# MATHEMATICS

## JEE (Mains + Advanced)

# Logarithm

- Theory
- DPP
- Practice Sheet
- Answer Key

# 09. LOGARITHM

## 1. SET THEORY

### 1.1 Basic Definition Of Set

A set is a well defined collection of objects. Here term 'well defined' means that there is some definite rule on the basis of which one can decide whether an object is in the collection or not. Sets are generally denoted by capital letters e.g., A, B, C, X, Y, Z etc. If an object 'a' is in the set A, then we say that 'a' is an element of set A or 'a' belongs to set A and we write $a \in A$. If 'a' is not in set A, then we say that 'a' does not belong to set A and we write $a \notin A$.

The collection of cricketers in the world who have played at least five test matches is a set. However, the collection of good cricket players of India is not a set, becasue the term "good player" is vague and it is not well defined.

**Illustration 1** Which of the following collection is a set ?

(a) The collection of all girls in your class.

(b) The collection of intelligent girls in your class.

(c) The collection of beautiful girls in your class.

(d) The collection of tall girls in your class.

**Ans. (a)**

**Solution** Clearly collections (b), (c) and (d) are not well defined collections. So, they do not form a set

A set is often described in one of the following two ways.

**a. Notation of a set :** Sets are denoted by capital letters like A, B, C or { } and the entries within the bracket are known as elements of set.

**b. Cardinal number of a set :** Cardinal number of a set X is the number of elements of a set X and it is denoted by n(X) e.g.
$X = \{x_1, x_2, x_3\}$ $\quad\therefore\quad$ $n(X) = 3$

### 1.2 REPRESENTATION OF SETS

**a. Set Listing Method (Roster Method) :**
In this method a set is described by listing all the elements, separated by commas, within braces { }

For example, the set of vowels of English Alphabet may be described as {a, e, i, o, u}.

The order in which the elements are written in a set makes no difference. Also, the repetition of an element has no effect.

**b. Set builder Method (Set Rule Method) :**
In this method, a set is described by characterizing property P(x) of its elements x. In such case the set is described by {x : P(x) holds} or {x | P(x) holds}, which is read as the set of all x such that P(x) holds. The symbol '|' or '**:**' is read as such that.

The set of all even integers can be written as
$E = \{x : x = 2n, n \in Z\}$

**llustration 1** Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

**Solution** The given equation can be written as
$(x - 1)(x + 2) = 0$, i.e., $x = 1, -2$
Therefore, the solution set of the given equation can be written in roster form as {1, –2}.

**llustration 2** Write the set {x : x is a positive integer and $x^2 < 40$} in the roster form.

**Solution** The required numbers are 1, 2, 3, 4, 5, 6. So, the given set in the roster form is {1, 2, 3, 4, 5, 6}.

**llustration 3** Write the set A = {1, 4, 9, 16, 25, ....} in set-builder form.

**Solution** We may write the set A as
A = {x : x is the square of a natural number}
Alternatively, we can write
$A = \{x : x = n^2$, whre $n \in N\}$

### 1.3 TYPE OF SETS

**I. Finite set :**
A set X is called a finite set if its element can be listed by counting or labeling with the help of natural numbers and the process terminates at a certain natural number n. i.e., n(X) = finite no.
e.g.
(1) A set of English Alphabets
(2) Set of soldiers in Indian Army

**II Infinite set :**
A set whose elements cannot be listed counted by the natural numbers {1, 2, 3.......n} for any number n, is called a infinite set. e.g.
(1) A set of all points in a plane
(2) $X = \{x : x \in R, 0 < x < 0.0001\}$
(3) $X = \{x : x \in Q, 0 \le x \le 0.0001\}$

**III Singleton set :**
A set consisting of a single element is called a Singleton set. i.e. n (X) = 1, E.g.
(1) $\{x : x \in N, 1 < x < 3\}$, (2) {{ }} : Set of null set,
(3) {f} is a set containing alphabet f.

**IV Null set :**
A set is said to be empty, void or null set if it has no element in it, and it is denoted by $\phi$, i.e., X is a null set if

n (X) = 0. e.g.

(1) $\{x : x \in R \text{ and } x^2 + 2 = 0\}$
(2) $\{x : x > 1 \text{ but } x < 1/2\}$
(3) $\{x : x \in R,\ x^2 < 0\}$

**V Equivalent Set :**

Two finite sets A and B are equivalent if their cardinal numbers are same i.e. n(A) = n(B).

**VI Equal Set :**

Two sets A and B are said to be equal if every element of A is a member of B and every element of B is a member of A. i.e. A = B, if A and B are equal and $A \neq B$, if they are not equal.

**llustration 1** State which of the following sets are finite or infinite.

(i) $\{x : x \in N \text{ and } (x-1)(x-2) = 0\}$
(ii) $\{x : x \in N \text{ and } x^2 = 4\}$
(iii) $\{x : x \in N \text{ and } 2x - 1 = 0\}$
(iv) $\{x : x \in N \text{ and } x \text{ is prime}\}$
(v) $\{x : x \in N \text{ and } x \text{ is odd}\}$

**Solution**

(i) Given set = {1, 2}. Hence, it is finite.
(ii) Given set = {2}. Hence, it is finite.
(iii) Given set = $\phi$. Hence, it is finite.
(iv) The given set is the set of all prime numbers and since set of prime numbers is infinite. Hence, the given set is infinite.
(v) Since there are infinite number of odd numbers, hence, the given set is infinite.

**VII UNIVERSAL SET**

It is a set which includes all the sets under considerations i.e. it is a super set of each of the given set. Thus, a set that contains all sets in a given context is called the universal set. It is denoted by U. e.g.,
If A = {1, 2, 3}, B = {2, 4, 5, 6} and C = {1, 3, 5, 7}, then
U = {1, 2, 3, 4, 5, 6, 7} can be taken as the universal set.

**VIII SUBSET**

A set A is said to be a subset of B if all the elements of A are present in B and is denoted by $A \subset B$
(read as A is subset of B) and symbolically written as :
$x \in A \Rightarrow x \in B \Leftrightarrow A \subset B$

**llustration 1** Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$ ?. If not, give an example

**Solution** No. Let A = {1}, B = {{1}, 2} and C = {{1}, 2, 3}. Here $A \in B$ as A = {1} and $B \subset C$.
But $A \not\subset C$ as $1 \in A$ and $1 \notin C$.
Note that an element of a set can never be a subset of itself.

**Number of subsets :**

Condiser a set X containing n elements as $\{x_1, x_2, ...., x_n\}$ then the total number of subsets of X = $2^n$

**Proof :** Number of subsets of above set is equal to the number of selections of elements taking any number of them at a time out of the total n elements and it is equal to $2^n$

$^nC_0 + {}^nC_1 + {}^nC_2 + ...... + {}^nC_n = 2^n$

**Types of Subsets :**

A set A is said to be a **proper subset** of a set B if every element of A is an element of B and B has at least one element which is not an element of A and is denoted by $A \subset B$.

The set A itself and the empty set is known as **improper subset** and is denoted as $A \subseteq B$.

e.g. If $X = \{x_1, x_2, .....,x_n\}$ then total number of proper sets = $2^n - 2$ (excluding itself and the null set). The statement $A \subset B$ can be written as $B \supset A$, then B is called the **super set** of A and is written as $B \supset A$.

**IX POWER SET**

The collection of all subsets of set A is called the power set of A and is denoted by P(A)
i.e. P(A) = {x : x is a subset of A}.

If $X = \{x_1, x_2, x_3, ..........x_n\}$ then $n(P(X)) = 2^n$ ; $n(P(P(x))) = 2^{2^n}$.

**llustration 1** Find the pairs of equal sets, if any, give reasons:
A = {0}, B = {x : x > 15 and x < 5},
C = {x : x – 5 = 0 }, D = {x: $x^2$ = 25},
E = {x : x is an integral positive root of the equation $x^2 - 2x - 15 = 0$}.

**Solution** Since $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that, $A \neq B, A \neq C, A \neq D, A \neq E$. Since B = $\phi$ but none of the other sets are empty. Therefore $B \neq C, B \neq D$ and $B \neq E$. Also C = {5} but $-5 \in D$, hence $C \neq D$. Since E = {5}, C = E. Further, D = {–5, 5} and E = {5}, we find that, $D \neq E$. Thus, the only pair of equal sets is C and E.

**llustration 2** Consider the sets $\phi$, A = { 1, 3 }, B = {1, 5, 9}, C = {1, 3, 5, 7, 9}. Insert the symbol , $\subset$ or $\not\subset$ between each of the following pair of sets :
(i) $\phi$ **. . .** B (ii) A . . . B (iii) A . . . C (iv) B . . . C

**Solution** (i) $\phi \subset B$ as $\phi$ is a subset of every set.
(ii) $A \not\subset B$ as $3 \in A$ and $3 \notin B$
(iii) $A \subset C$ as 1, 3 $\in$ A also belongs to C
(iv) $B \subset C$ as each element of B is also an element of C.

