mesh,

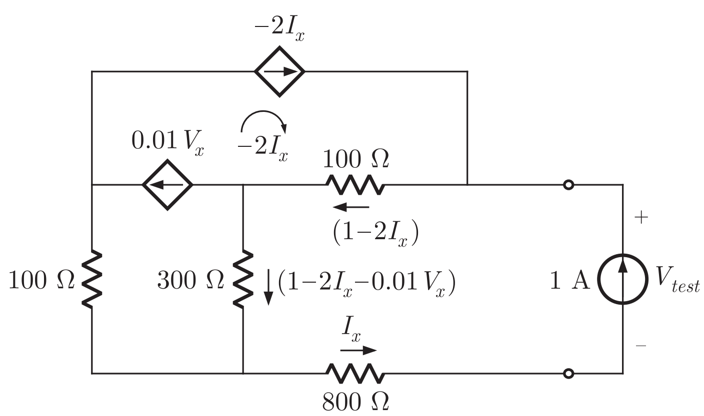
$$-24 - I_x - (I_x + 4) = 0$$

$$I_x = -14\text{ A}$$

So, $V = 1 \times (I_x + 4) = -14 + 4 = -10\text{ V}$

SOL 5.2.38 Correct answer is 100.

Writing currents into $100\,\Omega$ and $300\,\Omega$ resistors by using KCL as shown in figure.



$$I_x = 1\text{ A}, V_x = V_{test}$$

Writing mesh equation for bottom right mesh.

$$V_{test} = 100(1-2I_x) + 300(1-2I_x - 0.01V_x) + 800$$

$$= 100\text{ V}$$

$$R_{Th} = \frac{V_{test}}{1} = 100\,\Omega$$

SOL 5.2.39 Correct answer is 30.

For $R_L = 10\text{ k}\Omega$, $V_{ab1} = \sqrt{10\text{k} \times 3.6\text{m}} = 6\text{ V}$

For $R_L = 30\text{ k}\Omega$, $V_{ab2} = \sqrt{30\text{k} \times 4.8\text{m}} = 12\text{ V}$

$$V_{ab1} = \frac{10}{10 + R_{Th}} V_{Th} = 6 \qquad ...(1)$$

$$V_{ab2} = \frac{30}{30 + R_{Th}} V_{Th} = 12 \qquad ...(2)$$

Dividing equation (1) and (2), we get $R_{Th} = 30\text{ k}\Omega$. Maximum power will be transferred when $R_L = R_{Th} = 30\text{ k}\Omega$.

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