Multiplication of Matrices

$A = [a_{ij}]_{m \times n}$ $B = [b_{jk}]_{n \times p}$

n ↓ number of columns n ↓ Number of Rows

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} b_{11} & \cdots & \\ \vdots & \ddots & \\ & \cdots & b_{33} \end{bmatrix}$$

$C = A \times B$

$c_{ik} = a_{i1} b_{1k} + a_{i2} b_{2k} + \dots + a_{in} b_{nk} = \sum_{j=1}^{n} a_{ij} b_{jk}$

Eg:- $C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix}$

$$C \times D = \begin{bmatrix} 1\times2+(-1)\times(-1)+2\times5 & 1\times7+(-1)(1)+(2)\times(-4) \\ 0\times2+3\times-1+4\times5 & 0\times7+3\times1+4\times-4 \end{bmatrix} = \begin{bmatrix} 13 & -2 \\ 17 & -13 \end{bmatrix}$$

Eg:- $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 6 & 0 \\ 7 & 9 & 8 \end{bmatrix}$ ← 2

INVERTIBLE MATRICES

* Not all matrices are invertible.
* All invertible matrices are square matrices.
* Inverse of a square matrix is unique.

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$\boxed{[a_{ij}]^T_{m\times n} = [a_{ji}]_{n\times m}}$

▷ Let A^{-1} be the inverse of a square matrix A, then:

Def^n $\boxed{A A^{-1} = I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AI = A$, $A^{-1} I = A^{-1}$ & $A A^{-1} = I$

* $(A^{-1})^{-1} = A \iff$ If $B = A^{-1}$, then $A = B^{-1}$

Operations on a matrix (Transformation)

(i) Interchange any two rows or columns.

$R_i \longleftrightarrow R_j$ $C_i \longleftrightarrow C_j$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$R_2 \longleftrightarrow R_3$ ⇓ $C_1 \longleftrightarrow C_3$ ⇓

$B_1 = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix}$ $B_2 = \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$

(ii) Multiplication of a row or column by a non-zero number.

$R_2 \longrightarrow 3R_2$ (↳ Not zero) $\Rightarrow B_3 = \begin{bmatrix} 1 & 2 & 3 \\ 12 & 15 & 18 \\ 7 & 8 & 9 \end{bmatrix}$

(iii) $R_i \longrightarrow R_i + kR_j$, $k \neq 0$ $R_1 \longrightarrow R_1 - 2R_2$

$A^T = A'$

$$B_4 = \begin{bmatrix} 1-2(4) & 2-(2)(5) & 3-(2)6 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\Rightarrow B_4 = \begin{bmatrix} -7 & -8 & -9 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$R_1 \longrightarrow R_1 + R_3$

$$B_5 = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Eg:→ $A = \begin{bmatrix} 1 & 2 \\ \cdot & -1 \end{bmatrix}$ $A^{-1} = ?$ ←

$\begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Eg:→ $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ $A^{-1} = ?$ $I = BA$

$AA^{-1} = I$

$\Rightarrow I = BA$ $I = A^{-1}A$ $\therefore A^{-1} = B$

$A = IA$

$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ * Imp Now, I will try to make L.H.S = I

$R_2 \to R_2 - 2R_1$ (To both LHS & RHS) Imp

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

$R_2 \to -\frac{1}{5}R_2$

$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$

$R_1 \to R_1 - 2R_2$

$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I} = \underbrace{\begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}}_{A^{-1}} A$ $A^{-1}A = I$, $AA^{-1} = I$

$A^{-1} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

* NCERT
* Reference Book (RD. Sharma → Objective)
* Arihant Past Papers
43 year's IITJEE Topicwise Solved Papers

Matrices   Exercise NCERT H.W.
→ Doubt Session
→ JEE Session

NCERT → END
$n = 1, 2, 3, \ldots$
JEE
??
Revision

Syllabus → Doubt Sessions

Revision   CBSE (March)
JEE (April, May)

11^{th} 50 hrs of Classes   H.W. $\geq 2x$
12^{th} 50 hrs of Classes $x \to$ Class Hours.

H.W. is double the workload of C.W

* 2 Classes/week   12 months (11^{th} & 12^{th})
* 3 Classes/week   8 months (11^{th} & 12^{th})

₹6000/month
₹9000/month

CBSE :- ₹600/hour
JEE :- ₹750/hour

Eg) 24:→ $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Find A^{-1}

Eg) 25:→ $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

Find P^{-1}

Sol:→ Non-Invertible

$A = IA$

$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Imp

(Focus on making the non-diagonal)

le.

(Focus on making the non-diagonal elements zero.)

Sol:- $R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} =$$

$R_3 \longrightarrow R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} =$$

$C_3 \rightarrow C_3 - 2C_2$

$R_2 \rightarrow R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$C_3 \rightarrow C_3 + C_1$ $\qquad R_1 \rightarrow R_1 + \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 \rightarrow \frac{1}{2} R_3$

$C_3 \rightarrow \frac{1}{2} C_3$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

— Inverse doesn't exist.

HW :→ Ex-

Q -

Eg:- $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ ← Find the inverse

$\underline{-3.2}$

-15 to 22

* Properties of Multiplication of Matric

1. Associative Law :→ $(AB)C = A(BC)$
2. Distributive Law :→ $(A+B)C = AC+BC$
3. Existence of Multiplicative Identity :→ $\boxed{IA = }$

* Symmetric Matrices :→ $A^T = A$

or $A' = A$

Skew-symmetric Matrices :→ $A^T = -A$

or $A' = -A$

Theorem 1:- If $A + A'$ is symmetric, then $A-$

Theorem 2:→ Any square matrix can be ex
of a symmetric & a skew sy

$$A = \frac{1}{2}\underbrace{(A+A')}_{\text{symmetric}} + \frac{1}{2}\underbrace{(A-A')}_{\text{skew-symmetric}}$$

ces

$$\boxed{A I = A}$$

A' is skew symmetric.

pressed as the sum
mmetric matrix.

$$\boxed{\underline{H.W.}\ \ \underline{Ex-3.3}}$$