**1.4 INTERVALS**

The set of numbers between any two real numbers is called interval.
The following are the types of interval.

**I Closed Interval**

$x \in [a, b] \equiv \{x : a \le x \le b\}$


**II Open Interval**

$x \in (a, b)$ or $]a, b[ \equiv \{x : a < x < b\}$


**III Semi Open or Semi-Closed Interval**

$x \in [a, b[$ or $[a, b) = \{x : a \leq x < b\}$


$x \in ]a, b]$ or $(a, b] = \{x : a < x \leq b\}$


**llustration 1** Write the following as intervals :

(i) $\{x : x \in R, -4 < x \leq 6\}$
(ii) $\{x : x \in R, -12 < x < -10\}$
(iii) $\{x : x \in R, 0 \leq x < 7\}$
(iv) $\{x : x \in R, 3 \leq x \leq 4\}$

**Solution** (i) $(-4, 6]$ (ii) $(-12, -10)$
(iii) $[0, 7)$ (iv) $[3, 4]$

**llustration 2** Write the following intervals in set-builder form :

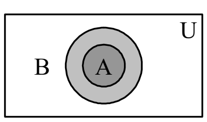
(i) $(-3, 0)$ (ii) $[6, 12]$
(iii) $(6, 12]$ (iv) $[-23, 5)$

**Solution** (i) $\{x : x \in R, -3 < x < 0\}$
(ii) $\{x : x \in R, 6 \leq x \leq 12\}$
(iii) $\{x : x \in R, 6 < x \leq 12\}$
(iv) $\{x : x \in R, -23 \leq x < 5\}$

### 1.5 VENN (EULER) DIAGRAMS

The diagrams drawn to represent sets are called Venn diagram or Euler-Venn diagrams. Here we represent the universal U as set of all points within rectangle and the subset A of the set U is represented by the interior of a circle. If a set A is a subset of a set B, then the circle representing A is drawn inside the circle representing B. If A and B are not equal but they have some common elements, then to represent A and B by two intersecting circles.

e.g. If A is subset of B then it is represented diagrammatically in fig.




## 1.6 OPERATIONS ON SETS

**I Union of sets :**

If A and B are two sets then union ($\cup$) of A and B is the set of all those elements which belong either to A or to B or to both A and B.

It is also defined as $A \cup B = \{x : x \in A \text{ or } x \in B\}$. It is represented through Venn diagram in fig.1 & fig.2

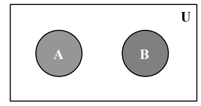



Fig. (1)

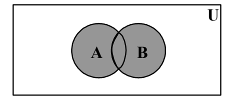



Fig. (2)

**llustration 1** If $A = \{x : x = 2n + 1, n \in Z\}$ and $B = \{x : x = 2n, n \in Z\}$, then $A \cup B =$

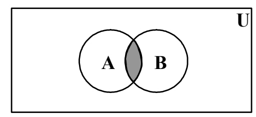
(A) N (B) Z (C) R (D) W

**Solution** We have,

$A \cup B = \{x : x = 2n + 1 \text{ or } x = 2n, n \in Z\}$
$= \{x : x \text{ is an integer}\} = Z$

**II Intersection of sets :**

If A and B are two sets then intersection ($\cap$) of A and B is the set of all those elements which belong to both A and B. It is also defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$ represented in Venn diagram (see fig.)




**llustration 1** If $A = \{x : x = 4n, n \in Z\}$ and
$B = \{x : x = 6n, n \in Z\}$, then $A \cap B =$

(A) $\{x : x = 2n, n \in Z\}$ (B) $\{x : x = 10n, n \in Z\}$
(C) $\{x : x = 12n, n \in Z\}$ (D) $\{x : x = 24n, n \in Z\}$

**Solution** We have,

$x \in A \cap B$
$\Rightarrow$ $x = 4n$ and $x = 6n, n \in Z$
$\Rightarrow$ x is a multiple of 4 and x is a multiple of 6
$\Rightarrow$ x is a multiple of 4 and 6 both
$\Rightarrow$ x is a multiple of 12
$\Rightarrow$ $x = 12n, n \in Z$

Hence, $A \cap B = \{x : x = 12n, n \in Z\}$

**NOTE**

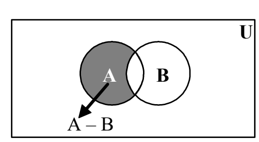
Sets A and B are said to be disjoint iff A and B have no common element or $A \cap B = \phi$. If $A \cap B \neq \phi$ then A and B are said to be intersecting or overlapping sets. **e.g.**

**(i)** If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ and $C = \{4, 7, 9\}$ then A and B are disjoint set where B and C are intersecting sets.

**(ii)** Set of even natural numbers and odd natural numbers are disjoint sets.

**III Difference of two sets :**

If A and B are two sets then the difference of A and B, is the set of all those elements of A which do not belong to B.




Thus, $A - B = \{x : x \in A \text{ and } x \notin B\}$
or $A - B = \{x \in A ; x \notin B\}$

Clearly $x \in A - B \Leftrightarrow x \in A$ and $x \notin B$

It is represented through the Venn diagrams.

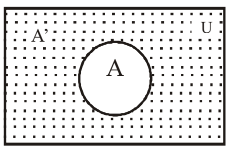
The difference B – A is the set of all those elements of B that do not belong to A

i.e. $B - A = \{x : x \in B$ and $x \notin A\}$.

**IV COMPLEMENTARY SET**

Complementary set of a set A is a set containing all those elements of universal set which are not in A. It is denoted by $\overline{A}$, $A^C$ or $A'$. So $A^C = \{x : x \in U$ but $x \notin A\}$.

e.g. If set $A = \{1, 2, 3, 4, 5\}$ and universal set $U = \{1, 2, 3, 4, .......50\}$ then $\overline{A} = \{6, 7, .......50\}$




(i) $U' = \phi$ (ii) $\phi' = U$

(iii) $(A')' = A$ (iv) $A \cup A' = U$

(v) $A \cap A' = \phi$

**NOTE**

All disjoint sets are not complementary sets but all complementary sets are disjoint.

**V Idempotent operation :**

For any set A, we have

(i) $A \cup A = A$ and (ii) $A \cap A = A$

**Proof :**

(i) $A \cup A = \{x : x \in A$ or $x \in A\} = \{x : x \in A\} = A$

(ii) $A \cap A = \{x : x \in A$ & $x \in A\} = \{x : x \in A\} = A$

**VI Identity operation :** For any set A, we have

(i) $A \cup \phi = A$ and

(ii) $A \cap U = A$ i.e. $\phi$ and U are identity elements for union and intersection respectively

**Proof :**

(i) $A \cup \phi = \{x : x \in A$ or $x \in \phi\} = \{x : x \in A\} = A$

(ii) $A \cap U = \{x : x \in A$ and $x \in U\} = \{x : x \in A\} = A$

**VII Commutative operation :**

For any set A and B, we have

(i) $A \cup B = B \cup A$ and

(ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

**VIII Associative operation :**

If A, B and C are any three sets then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

**IX Distributive operation :**

If A, B and C are any three sets then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

**X De-Morgan's Principle :**

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

**Proof :**

(i) Let x be an arbitrary element of $(A \cup B)'$. Then $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B) \Rightarrow x \notin A$ and $x \notin B \Rightarrow x \in A' \cap B'$

Again let y be an arbitrary element of $A' \cap B'$. Then $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$ $\Rightarrow$ $y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$ $\Rightarrow$ $y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$.

Hence $(A \cup B)' = A' \cap B'$

Similarly (ii) can be proved.

**SOME IMPORTANT RESULTS**

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint non-void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$

i.e, $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \cup B \cup C)$

$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(v) No. of elements is exactly two of the sets A, B, C =

$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$.

**2. NUMBER SYSTEM**

**2.1 Natural Numbers**

Counting numbers 1, 2, 3, 4, 5,..... are known as natural numbers. The set of all natural numbers can be represent by

$N = \{1, 2, 3, 4, 5, ....\}$

**2.2 Whole Numbers**

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5..... are called whole numbers. The set of whole numbers can be represented by

$W = \{0, 1, 2, 3, 4, 5, ....\}$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

**2.3 Integers**

All counting numbers and their negatives including zero are known as integers. The set of integers can be represented by

$Z = \{.... -4, -3, -2, -1, 0, 1, 2, 3, 4, ....\}$

**2.4 Positive integers**

The set $I^+ = \{1, 2, 3, 4, ....\}$ is the set of all positive integers, Clearly, positive integers and natural numbers are synonyms.

**2.5 Negative integers**

The set $I^- = \{-1, -2, -3, ....\}$ is the set of all negative integers. 0 is neither positive nor negative.

**2.6 Non-negative integers**

The set $\{0, 1, 2, 3, ....\}$ is the set of all non-negative integers.

**2.7 Even or Odd Numbers**

Number which are divisible by 2 are called even and which are not divisible by 2 called odd number. In general, even numbers can be represented by $2n$ and odd numbers can be represented by $2n \pm 1$, where $n$ is an integer

**Points to Remember**

(i) The sum and product of any number of even numbers is an even number.

