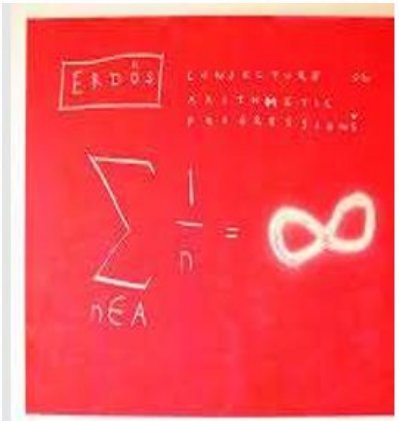
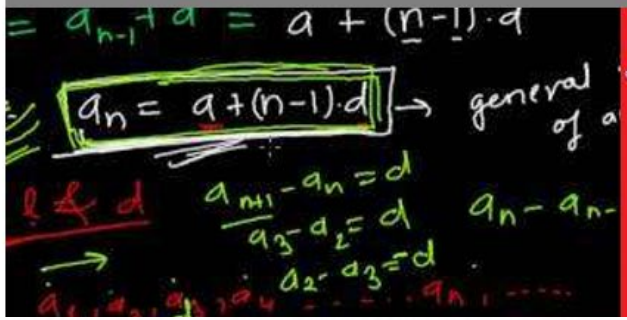


MATHEMATICS

Arithmetic Progressions

nth term = $a + (n - 1)d$



By the end of this session you will be able to

1. Differentiate between sequence and series
2. Types of progressions
3. Define Arithmetic Sequence
4. Define terminology related to Arithmetic sequence

   like first term , common difference.
5. How to find write the nth term formula


Let's
ACE it!

Differentiate between Sequence and Series

Sequence	Series
Definition : It is an ordered list, it contains members	Definition: Associated with every sequence , is a SERIES the indicated sum of the sequence.
Types of sequence : infinite and finite	
Notation: x1,x2,x3,……,xn	Notation : $\sum xk = x1+x2+x3+\ldots\ldots+xn$

Introduction

What do you observe in the following pictures?

A certain **pattern** has been followed while creating these things.

Example of Arithmetic Sequence in real life

Namita's school offered her a scholarship of Rs. 1000 when she was in class 6 and increased the amount by Rs. 500 each year till class 10.
The amounts of money (in Rs) Namita received in class 7th,8th,9th and 10th were respectively: 1500, 2000, 2500 and 3000

Each of the numbers in the list is called a **term**.
Here we find that the succeeding terms are obtained by adding a fixed number.

Arithmetic Progressions

Consider the following lists of numbers :

1, 3, 5, 7, 9, ...	each term is obtained by adding 2 to the previous term
10, 8, 6, 4, 2, ...	each term is obtained by adding -2 to the previous term
- 3, -2, -1, 0, ...	each term is obtained by adding 1 to the previous term
5, 5, 5, 5, 5, ...	each term is obtained by adding 0 to the previous term

Each list follows a pattern or rule.

Arithmetic Progressions

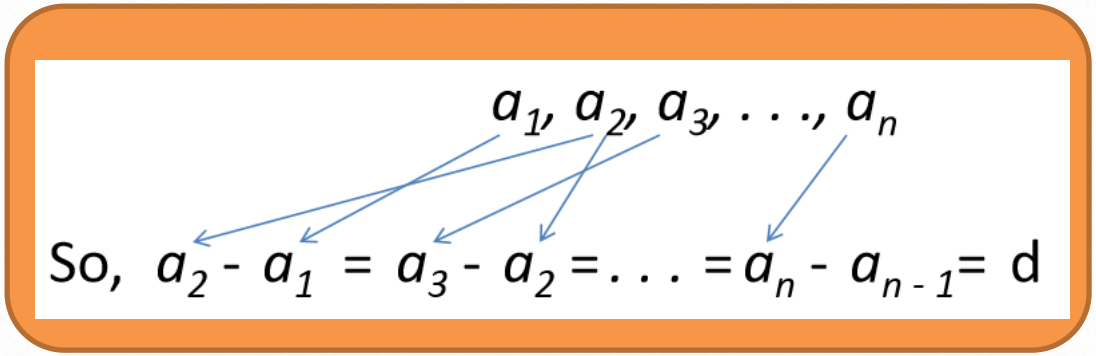
An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the previous term except the first term.

This fixed number is called the common difference of the AP.

