① One - One ; Onto ; Bijective ;
Many - One ; Surjective ; Into ; $(n>m)$

② $n(A) = m$; $n(B) = n$; No. of functions $= n^m$
$n \ge m \Rightarrow$ No. of one - one $= {}^nP_m$
$n < m \Rightarrow$ No. of one - one $= 0$

③ Even & Odd Function.

④ fog or gof will be even one of them is even.

⑤ $f(x+T) = f(x) \rightarrow$ Periodic Function

$f(x) \rightarrow T_1$ & $g(x) \rightarrow T_2$
$\Rightarrow \left.\begin{array}{l} f(x) \pm g(x) \\ f(x).g(x) \ \& \ f(x)/g(x) \end{array}\right\}$ L.C.M. of (T_1, T_2)

① L-Hospital Rule if $\lim_{x\to a} f(x)/g(x) = 0/0, \infty/\infty$

② $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a$; $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
$\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$; $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$

③ Differentiability at $x = a$ iff $\frac{f(x) - f(a)}{x-a}$ exist finitely

$\frac{f(a+h) - f(a)}{h} = \frac{f(a-h) - f(a)}{-h} = f'(a)$

④ Rolle's Theorem: $f(a) = f(b)$ in $[a,b]$
$\Rightarrow f'(c) = 0$; c in (a,b)

⑤ MVT : $f'(c) = \frac{f(b) - f(a)}{b-a}$

$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

① $\sin^{-1}(-x) = -\sin^{-1}(x)$; $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

② $\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$

③ $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$
$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$
$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

④ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
$2\cos A\sin B = \sin(A+B) - \sin(A-B)$
$2\cos A\cos B = \cos(A+B) + \cos(A-B)$
$-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

⑤ $\sin A + \sin B = 2\sin(A+B/2)\cos(A-B/2)$
$\sin A - \sin B = 2\cos(A+B/2)\sin(A-B/2)$
$\cos A + \cos B = 2\cos(A+B/2)\cos(A-B/2)$
$\cos A - \cos B = -2\sin(A+B/2)\sin(A-B/2)$

① $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{1-x^2}}$

② Take log in $(f(x))^{g(x)}$

③ $\Delta y = f(x+\Delta x) - f(x)$
& $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

④ 1^{st} Derivative Test

⑤ 2^{nd} Derivative Test

⑥ Absolute Maxima & Minima.

$|\vec{a}\times\vec{b}|^2 + |\vec{a}.\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$

① Equivalent Matrices: Same Order

② $(A')' = A$; $(kA)' = kA'$; $(AB)' = B'A'$

③ Idempotent : $A^2 = A$
Nilpotent of order 'k' iff $A^k = 0$.
Involutory : $A^2 = I$
Orthogonal : $AA^T = A^TA = I$

④ Trace of a Matrix $= \sum$ Diagonal elements.

⑤ $A(\text{adj}A) = (\text{adj}A)A = |A|I_n$
$|\text{adj}A| = |A|^{n-1}$; $|\text{adj}(\text{adj}A)| = |A|^{(n-1)^2}$
$\text{adj}(\text{adj}A) = |A|^{n-2}A$; $\text{adj}(A^{-1}) = (\text{adj}A)^{-1}$
$\text{adj}(AB) = \text{adj}(B).\text{adj}(A)$

⑥ $A^{-1} = \frac{1}{|A|}\text{adj}A$; $|A| \neq 0$ (Non singular)

① $\int [xf'(x) + f(x)].dx = x.f(x) + C$

② $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$
$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x+\sqrt{x^2-a^2}| + C$
$\int \frac{dx}{\sqrt{a^2+x^2}} = \log|x+\sqrt{x^2+a^2}| + C$

$AX = B$

① $D \neq 0 \rightarrow$ One of $D_1, D_2, D_3 \neq 0 \rightarrow$ Consistent & Unique
$\rightarrow D_1 = D_2 = D_3 = 0 \rightarrow$ Consistent & Trivial (0).

② $D = 0 \rightarrow D_1 = D_2 = D_3 = 0 \rightarrow$ Consistent & Infinite
$\rightarrow$ One of $D_1, D_2, D_3 \neq 0 \rightarrow$ Inconsistent & No Sol^n

$AX = 0$

① $|A| \neq 0 \Rightarrow X = 0 \Rightarrow$ Trivial Sol^n

② $|A| = 0 \Rightarrow$ Infinite Sol^n (Trivial & Non Trivial)
and $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

① $\int \sqrt{a^2-x^2}.dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \sqrt{x^2-a^2}.dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$
$\int \sqrt{x^2+a^2}.dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

Trigonometry Substitution

② $\int_a^b f(x).dx = \int_a^b f(b+a-x).dx$
$\int_{-a}^{a} f(x).dx \begin{cases} = 0 \rightarrow f(x) \text{ is odd} \\ = 2\int_0^a f(x).dx \rightarrow f(x) \text{ is even} \end{cases}$
$\int_0^a f(x).dx = \int_0^{a/2} [f(x) + f(a-x)]$
$= \begin{cases} 0 ; \text{ if } f(x) = -f(a-x) \\ 2f(x) ; \text{ if } f(x) = f(a-x) \end{cases}$

① No. of times we differentiate = No. of arbitrary constant in eqⁿ

② Homogeneous D. Eqⁿ : $y = vx$

③ Linear D. Eqⁿ : $\frac{dy}{dx} + Py = Q$

$\Rightarrow$ I.F. $= e^{\int P.dx} \Rightarrow y(\text{I.F.}) = \int Q(\text{I.F.})dx$

④ Exact D. Eqⁿ : $x.dy + y.dx = 0$

$\Rightarrow xy = C$

⑤ CAFE Approach

Follow → Exception Case

Ask for method

Convert into dy/dx

---

① Section Formula ; Mid-Point

② $\perp$ to $\vec{a}$ & $\vec{b}$ is $\vec{a} \times \vec{b}$.