(ii) The difference of two even numbers is an even number.

(iii) The sum of odd numbers depends on the number of numbers.

(a) If the number of numbers is odd, then sum is an odd number.

(b) If the number of numbers is even, then sum is an even number.

(iv) If the product of a certain number is even, then atleast one of the number has to be even.

**2.8 Rational Numbers**

The numbers of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$, are known as rational numbers.

e.g. $\frac{4}{7}, \frac{3}{2}, \frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc. The set of all rational numbers is denoted by Q.

i.e. $Q = \left\{x : x = \frac{p}{q}; p, q \in I, q \neq 0\right\}$

Since every natural number a can be written as $\frac{a}{1}$, so a is rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$, so it is also a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or non-terminating repeating decimals.

**(a) Terminating Decimal :**

Let x be a rational number whose decimal expansion terminates. Then, x can be repressed in the form $\frac{p}{q}$, where p and q are co-primes, and prime factorizations of q is of the form $2^m \times 5^n$, where m, n are non-negative integers. In this a finite number of digit occurs after decimal.

For example : $\frac{1}{2} = 0.5, \frac{11}{16} = 0.6875, \frac{3}{20} = 0.15$ etc.

**(b) Non-Terminating and Repeating (Recurring Decimal)**

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating. In this a set of digits or a digit is repeated continuously.

For example : $\frac{2}{3} = 0.6666.... = 0.\overline{6}$

and $\frac{5}{11} = 0.454545.... = 0.\overline{45}$

**2.9 Irrational numbers**

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, etc.

**Note** That the exact value of $\pi$ is not $\frac{22}{7}$. $\frac{22}{7}$ is rational while $\pi$ is irrational numbers. $\frac{22}{7}$ is approximate value of $\pi$. Similarly, 3.14 is not an exact value of it.

**2.10 Prime Number**

Except 1 each natural number which is divisible by only 1 and itself is called as prime number e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ..... etc.

**2.11 Co-prime**

A pair of two natural numbers having no common factor, other than 1, is called a pair of co-primes.
For Ex. (3, 5), (4, 5), (5, 6), (7, 9), (6, 7) etc., are co-primes

**2.12 Twin primes**

Prime numbers differing by two are called twin primes, e.g. (3, 5), (5, 7), (11, 13) etc, are called twin primes.

**2.13 Prime triplet**

A set of three consecutive primes differing by 2, such as (3, 5, 7) is called a prime triplet.

**"every prime number except 2 is odd
but every odd number need not be prime"**

**2.14 Fractions**

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator < Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction : Numerator > Denominator i.e., $\frac{5}{3}$.

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $3\frac{2}{7}$

(f) Compound fraction : Fraction whose number and denominator themselves are fractions i.e. $\frac{2/3}{5/7}$

**2.15 Composite numbers**

All natural numbers, which are not prime are composite numbers. If C is the set of composite numbers then C = {4, 6, 8, 9, 10, 12.....}

**2.16 Imaginary numbers**

All the numbers whose square is negative are called imaginary numbers. e.g. 3i, – 4i, ....., where $i = \sqrt{-1}$

**2.17 Complex numbers**

The combined form of real and imaginary numbers is known as complex number.

It is denoted by z = a + ib, where a is real and b is imaginary part of z and a, b $\in$ R.

## DPP 1

**Total Marks : 39** **Time 25 min**

**Instructions**

1. **Question – 3, 4, 5, 8, 9 marking scheme : +3 for correct answer, –1 in all other cases. [5 × 3 = 15]**
2. **Question – 1, 2, 6, 7, 10, 11 marking scheme : +4 for correct answer, 0 in all other cases. [4×6=24]**

**1.** Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\right\}$ in the set-builder form.

**2.** Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form :

(i) {P, R, I, N, C, A, L} (a) { x : x is a positive integer and is a divisor of 18}

(ii) { 0 } (b) { x : x is an integer and $x^2 - 9 = 0$}

(iii) {1, 2, 3, 6, 9, 18} (c) {x : x is an integer and x + 1= 1}

(iv) {3, –3} (d) {x : x is a letter of the word PRINCIPAL}

**3.** If B is the set whose elements are obtained by adding to 1 to each of the even numbers, then the set builder notation of B is

(A) B = {x : x is even}
(B) B = {x : x is odd and x > 1}
(C) B = {x : x is odd and x $\in$ Z}
(D) B = {x : x is an integer}

**4.** Which of the following is the empty set ?

(A) {x : x is a real number and $x^2 - 1 = 0$}
(B) {x : x is a real number and $x^2 + 1 = 0$}
(C) {x : x is a real number and $x^2 - 9 = 0$}
(D) {x : x is a real number and $x^2 = x + 2$}

**5.** Which of the following sets is not finite ?

(A) $\{(x, y) : x^2 + y^2 \le 1 \le x + y, x, y \in R\}$
(B) $\{(x, y) : x^2 + y^2 \le 1 \le x + y, x, y \in Z\}$
(C) $\{(x, y) : x^2 \le y \le |x|, x, y \in Z\}$
(D) $\{(x, y) : x^2 + y^2 = 1, x, y \in Z\}$

**6.** Which of the following pairs of sets are equal ? Justify your answer.

(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".

(ii) $A = \{n : n \in Z \text{ and } n^2 \le 4\}$ and B $= \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$.

**7.** Let A = { a, e, i, o, u} and B = { a, b, c, d}. Is A a subset of B ? No. (Why ?). Is B a subset of A ? No. (Why ?)

**8.** The collection of intelligent students in a class is :

(A) a null set (B) a singleton set
(C) a finite set (D) not a well defined collection.

**9.** If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n - 1) : n \in N\}$, then

(A) $X \subset Y$ (B) $Y \subset X$
(C) X = Y (D) None of these

**10.** Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}. Find $A'$, $B'$, $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$.

**11.** Let A = { 1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8 }. Find A – B and B – A.

**Result Analysis**

1. 30 to 39 Marks : **Advance Level.**
2. 20 to 29 Marks : **Main Level.**
3. < 20 Marks : **Below Averaege**

**(Please go through this artical again.)**

## 3. INEQUALITIES

### 3.1 INEQUATIONS

A statement involving variable (s) and the sign of inequality viz., > or, < or, ≥ or, ≤ is called an inequation.

An inequation may contian one or more variables. Also, it may be linear or quadratic or cubic etc.,

If a is a non-zero real number and x, y are variables, then $ax + b < 0$, $ax + b \le 0$, $ay + b > 0$ and $ay + b \ge 0$ are linear inequations in one variable.

If $a \ne 0, b \ne 0$, c real numbers and x, y, z are variables, then $ax + by < c$, $ay + bz \le c$, $ax + by > c$ and $ax + by \ge c$ are linear inequations in two variables.

Inequations $2x - 3 < 0$,

$3y \ge 5, \frac{x}{2} - 7 \le 0, \frac{2x-3}{4} \ge -\frac{x}{2} + 7$ and $-2 + t \le \frac{5}{3}$ are linear inequations in one variables. Inequations of the form

$\frac{2x-3}{x-1} < 4, x^2 - 5x + 6 \ge 0$ etc are not linear inequations.

**Solution** A solution of an inequation is the value(s) of the variable(s) that makes it a true statement.

For example, x = 9 is a solution of the inequation

$\frac{3-2x}{5} < \frac{x}{3} - 4,$ because for $x = 9$ it reduces to $-3 < -1$ which is a true statement. But, x = 6 is not its solution. For x = 6, the inequation reduces to $-\frac{9}{5} < -2$ which is not true. For any real number x, we have $x^2 + 1 > 0$. So, the solution set of the inequation $x^2 + 1 > 0$ is the set R of all real numbers and the solution set of $x^2 + 1 < 0$ is the null set $\phi$.

For solving an inequation, we use the following rules :

**RULE 1** Same number may be added to (or substracted from) both sides of an in equation without changing the sign of inequality.

**RULE 2** Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.

**RULE 3** Any term of an ineqaution may be taken to the other side with its sign changed without affecting the sign of inequality.