It can be positive, negative or zero.

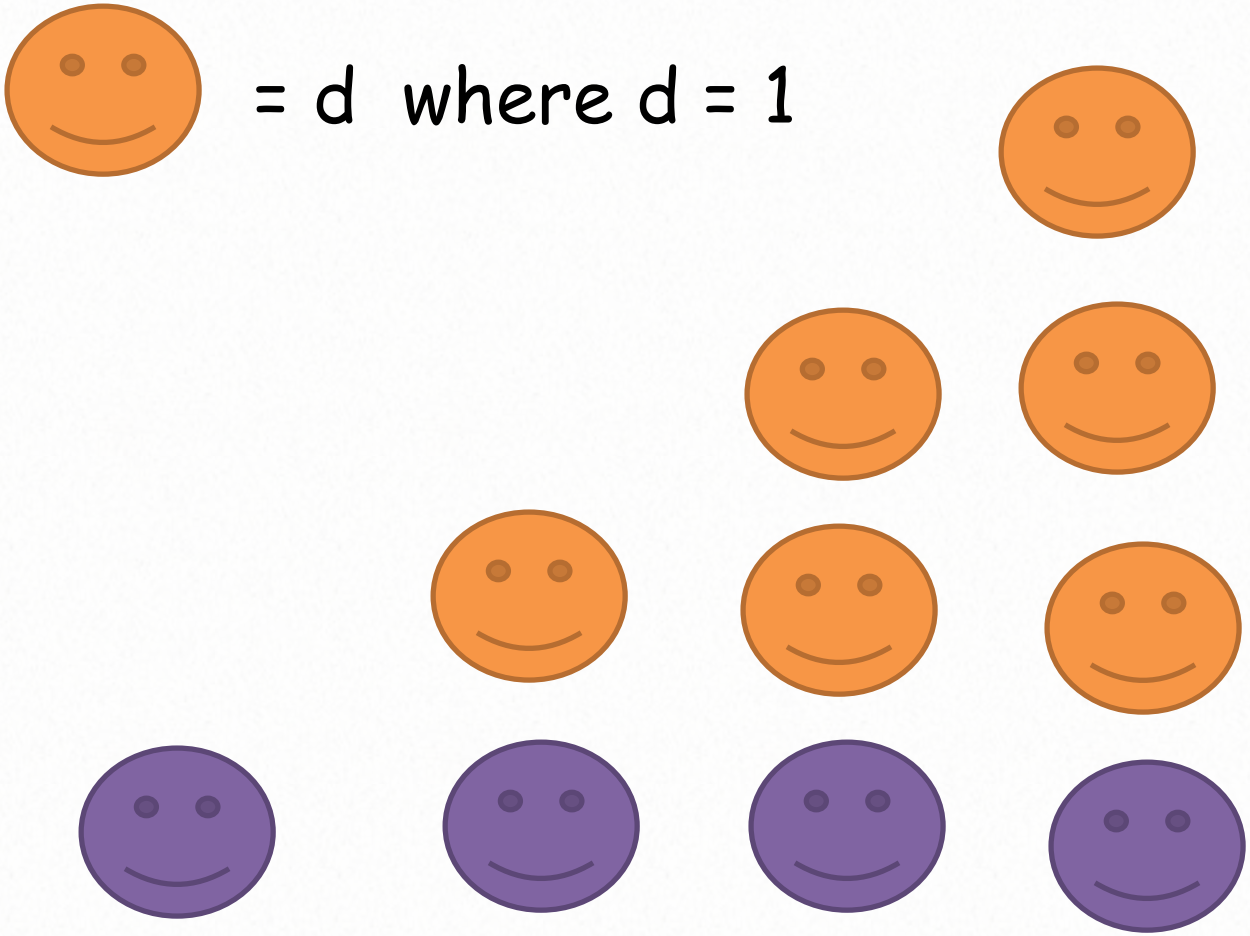
Formula for Common Difference

Let us denote the first term of an AP by a_1, second term by a_2, . . ., nth term by a_n and the common difference by d. Then the AP becomes



$$a_n - a_{n-1} = d$$

Illustrative example of Arithmetic Progression



General Form of an AP

We can see that
$a, a + d, a + 2d, a + 3d,$

represents an **arithmetic progression** where **a** is the **first term** and **d** the common difference.
This is called the **general form** of an **AP**.