③ ar(∥gm) $= |\vec{a} \times \vec{b}|$ or $\frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

④ ar(Δ) $= \frac{1}{2}|\vec{a} \times \vec{b}|$

⑤ Scalar Triple product $[\vec{a}\ \vec{b}\ \vec{c}]$

$(\vec{a} \times \vec{b}).\vec{c} = |\vec{a}|\,|\vec{b}|\,|\vec{c}| \sin\theta \cos\phi$

\# Volume of Parallelopiped $= [\vec{a}\ \vec{b}\ \vec{c}]$

" " Tetrahedron $= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

\# $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow$ any two are equal or all three coplanar.

⑥ Vector Triple Product

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

---

① $l^2 + m^2 + n^2 = 1$

② $\vec{r} = \vec{a} + \lambda\vec{b}$; $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

③ Distance b/w two parallel lines

$|\vec{b} \times (\vec{a_2} - \vec{a_1})| / |\vec{b}|$

④ Distance b/w two skew lines

$|(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})| / |\vec{b_1} \times \vec{b_2}|$

⑤ $\vec{r}.\hat{n} = d$; $(\vec{r} - \vec{a})\hat{n} = 0$

⑥ Plane through 3 non-collinear points

$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})]$

OR
$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

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⑦ Plane through intersection of two planes

$P_1 + \lambda P_2 = 0$

⑧ Two lines to be coplanar

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

⑨ Eqⁿ of a plane containing two lines

$(\vec{r} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0$

⑩ Angle b/w a line & a plane

$\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}|\,|\vec{n}|}$

---

① Complementary Event : A^c or $A' = S - A$

② Mutually Exclusive : $A \cap B = \phi$

③ Exhaustive Events : their 'U' gives 'S'.

④ Equally Likely Events

⑤ If A & B are independent events, then A' & B ; A & B' ; A' & B' are also independent.

⑥ Dependent Events : $P(A \cap B) = P(A).P(B|A)$

Independent Events : $P(A \cap B) = P(A).P(B)$

⑦ A : event that has already occurred

$$P(E_i|A) = \frac{P(A|E_i).P(E_i)}{P(A|E_1).P(E_1) + P(A|E_2).P(E_2) + \cdots}$$

---

⑧ Random Variable $X : S \rightarrow R$

↓

Probability Distribution $P(X)$

↓

Mean or Expectation $E(X) = \sum p_i x_i = \mu$

↓

Variance : $E(X^2) - (E(X))^2$

Standard Deviation : $\sigma = +\sqrt{Var(X)}$

⑨ Bernoulli Trials : $p + q = 1$

⑩ Binomial Distribution

n (no. of trials) → r success ; → $(n-r)$ failure

$\Rightarrow P(X = r) = {}^nC_r\, p^r q^{n-r}$

p → Success ; q → Failure.

---

① α → Conductors (+) & SemiConductors (−)

② Semi-Conductor : $\Delta E_g < 2eV$

③ Intrinsic : $I = I_e + I_h$; $\sigma = ne\mu$; $\sigma = \sigma_e + \sigma_h$

\# $n_e n_h = n_i^2$

④ p-n Junction : Depletion layer free from charges

⑤ Forward & Reverse Biasing ; ($r_d = \Delta V/\Delta I$).

⑥ p-n junction diode as Rectifier → DC to AC.

⑦ Zener-Diode : Reverse Biased & Voltage Stabilizer

⑧ Field Ionisation after Breakdown Voltage.

⑨ LED : $[\lambda_{light} = 12431/\Delta E_g$ (eV) & λ in Å.

⑩ Transistor : E (Heavily & moderate) ; (Dope & Size)

B (Low & Thin) ; C (Moderate & Large) ; $\boxed{\beta = \frac{I_c}{I_B}}$

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① $d = \sqrt{2RH}$; $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

② Length of antenna (min.) $= \lambda/4$

③ Ground, Sky & Space Wave

(< 3MHz) (3−30 MHz) (> 30 MHz)

④ Power Radiated from antenna : $P \propto (l/\lambda)^2$

⑤ Modulated Wave

Amplitude : $C_m = (A_c + A_m \sin\omega_m t)$

Frequency : $\omega = \omega_c$ (Same as Carrier)

$C_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$

\# Amplitude Modulation Index : $\boxed{\mu = A_m/A_c}$

\# $\omega_{LSB} = \omega_c - \omega_m$; $\omega_{USB} = \omega_c + \omega_m$

\# Bandwidth of AM : $\Delta\omega = 2\omega_m$