**Illustration 1** The solution of the inequation

$\frac{1}{2}\left(\frac{3}{5}x + 4\right) \ge \frac{1}{3}(x - 6)$ is

(A) $[120, \infty)$ (B) $(-\infty, 120]$
(C) $[0, 120]$ (D) $[-120, 0]$

**Ans. (B)**

**Solution** We have,

$$\frac{1}{2}\left(\frac{3}{5}x + 4\right) \ge \frac{1}{3}(x - 6)$$

$$\Rightarrow \frac{1}{2}\left(\frac{3x + 20}{5}\right) \ge \frac{1}{3}(x - 6)$$

$$\Rightarrow \frac{3x + 20}{10} \ge \frac{x - 6}{3}$$

$\Rightarrow 3(3x + 20) \ge 10(x - 6)$

[Multiplying both sides by 30, i.e., LCM of 10 and 3]

$\Rightarrow 9x + 60 \ge 10x - 60$

$\Rightarrow 9x - 10x \ge -60 - 60$

$\Rightarrow -x \ge -120$

$\Rightarrow x \le 120$

$\Rightarrow x \in (-\infty, 120]$

Hence, the solution set of the given inequation is $(-\infty, 120]$

**Remark** The solution set of simultaneous inequations is the intersection of their solution sets.

**Illustration 2** If $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ and $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ then x belongs to the interval

(A) $(3, \infty)$ (B) $(0, \infty)$
(C) $(-\infty, 3)$ (D) $(-\infty, 0)$

**Ans (A)**

**Solution** We have,

$$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \text{ and } \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$$

$$\Rightarrow \frac{10x + 3x}{8} > \frac{39}{8} \text{ and } \frac{2x - 1 - 4x + 4}{12} < \frac{3x+1}{4}$$

$$\Rightarrow \frac{13x}{8} > \frac{39}{8} \text{ and } \frac{-2x + 3}{12} < \frac{3x+1}{4}$$

$\Rightarrow 13x > 39$ and $-2x + 3 < 9x + 3$

$\Rightarrow x > 3$ and $-11x < 0$

$\Rightarrow x > 3$ and $x > 0$

$\Rightarrow x \in (3, \infty)$ and $x \in (0, \infty)$ $\Rightarrow x \in (3, \infty)$

### 3.2 SOLVING RATIONAL ALGEBRAIC INEQUATIONS

If P(x) and Q(x) are polynomials in x, then the inequations

$\frac{P(x)}{Q(x)} > 0, \frac{P(x)}{Q(x)} < 0, \frac{P(x)}{Q(x)} \ge 0$ and $\frac{P(x)}{Q(x)} \le 0$ are known as rational algebraic inequations. To solve these inequations we use the sign method as explained in the following algorithm.

**ALGORITHM**

**STEP 1** Obtain P(x) and Q(x)

**STEP 2** Factorise P(x) and Q(x) into linear factors.

**STEP 3** Make the coefficient of x positive in all factors.

**STEP 4** Obtian critical points by equating all factors to zero.

**STEP 5** Plot the critical points on the number line. If there are n critical points, they divide the number line into (n + 1) regions.

**STEP 6** In the right most region the expression $\frac{P(x)}{Q(x)}$ bears positive sign and in other regions the expression bears alternate negative and positive signs.

**Illustration 1** If $\frac{x-1}{x} \geq 2$, then x belongs to the interval

(A) (1, 2) (B) (0, 1)

(C) [–1, 0) (D) (1, ∞)

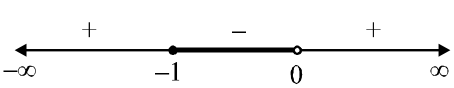
**Ans. (C)**

**Solution** We have,

$$\frac{x-1}{x} \geq 2 \;\Rightarrow\; \frac{x-1}{x} - 2 \geq 0 \;\Rightarrow\; \frac{-1-x}{x} \geq 0$$

$$\Rightarrow \frac{x+1}{x-0} \leq 0 \qquad .....(i)$$

On equating x + 1 and x – 0 to zero, we obtain x = –1 and x = 0 as critical points. These points when plotted on number line divided it into there regions. Marking alternatively positive and negative from the right most region, we obtain the sign of $\frac{x+1}{x}$ for different values of x as shown in the following figure.




Since the expression in (i) is non-positive. So, the solution set of (i) is [–1, 0).

### 3.3 Generalized Method of Intervals

Let $F(x) = (x-a_1)^{k_1}(x-a_2)^{k_2}...(x-a_{n-1})^{k_{n-1}}(x-a_n)^{k_n}$.

Here, $k_1, k_2, k_n \in Z$ and $a_1, a_2, ...., a_n$ are fixed real numbers satisfying the condition

$a_1 < a_2 < a_3 < .... a_{n-1} < a_n$

For solving F(x) > 0 or F(x) < 0, consider the following algorithm.

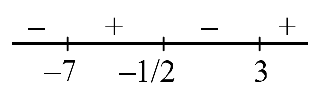
- We mark the numbers $a_1, a_2,..., a_n$ on the number axis and put plus sign in the interval on the right of the largest of these numbers, i.e., on the right of $a_n$.
- Then we put plus sign in the interval on the left of $a_n$ if $k_n$ is an even number and minus sign of $k_n$ is an odd number. In the next interval, we put a sign according to the following rule :
- When passing through the point $a_{n-1}$ the polynomial F(x) changes sign if $k_{n-1}$ is an odd number.

  Then we consider the next interval and put a sign in it using the same rule.
- Thus we consider all the intervals. The solution of the inequality F(x) > 0 is the union of all intervals in which we put plus sign and the soultion of the inequality F(x) < 0 is the union of all intervals in which we minus sign.

**llustration 1** Solve (2x + 1) (x – 3) (x + 7) < 0.

**Solution** (2x + 1) (x – 3) (x + 7) < 0

Sign scheme of (2x + 1) (x – 3) (x + 7) is as follows :




Hence, solution is (–∞, –7) ∪ (–1/2, 3).

**llustration 2** Solve $\frac{2}{x} < 3$

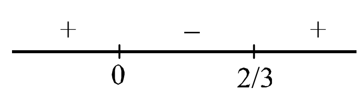
**Solution** $\frac{2}{x} < 3$

$$\Rightarrow \frac{2}{x} - 3 < 0 \;\Rightarrow\; \frac{2-3x}{x} < 0$$

(we cannot cross multiply with x, as x can be negative or positive)

$$\Rightarrow \frac{3x-2}{x} > 0 \;\Rightarrow\; \frac{(x-2/3)}{x} > 0$$

Sign scheme of $\frac{(x-2/3)}{x}$ is as follows :




$\Rightarrow \quad x \in (-\infty, 0) \cup (2/3, \infty)$

**llustration 3** Solve $\frac{2x-3}{3x-5} \geq 3$

**Solution** $\frac{2x-3}{3x-5} \geq 3$

$\Rightarrow \dfrac{2x-3}{3x-5} - 3 \geq 0 \qquad \Rightarrow \dfrac{2x-3-9x+15}{3x-5} \geq 0$

$\Rightarrow \dfrac{-7x+12}{3x-5} \geq 0 \qquad \Rightarrow \dfrac{7x-12}{3x-5} \leq 0$

Sign scheme of $\dfrac{7x-12}{3x-5}$ is as follows :

+    –    +

——|———|——
5/3    12/7

$\Rightarrow x \in (5/3, 12/7]$

x = 5/3 is not included in solution as at x = 5/3.

denominator becomes zero.

$x \in \left(\dfrac{5}{3}, \dfrac{12}{7}\right]$

**llustration 4** Solve $(1-x)^2 (x+4) < 0$

**Solution** $(x-1)^2 (x+4) < 0$ ... (i)

Sign scheme of $(x-1)^2 (x+4)$ is as follows :

–    +    +

——|———|——
–4    1

Sign of expression does not change at x = 1 as (x – 1) factor has even power.

Hence, solution of (i) is $x \in (-\infty, -4)$

## DPP 2

**Total Marks : 38** **Time 25 min**

**Instructions**

1. **Question – 4, 5 marking scheme : +3 for correct answer, –1 in all other cases. [3 × 2 = 6]**
2. **Question – 1, 2, 3, 6, 7, 8, 9, 10 marking scheme : +4 for correct answer, 0 in all other cases. [4×8=32]**

1. Solve $4x + 3 < 6x + 7$

2. Solve $\dfrac{5-2x}{3} \leq \dfrac{x}{6} - 5$

3. Solve 7x + 3 < 5x + 9. Show the graph of the solutions on number line.

4. The solution set of the inequation $\dfrac{x-1}{x-2} > 2$, is

   (A) (2, 3) (B) [2, 3]
   (C) $(-\infty, 3) \cup (3, \infty)$ (D) None of these

5. The set of values of x satisfying the system of inequations

   $5x + 2 < 3x + 8$ and $\dfrac{x+2}{x-1} < 4$ is,

   (A) $(-\infty, 1)$ (B) (2, 3)
   (C) $(-\infty, 3)$ (D) $(-\infty, 1) \cup (2, 3)$

6. Solve $-8x \leq 5x - 3 < 7$.

7. Solve $-5 \leq \dfrac{5-3x}{2} \leq 8$

8. Solve $\dfrac{x-1}{x} - \dfrac{x+1}{x-1} < 2$

9. Solve $\dfrac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} > 1$

10. Solve $\dfrac{(x-4)^{2005}.(x+8)^{2008}(x+1)}{x^{2006}(x-2)^3.(x+3)^5.(x-6)(x+9)^{2010}} \leq 0$

**Result Analysis**

1. 30 to 38 Marks : **Advance Level.**
2. 20 to 29 Marks : **Main Level.**
3. < 20 Marks : **Below Averaege**
   **(Please go through this artical again.)**

**LOGARITHM**

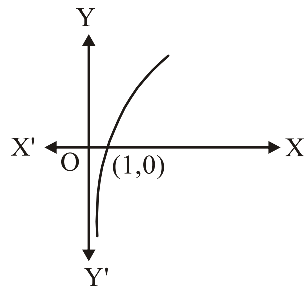
Every positive real number N can be expressed in exponential form as $a^x = N$ where 'a' is also a positive real different than unity and is called the base and 'x' is called an exponent. We can write the relation $a^x = N$ in logarithmic form as $\log_a N = x$.