The behavior of the arithmetic progression depends on the common difference d. If the common difference is:

- **Positive**, the members (terms) will grow towards positive infinity.
- **Negative**, the members (terms) will grow towards negative infinity.

Types of Arithmetic Sequence

Finite AP

- Number of students in class 5th to 10th are 25, 23, 21, 19, 17, 15.
- There are only a finite number of terms.
- They have a last term

Infinite AP

. 2, 7, 12, 17, 22, ...
- There are infinite number of terms.
- They do not have a last term.

Find the nth term of an AP:

Let us consider an A.P. with
first term 'a' and common difference 'd' ,then

- The first term = $a_1 = a + 0\,d = a + (1-1)d$
- The second term = $a_2 = a + d = a + (2-1)d$
- The third term = $a_3 = a + 2d = a + (3-1)d$
- The fourth term = $a_4 = a + 3d = a + (4-1)d$

The nth term = $a_n = a + (n-1)d$

Practice:

Q1 To check that a given term is in A.P. or not.
2, 6, 10, 14….

(i) Here , first term a = 2,

find differences in the next terms

$a_2 - a_1 = 6 - 2 = 4$
$a_3 - a_2 = 10 - 6 = 4$
$a_4 - a_3 = 14 - 10 = 4$

Since the differences are common.

Hence the given terms are in A.P.

Problem :Find 10th term of A.P. 12, 18, 24, 30……

Solution: Given A.P. is 12, 18, 24, 30..

First term is a = 12

Common difference is d = 18- 12 = 6

nth term is $a_n = a + (n-1)d$

Put n = 10,

$a_{10} = 12 + (10-1)6$
$= 12 + (9 \times 6)$
$= 12 + 54$

$a_{10} = 66$

Problem :Find how many two digits numbers are divisible by 6?

Solution:
The sequence is : 12,18,24,……96

First term is a = 12

Common difference is d = 18- 12 = 6
nth term is $a_n = 96$
a + (n-1)d = 96

12+(n-1) 6 = 96

(n-1) 6 = 96-12

(n-1) 6 = 84
(n-1) = 84/6= 14
N = 15
There are 15 two digits numbers divisible 6.

The sum of n terms of an AP

To find the sum of n terms,

we find as,

Sum = n X [(first term + last term) / 2]

Now last term will be = a + (n-1) d

Therefore,

✓ $S_n = \frac{1}{2} n [2a + (n - 1)d]$

It can also be written as

✓ $S_n = \frac{1}{2} n [a + an]$

Problem 1. Find the sum of 30 terms of given A.P. 12 + 20 + 28 + 36……

Solution : Given A.P. is 12 , 20, 28 , 36

Its first term is a = 12 Common difference is d = 20 – 12 = 8

The sum to n terms of an arithmetic progression

$S_n = \frac{1}{2} n [2a + (n - 1)d] = \frac{1}{2} \times 30 [2 \times 12 + (30-1) \times 8]$

$= 15 [24 + 29 \times 8]$
$= 15[24 + 232]$
$= 15 \times 246$
$= 3690$

Problem 1:Given a = 2, d = 8, Sn = 90, find n and an.

Solution :

Its first term is a = Common difference is d

$Sn = \frac{1}{2} n [2a + (n - 1)d]$

$90 = \frac{1}{2} n[2 \times 2 + (n-1)\ 8]$

$180 = n[4 + 8n - 8]$

$180 = n[8n - 4]$

$180 = 8n^2 - 4n$

RECAP

The general forms of an AP is
a,(a+d), (a+2d),. .. , a + (m - 1)d.

i. Nth term of the AP is
Tn =a+(n-1)d

iii. Sum of 1st n term of an AP is

Sn=N/2{2a=(n-1)d}.

ii. Nth term form the end
={l-(n-1)d}, where l is the last
term of the word.

iv. Also Sn=n/2 (a+l)


ANY
questions?

Thank You!

Practice

Question: 1. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Solution: $a_{22} = 149 = a + 7 \times 21$

$\Rightarrow 149 - 147 = 2 = a$

Now, $S_{22} = \frac{22}{2}(2 + 149) = 11 \times 151 = 1661$

Question: 2. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution: Here d = 4 and a = 14 - 4 = 10

So, $a_{51} = 10 + 4 \times 50 = 210$

Now, $S_{51} = \frac{51}{2}(10 + 210) = 51 \times 110 = 5610$