Hence $a^x = N \Leftrightarrow \log_a N = x$.

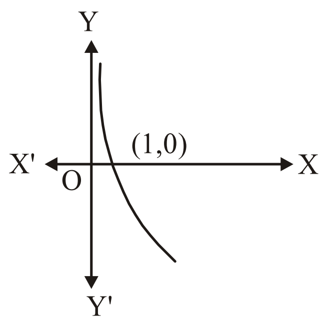
Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number.

**GRAPH OF LOGARITHMIC FUNCTIONS**

**1.** Graph of $y = \log_a x$, if $a > 1$ and $x > 0$




**2.** Graph of $y = \log_a x$, if $0 < a < 1$ and $x > 0$.

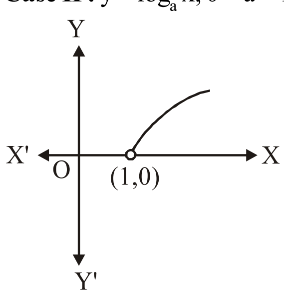


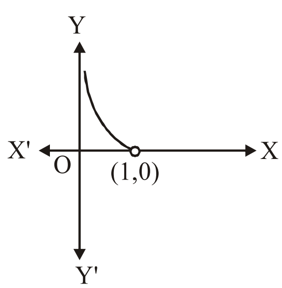

**Points to Consider**

**1.** If the number x and the base 'a' are on the same side of the unity, then the logarithm is positive.

**Case I :** $y = \log_a x, a > 1, x > 1$

**Case II :** $y = \log_a x, 0 < a < 1, 0 < x < 1'$

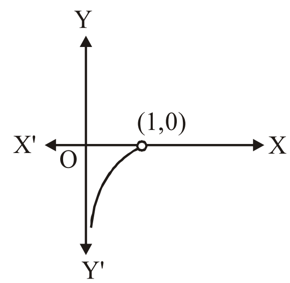


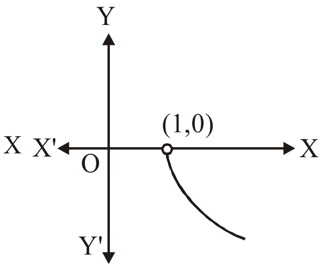




**2.** If the number x and the base a are on the opposite sides of the unity, then the logarithm is negative.

**Case I** $y = \log_a x, a > 1, 0 < x < 1$

**Case II** $y = \log_a x, 0 > a < 1, x > 1$




## NATURAL LOGARITHM

**Natural logarithm** is the logarithm to the base e, where e is an irrational constant approximately equal to 2.718281828. Here, e is an irrational number. Also, e is defined exactly as $e = (1 + 1/m)^m$ as m increases to infinity. We can see how this definition produces e by inputting a large value of m such as m = 10,000,000 to get $(1 + 1/100000000)^{10000000} =$ 2.7182817 (rounded), which is very close to the actual value. The natural logaritum is generally written as ln (x) or $\log_e(x)$.

**Common logarithm** is the logarithm with base 10. It is also known as the **decadic logaritm,** named after its base. It is indicated by $\log_{10}(x)$. On calculator, it is usually written as "log," but mathematicians usually mean natural logarithm, and not common logarithm, when they write "log". To mitigate this ambiguity, the ISO specification is that $\log_{10}(x)$ should be log(x) and **$\log_e(x)$** should be **ln (x).**

**llustration 1** Find x : (i) $\log_2 x = 0$ (ii) $\log_3(\log_2 x) = 1$

**Solution(i)** $\log_2 x = 0 \quad \Rightarrow \quad x = 2^o = 1$

**(ii)** $\log_3(\log_2 x) = 1 \quad \Rightarrow \quad \log_2 x = 3^1$

$x = 2^3 = 8$

Limitations of logarithm : $\log_a N$ is defined only when

(1) $N > 0$ (2) $a > 0$ (3) $a \neq 1$

**NOTE**

**1.** For a given value of N, $\log_a N$ will give as a unique value.

**2.** log of zero does not exist.

**3.** logarithm of (–)ve reals are not defined in the system of real numbers.

**FUNDAMENTAL IDENTITIES :**

Using the basic definition of logarithm we have 3 important deductions : $(N > 0, N \neq 1)$

(a) $\text{Log}_N N = 1$ i.e., logarithm of a number of the same base is 1.

(b) $\log_{\frac{1}{N}} N = -1$ i.e., logarithm of a number to its reciprocal is –1.

(c) $\text{Log}_N 1 = 0$ i.e., logarithm of unity of any base is zero.

**llustration 2** Find the the value of the following

(i) $\log_9 27$ (ii) $\log_{3\sqrt{2}} 324$

(iii) $\log_{1/9}\left(27\sqrt{3}\right)$ (iv) $\log_{(5+2\sqrt{6})}\left(5-2\sqrt{6}\right)$

(v) $\log_{0.2} 0.008$

**Solution** (i) Let $x = \log_9 27$

$\Rightarrow 9^x = 27 \quad \Rightarrow \quad 3^{2x} = 3^3 \Rightarrow 2x = 3$

$\therefore \quad x = \frac{3}{2}$

(ii) Let $x = \log_{3\sqrt{2}} 324$

$\Rightarrow \left(3\sqrt{2}\right)^x = 324 = 2^2 . 3^4 \quad \Rightarrow \left(3\sqrt{2}\right)^x = \left(3\sqrt{2}\right)^4$

$\therefore \quad x = 4$

(iii) Let $x = \log_{1/9}\left(27\sqrt{3}\right)$

$\Rightarrow \left(\frac{1}{9}\right)^x = 27\sqrt{3}$

$\Rightarrow 3^{-2x} = 3^{7/2} \Rightarrow -2x = 7/2$

$\therefore \quad x = -\frac{7}{4}$

(iv) $\because \left(5+2\sqrt{6}\right)\left(5-2\sqrt{6}\right) = 1$

or $\quad 5+2\sqrt{6} = \frac{1}{5-2\sqrt{6}}$

Now, let x $= \log_{(5+2\sqrt{6})}\left(5-2\sqrt{6}\right)$

$= \log_{1/(5-2\sqrt{6})} 5-2\sqrt{6} = -1$

(v) Let $x = \log_{0.2} 0.008$

$\Rightarrow (0.2)^x = 0.008$

$\Rightarrow (0.2)^x = (0.2)^3$

$\therefore \quad x = 3$

**llustration 3** Find the value of the following :

(i) $\log_{\tan 45^o} \cot 30^o$ (ii) $\log_{(\sec^2 60^o - \tan^2 60^o)} \cos 60^o$

(iii) $\log_{(\sin^2 30^o + \cos^2 30^o)} 1$ (iv) $\log_{30} 1$

**Solution** (i) Here, base = tan 45° = 1

$\therefore$ log is not defined.

(ii) Here, base = $\sec^2 60^o - \tan^2 60^o = 1$

$\therefore$ log is not defined.

(iii) $\because \log_{(\sin^2 30^o + \cos^2 30^o)} 1 = \log_1 1 \neq 1$

$\because$ Here, base = 1

$\because$ log is not defined

(iv) $\log_{30} 1 = 0$

**llustration 4** Find all values of x for which the following equalities hold true ?

(i) $\log_2 x^2 = 1$

(ii) $\log_3 x = \log_3 (2-1)$

(iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2} (2x+1) = \log_{1/2} (x+1)$

(v) $\log_{1/3} (x^2+8) = -2$

**Solution** (i) $\log_2 x^2 = 1 \quad \Rightarrow x^2 = 2$

$x = \pm\sqrt{2}$

(ii) $\log_3 x = \log_3(2-1)) = \log_3 1 = 0$

$x = 3^o = 1$

(iii) $\log_4 x^2 = \log_4 x \quad \Rightarrow x^2 = x$

$x(x-1) = 0 \quad \Rightarrow x = 0, 1$

then $x = 1, x > 0 \quad$ ($x = 0$ is rejected)

(vi) $\log_{1/2}(2x+1) = \log_{1/2}(x+1)$

$2x+1 = x+1 \quad \Rightarrow x = 0$

(v) $\log_{1/3}(x^2+8) = -2$

$x^2 + 8 = (1/3)^{-2} \quad \Rightarrow x^2 + 8 = 9$

$x^2 = 1 \quad \Rightarrow x = \pm 1$

## DPP 3

**Total Marks 32** **Time 20 Minute**

Question Number 1 to 5. **Marking Scheme** : +2 for correct answer 0 in all other cases.

**Find the value of following :** **[14 marks]**

**1.** (i) $\log_{\sin 30^o}\left(\cos 60^o\right)$

(ii) $\log_{\frac{4}{3}}\left(1.\overline{3}\right)$

(iii) $\log_{10}(\cos 0^o)$

(iv) $\log_2(\sin^2 x + \cos^2 x)$

(v) $\log_{1.5}\left(0.\overline{6}\right)$

(vi) $\log_{0.125}(8)$

(vii) $\log_{2-\sqrt{3}}\left(2+\sqrt{3}\right)$

**2.** (i) $\log_{\sqrt[3]{7}}\left(2401\right)$ **[6 marks]**

(ii) $\log_{5\sqrt{5}}\left(125\right)$

(iii) $\log_2[\log_2\{\log_3(\log_3 27^3)\}]$

**3.** (i) $\log \tan 1^\circ . \log \tan 2^\circ . \log \tan 3^\circ$ ......... $\log \tan 89^\circ$

(ii) $\log \sin 1^\circ . \log \sin 2^\circ . \log \sin 3^\circ$ ......... $\log \sin 90^\circ$

**[4 marks]**

**4.** Use basic definition of logarithm to prove that :

$\log_2 10 - \log_8 125 = 1$

**5.** Find all the values of x for which the following equalities hold true ?

(i) $\log_2 x^2 = 2$ **[6 marks]**

(ii) $\log_{1/4} x^2 = 1$

(iii) $\log_{1/2} x = \log_{1/2} (3 - x)$

**Result Analysis**

1. 24 to 32 Marks : **Advanced Level.**
2. 18 to 23 Marks : **Main Level**
3. < 18 Marks : **Below Averaege**

(**Please go through this articles again**)

## FUNDAMENTAL LAWS OF LOGARITHMS

**1.** For m, n, a > 0, a ≠1; $\log_a (mn) = \log_a m + \log_a n$

Proof : Let $\log_a m = x$ and $\log_a n = y$.

$\log_a m = x \Rightarrow a^x = m$.

and $\log_a n = y \Rightarrow a^y = n$

$\therefore mn = a^x . a^y$

or $mn = a^{x+y} \Rightarrow \log_a (mn) = x + y$

$\Rightarrow \log_a (mn) = \log_a m + \log_a n$

In general, if $x_1, x_2, ....., x_n$ are positive real number, then $\log_a (x_1 x_2 ... x_n) = \log_a x_1 + \log_a x_2 + .... + \log_a x_n$.

**2.** For m, n, a > 0, a ≠1 ; $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

**Proof :** Let $\log_a m = x \Rightarrow a^x = m$ and

$\log_a n = y \Rightarrow a^y = n$

$\therefore \frac{m}{n} = \frac{a^x}{a^y}$ or $\frac{m}{n} = a^{x-y}$

or $\log_a \left(\frac{m}{n}\right) = x - y = \log_a m - \log_a n$

**3.** For m, n, a > 0, a ≠ 1; $\log_a (m^n) = n \log_a m$

**Proof** : Let $\log_a m = x$

or $a^x = m$ or $(a^x)^n = m^n$

or $a^{xn} = m^n$ or $\log_a (m^n) = nx$

$\Rightarrow \log_a (m^n) = n \log_a m$

**4.** **Base change theorem**

If m, a, b > 0 and a ≠ 1, b ≠ 1 then $\log_a m = \frac{\log_b m}{\log_b a}$

**Proof :** Let $\log_a m = x$, Then $a^x = m$.

Now, $a^x = m$ or $\log_b (a^x) = \log_b m$

[taking log to the base b]

or $x \log_b a = \log_b m$

or $\log_a m . \log_b a = \log_b m$ $[\because \log_m m = 1]$

or $\log_a m = \frac{\log_b m}{\log_b a}$

Replacing b by m in the above result, we get

$\log_a m = \frac{\log_m n}{\log_m a} \Rightarrow \log_a m = \frac{1}{\log_m a}$ $[\because \log_m m = 1]$

**5.** For a, n > 0 and a ≠ 1 ; $a^{\log_a n} = n$

**Proof :** Let $\log_a n = x$. Then $a^x = n$. Therefore, $a^{\log_a n} = n$.

[Putting the value of x in $a^x = n$]

For example, $3^{\log_3 8} = 8, 2^{3\log_2 5} = 2^{\log_2 5^3} = 5^3$,

$5^{-2\log_5 3} = 5^{\log_5 3^{-2}} = 3^{-2} = 1/9$

**6.** $\log_{a^q} n^p = \frac{p}{q} \log_a n$, where a, n > 0, a ≠ 1

**Proof :** Let $\log_{a^q} n^p = x$ and $\log_a n = y$. Then,

$(a^q)^x = n^p$ and $a^y = n$

$\therefore a^{qx} = n^p$ and $a^y = n$

$\Rightarrow a^{qx} = n^p$ and $(a^y)^p = n^p$

$\Rightarrow a^{qx} = (a^y)^p$ or $a^{qx} = a^{yp}$

or $qx = yp \Rightarrow x = \left(\frac{p}{q}\right) y$

$\Rightarrow \log_{a^q} n^p = \frac{p}{q} \log_a n$

**7.** $a^{\log_b c} = c^{\log_b a}$

**Proof :** $a^{\log_b c} = p \Rightarrow \log_b c = \log_a p$

$\Rightarrow \frac{\log c}{\log b} = \frac{\log p}{\log a} \Rightarrow \frac{\log a}{\log b} = \frac{\log p}{\log c}$

$\Rightarrow \log_b a = \log_c p \Rightarrow p = c^{\log_b a} \Rightarrow a^{\log_b c} = c^{\log_b a}$

**llustration 1** What is logarithm of $32\sqrt[5]{4}$ to the base $2\sqrt{2}$ ?

**Solution** $\log_{2\sqrt{2}} 32\sqrt[5]{4} = \log_{(2^{3/2})} \left(2^5 4^{\frac{1}{5}}\right)$

$$\log_{(2^{3/2})} \left(2^{5+\frac{2}{5}}\right) = \frac{2}{3}\frac{27}{5} \log_2 2 = \frac{18}{5} = 3.6$$

**llustration 2** Find the value of $\log_5 \log_2 \log_3 \log_2 512$.

**Solution** $\log_5\log_3\log_2 2^9 = \log_5\log_2\log_3 (9 \log_2 2)$

$= \log_5\log_2\log_3 3^2 = \log_5 \log_2 2 = \log_5 1 = 0$

**llustration 3** If $\log_{\sqrt{8}} b = 3\frac{1}{3}$, then find the value of b.

**Solution** $\log_{\sqrt{8}} b = 3\frac{1}{3} \Rightarrow \frac{2}{3}\log_2 b = \frac{10}{3}$

or $\log_2 b = 5$ or $b = 2^5 = 32$.

**llustration 4** Find the value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

**Solution** $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= \left(3^4\right)^{\log_3 5} + \left(3^3\right)^{\log_{3^2}\left(6^2\right)} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + \left(3^3\right)^{\log_3(6)} + 3^{4\log_{3^2} 7}$

$= 5^4 + 3^{\log_3 6^3 + 3^{2\log_3 7}}$

$= 5^4 + 6^3 + 3^{\log_3 7^2} = 625 + 216 + 7^2 = 890$

**llustration 5** Find the value of $\log_3 4 \log_4 5 \log_5 6 \log_6 7 \log_7 8 \log_8 9$.

**Solution** $\log_3 4\log_4 5\log_5 6 \log_6 7 \log_7 8 \log_8 9$

$$= \frac{\log 4}{\log 3}\times\frac{\log 5}{\log 4}\times\frac{\log 6}{\log 5}\times\frac{\log 7}{\log 6}\times\frac{\log 9}{\log 8} = \frac{\log 9}{\log 3} = \log_3 9 = 2$$

**llustration 6 Simplify :** $\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab}$.

**Solution** $\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab}$

$$= \frac{\log a}{\log a + \log b + \log c} + \frac{\log b}{\log a + \log b + \log c} + \frac{\log c}{\log a + \log b + \log c} = 1$$

**llustration 7** If $\log_{12} 27 = a$, then find $\log_6 16$ in terms of a.

(A) $2\left(\frac{3-a}{3+a}\right)$ (B) $3\left(\frac{3-a}{3+a}\right)$

(C) $4\left(\frac{3-a}{3+a}\right)$ (D) $5\left(\frac{3-a}{3+a}\right)$

**Solution** Since $a = \log_{12}27 = \log_{12}(3)^3 = 3 \log_{12} 3$, we get

$$\frac{3}{\log_3 12} = \frac{3}{1+\log_3 4} = \frac{3}{1+2\log_3 2}$$

$\therefore \log_3 2 = \frac{3-a}{2a}$

Then, $\log_6 16 = \log_6 2^4 = 4 \log_6 2 = \frac{4}{\log_2 6} = \frac{4}{1+\log_2 3}$

$$= \frac{4}{1+\frac{2a}{3-a}} = 4\left(\frac{3-a}{3+a}\right)$$

**llustration 8** Simply : (a) $\log_{1/3} \sqrt[4]{729\sqrt[3]{9^{-1}.27^{-4/3}}}$ ;

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}}$

**Solution** (a) $\log_{\frac{1}{3}} \sqrt[4]{729.\sqrt[3]{9^{-1}.27^{-4/3}}}$

$= \log_{\frac{1}{3}} \sqrt[4]{729.\sqrt[3]{3^{-2}.3^{-2}}} = \log_{\frac{1}{3}} \sqrt[4]{729\times 3^{-3}}$

$\log_{\frac{1}{3}} \sqrt[4]{3^6\times 3^{-2}} = \log_{\frac{1}{3}}(3) = -1$

(b) $a^{\frac{\log_b(\log_b N)}{\log_b a}} = a^{\log_a(\log_b N)} = \log_b N$

**llustration 9** If $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log_e a + \log_e b\right)$, then find the relation between a and b.

**Solution** $\log_e\left(\frac{a+b}{2}\right) = \frac{1}{2}\left(\log_e a + \log_e b\right) = \left(\log_e \sqrt{ab}\right)$

$\Rightarrow \frac{a+2}{2} = \sqrt{ab}$ or $a + b - 2\sqrt{ab} = 0$ $\left(\sqrt{a}-\sqrt{b}\right)^2 = 0$

or $a = b$

**llustration 10** If $a^x = b$, $b^y = c$, $c^z = a$, then find the value of xyz.

**Solution** $a^x = b$, $b^y = c$, $c^z = a$

$\Rightarrow x = \log_a b$, $y = \log_b c$, $z = \log_c a$

$\Rightarrow xyz = (\log_a b)(\log_b c)(\log_c a) = \frac{\log b \log c \log a}{\log a \log b \log c} = 1$

**llustration 11** Suppose x, y, z > 0 and are not equal to 1 and $\log x + \log y + \log z = 0$. Find the value of

$x^{\frac{1}{\log y}+\frac{1}{\log z}} \times y^{\frac{1}{\log z}+\frac{1}{\log x}} \times z^{\frac{1}{\log x}+\frac{1}{\log y}}$ (base 10)

**Solution** Let $K = x^{\frac{1}{\log y}+\frac{1}{\log z}} \times y^{\frac{1}{\log z}+\frac{1}{\log x}} \times z^{\frac{1}{\log x}+\frac{1}{\log y}}$

$$\log K = \log x\left[\frac{1}{\log y}+\frac{1}{\log z}\right] + \log y\left[\frac{1}{\log z}+\frac{1}{\log x}\right] + \log z\left[\frac{1}{\log x}+\frac{1}{\log y}\right]$$

Putting $\log x + \log y + \log z = 0$ (given), we get

$$\frac{\log x}{\log y}+\frac{\log z}{\log y}=-1;\frac{\log y}{\log x}+\frac{\log z}{\log x}=-1;\frac{\log x}{\log z}+\frac{\log y}{\log z}=-1$$

$\therefore$ R.H.S. of Eq. (i) = –3 $\Rightarrow \log_{10}K = -3$ or $K = 10^{-3}$

**llustration 12** If $y^2 = xz$ and $a^x = b^y = c^z$, then prove that,

$\log_b a = \log_c b$.

**Solution** $a^x = b^y = c^z \Rightarrow x \log a = y \log b = z \log c$

$$\therefore \quad \frac{y}{x}=\frac{z}{y} \Rightarrow \frac{\log a}{\log b}=\frac{\log b}{\log c}$$

$\Rightarrow \log_b a = \log_c b$.

**llustration 13**

$$A = \log_{11}\left(11^{\log_{11}1331}\right); B = \log_{385}5+\log_{385}7+\log_{385}11;$$

$$C = \log_4\left(\log_2\left(\log_5 625\right)\right); D = 10^{\log_{100}6}.$$

Find the value of $\frac{AB}{CD}$.

**Solution** $A = \log_{11}11^{\log_{11}1331} = \log_{11}1331 = \log_{11}(11)^3 = 3$

$$B = \log_{385}5+\log_{385}7+\log_{385}11$$

$$= \log_{385}(5\times7\times11) = \log_{385}385 = 1$$

$$C = \log_4\left[\log_2\left(\log_5(625)\right)\right]$$

$$= \log_4 \log_2\left(\log_5 5^4\right)$$

$$= \log_2 \log_2 4$$

$$= \log_4 2 = \log_{2^2} 2 = 1/2$$

$$D = 10^{\log_{100}16} = 10^{\frac{1}{2}\log_{10}16} = 10^{\log_{10}4} = 4$$

$$= \frac{AB}{CD} = \frac{3\times1}{1/2\times4} = \frac{3}{2}$$

**llustration 14** If $\log_b a \log_c a + \log_a b \log_c b + \log_a c \log_b c = 3$ (where a, b, c are different positive real numbers $\neq 1$), then find the value of abc.

**Solution** $\log_b a \log_c a + \log_a b \log_c b + \log_a c \log_b c = 3$

or $\quad \frac{\log a}{\log b}\frac{\log a}{\log c}+\frac{\log b}{\log a}\frac{\log b}{\log c}+\frac{\log c}{\log a}\frac{\log c}{\log b}=3$

or $\quad (\log a)^3 + (\log b)^3 + (\log c)^3 = 3(\log a)(\log b)(\log c)$

or $\quad \log a + \log b + \log c = 0$ (as a, b, c are different)

$\Rightarrow \log abc = 0$ or $abc = 1$

**llustration 15** If $\log_a x = b$ for permissible values of a and x then identify the statement (s) which can be correct :

(A) If a and b are two irrational numbers then x can be rational

(B) If a rational and b irrational then x can be rational

(C) If a irrational and b rational then x can be rational

(D) If a rational and b rational then x can be rational

**Solution (A, B, C, D)**

$\log_a x = b$ or $x = a^b$

(A) $(\sqrt{2})^{\log_2 49}$

(B) For $a = 2 \in Q$ and $b = \log_2 3 \notin Q$; $x = 3$ which is rational.

(C) For $a = \sqrt{2}$ and $b = 2$; $x = 2$.

(D) The option is obviously correct $2^2$.

**llustration 16** If $a^x = b^y = c^z = d^w$, then prove that $\log_a(bcd) = \left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)x$

**Solution** $a^x = b^y$, $a^x = c^z$, $a^x = d^w$

take log base a both side then add all

$$\log_a d = \frac{x}{w}\cdot\log_a a$$

$$\log_a b = \frac{x}{y}\cdot\log_a a$$

$$\log_a c = \frac{x}{z}\cdot\log_a a$$

$$\log_a(bcd) = \left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)x$$

**llustration 17** If $\log_x \log_{18}\left(\sqrt{2}+\sqrt{8}\right)=\frac{1}{3}$, then the value of 32 x =

**Solution** $\log_x \log_{18}\left(\sqrt{2}+\sqrt{8}\right)=\frac{1}{3}$,

$$\log_x \log_{\left(\sqrt{2}+\sqrt{8}\right)}\left(\sqrt{2}+\sqrt{8}\right)=\frac{1}{3}$$

$$\log_x \frac{1}{2}=\frac{1}{3} \quad \Rightarrow \quad \frac{1}{2}=x^{\frac{1}{3}}$$

$$\left(\frac{1}{2}\right)^3 = x \quad \Rightarrow \quad 32x = \frac{32}{8} = 4$$

**llustration 18** Let $P = \dfrac{5}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}+\frac{1}{\log_5 x}}$ and $(120)^P = 32$, then the value of x be :

**Solution**

$$P = \frac{5}{\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}+\frac{1}{\log_5 x}}$$

$$P = \frac{5}{\log_x 2+\log_x 3+\log_x 4+\log_x 5}$$

$$P=\frac{5}{\log_x(2\times3\times4\times5)}$$

$$P=\frac{5}{\log_x 120} \Rightarrow P\log_x 120=5$$

$\log_x(120)^p=5$

given that $(120)^p=32$

$\log_x 32=5 \Rightarrow 32=x^5 \Rightarrow x=2$

**llustration 19** If x,y,z be positive real numbes such that $\log_{2x}(z)=3$, $\log_{5y}(z)=6$ and $\log_{xy}(z)=\frac{2}{3}$ then the value of z is :

**Solution**

$\log_2 2x=1/3 \Rightarrow \log_2 \Rightarrow 5y=1/6 \Rightarrow \log_2 xy=3/2$

$$\log_2 2x+\log_2 5y+\log_2 xy=\frac{1}{3}+\frac{1}{6}+\frac{3}{2}=\frac{2+1+9}{6}$$

$\Rightarrow \log_2 10x^2y^2=2 \Rightarrow 10x^2y^2=z^2$ .....(i)

$\log_{xy}z=2/3 \Rightarrow z=(xy)^{2/3}$

$z^3=x^2y^2$ .....(ii)

Equation (i) & (ii)

$10z^3=z^2$

$\Rightarrow z=1/10$

**llustration 20** Sum of values of x and y satisfying $\log_x(\log_3(\log_x y)=0$ and $\log_y 27=1$ is :

**Solution (B)**

$\log_x(\log_3(\log_x y))=0$, $\log_y 27=1 \Rightarrow y=27$

$\log_3(\log_x y)=x^o=1$

$\log_x y=3 \Rightarrow y=x^3$

$\log_{3x}27=1 \Rightarrow 3^3=x^3$

$x=3 \Rightarrow y=27$

$x+y=27+3=30$

**llustration 21** Simplify the following :

(a) $4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}$

(b) $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}}3}}}{409}\cdot\left((\sqrt{7})^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}\right)$

(c) $5^{\log_{1/5}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\dfrac{4}{\sqrt{7}+\sqrt{3}}\log_{1/2}\dfrac{1}{10+2\sqrt{21}}$

(d) $49^{(1-\log_7 2)}+5^{-\log_5 4}$

**Solution** (a) Let $E=4^{5\log_{4\sqrt{2}}(3-\sqrt{6})-6\log_8(\sqrt{3}-\sqrt{2})}=4^y$.

where $y=5\log_{4\sqrt{2}}\left(3-\sqrt{6}\right)-6\log_8\left(\sqrt{3}-\sqrt{2}\right)$

$=5\log_{2^{\frac{5}{2}}}\left(\sqrt{3}\cdot\left(\sqrt{3}-\sqrt{2}\right)\right)-6\log_{2^3}\left(\sqrt{3}-\sqrt{2}\right)$

$=2\cdot\log_2\left(\sqrt{3}\cdot\left(\sqrt{3}-\sqrt{2}\right)\right)-2\cdot\log_2\left(\sqrt{3}-\sqrt{2}\right)$

$=2\cdot\log_2\sqrt{3}+2\log_2\left(\sqrt{3}-\sqrt{2}\right)-2\log_2\left(\sqrt{3}-\sqrt{2}\right)$

$=2\cdot\log_2\sqrt{3}=\log_2 3$

Now, $E=4^y=4^{\log_2 3}=\left(2^2\right)^{\log_2 3}=2^{\log_2 9}=9$ **Ans.**

(b) Let $E=\left(\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}}3}}}{409}\right)\cdot\left(\dfrac{(\sqrt{7})^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}}{\text{say}(B)}\right)$

$=\dfrac{AB}{409}(\text{say})$

where $A=81^{\log_9 5}+3^{3\log_3\sqrt{6}}=\left(9^2\right)^{\log_9 5}+3^{\log_3(\sqrt{6})^3}$

$=9\log_9(5)^2+3^{\log_3(\sqrt{6})^3}=(5)^2+\left(\sqrt{6}\right)^3=\left(25+6\sqrt{6}\right)$

Now, $B=\left(\sqrt{7}\right)^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}$

$=\left(\sqrt{7}\right)^{2\cdot\log_7 25}-5^{3\cdot\log_{5^2}6}=7^{(\log_7 25)}-5^{\log_5(6)^{3/2}}$

$=\left(25-6\sqrt{6}\right)$

Hence, $E=\dfrac{A\cdot B}{409}=\dfrac{\left(25+6\sqrt{6}\right)\left(25-6\sqrt{6}\right)}{409}$

$=\dfrac{(25)^2-\left(6\sqrt{6}\right)^2}{409}=\dfrac{625-216}{409}=\dfrac{409}{409}=1$

(c) Let $E=5^{\log_{\frac{1}{3}}\left(\frac{1}{2}\right)}+\log_{\sqrt{2}}\left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right)+\log_{\frac{1}{2}}\left(\dfrac{1}{10+2\sqrt{21}}\right)$

$=5^{\log_5 2}+2\log_2\left(\dfrac{4\cdot\left(\sqrt{7}-\sqrt{3}\right)}{7-3}\right)+\log_2\left(10+2\sqrt{21}\right)$

$=2+\log_2\left(\sqrt{7}-\sqrt{3}\right)^2+\log_2\left(10+2\sqrt{21}\right)$

$=2+\log_2\left(10-2\sqrt{21}\right)+\log_2\left(10+2\sqrt{21}\right)$

$=2+\log_2(100-84)=2+\log_2 16=2+4=6.$

(d) Let $E=49^{(1-\log_7 2)}+5^{-\log_5 4}=49^{(\log_7 7-\log_7 2)}+5^{\log_5(4)^{-1}}$

$=49^{\log_7(7/2)}+\left(\dfrac{1}{4}\right)=7^{\log_7\left(\frac{7}{2}\right)^2}+\left(\dfrac{1}{4}\right)=\left(\dfrac{7}{2}\right)^2+\left(\dfrac{1}{4}\right)=\dfrac{25}{2}$

## DPP 4

**Total Marks 56** **Time 40 Minute**

Question Number 1 to 14. **Marking Scheme** : +4 for correct answer 0 in all other cases.

**1.** If $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \ldots\ldots \log_n (n+1) = 10$, find n.

**2.** Simplify : $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

**3.** Prove that : $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80} = 1$

**4.** The value of x + y + z satisfying the system of equations

$\log_2 x + \log_4 y + \log_4 z = 2$ is

$\log_3 y + \log_9 z + \log_9 x = 2$

$\log_4 z + \log_{16} x + \log_{16} y = 2$

**5.** Prove that : $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} = 0$

**6.** (a) Let a > 1 be a real number. Solve $a^{2\log_2 x} = 5 + 4x^{\log_2 a}$

(b) Solve for a, $a^{2+\log_a 4} = a^2 + 27$.

**7.** Prove that : $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$

**8.** Prove that : $\frac{\log_3 12}{\log_{36} 3} - \frac{\log_3 4}{\log_{108} 3} = 2$

**9.** If $a > 0,\ c > 0,\ b = \sqrt{ac},\ a,\ a, c$ and $ac \neq 1, N > 0$ prove that

$$\frac{\log_a N}{\log_c N} = \frac{\log_a N - \log_b N}{\log_b N - \log_c N}$$

**10.** Let M denotes antilog$_{32}$ 0.6 and N denote the value of $49^{(1-\log_7 2)} + 5^{-\log_5 4}$. Then M.N. is :

(A) 100 (B) 400 (C) 50 (D) 200

**11.** If $\log_4 5 = a$ and $\log_5 6 = b$, then $\log_3 2$ is equal to :

(A) $\frac{1}{2a+1}$ (B) $\frac{1}{2b+1}$ (C) 2ab + 1 (D) $\frac{1}{2b-1}$

**12.** If $x = \log_a bc$ ; $y = \log_b ac$ and $z = \log_c ab$ then which of the following is equal to unity ?

(A) x + y + z (B) x yz

(C) $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ (D) (1 + x) + (1 + y) + (1 + z)

**13.** $x^{\log_x a \cdot \log_a y \cdot \log_y z}$ is equal to :

(A) x (B) y (C) z (D) a

**14.** If $7\log_a\frac{16}{15} + 5\log_a\frac{25}{24} + 3\log_a\frac{81}{80} = 8$, then a =

(A) $2^{1/8}$ (B) $(10)^{1/8}$ (C) $(30)^{1/8}$ (D) 1

**Result Analysis**

**1.** 56 to 40 Marks : **Advanced Level.**

**2.** 40 to 28 Marks : **Main Level**

**3.** < 28 Marks : **Below Averaege**

(**Please go through this articles again**)

## LOGARITHMIC EQUATION

While solving logarithmic equation,we tend to simplify the equation. Solving equation after simplification may give some root which may not define all the terms in the initial equation. Thus, while solving equations involving logarithmic function, we must take care of domain of the equation.

**llustration 1** Solve $\log_4 8 + \log_4(x+3) - \log_4(x-1) = 2$

**Solution** $\log_4 8 + \log_4(x+3) - \log_4(x-1) = 2$

or $\log_4\frac{8(x+3)}{(x-1)} = 2$ or $\frac{8(x+3)}{x-1} = 4^2$

or $x + 3 = 2x - 2 \Rightarrow x = 5$

Also for x = 5 all terms of the equation are defined.

**llustration 2** Solve log (–x) = 2 log (x + 1).

**Solution** by definition, x < 0 and x + 1 > 0, ⇒ –1 < x < 0.

Now log(–x) = 2 log (x + 1)

or $-x = (x+1)^2$

or $x^2 + 3x + 1 = 0$

or $x = \frac{-3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}$

Hence, $x = \frac{-3+5}{2}$ is the only solution.

**llustration 3** Solve $\log_4(2 \times 4^{x-2} - 1) + 4 = 2x$.

**Solution** $\log_4(2 \times 4^{x-2} - 1) + 4 = 2x$

or $\log_4(2 \times 4^{x-2} - 1) = 2x - 4$

or $2 \times 4^{x-2} - 1 = 4^{2x-4}$ or $2y - 1 = y^2$

or $y^2 - 2y + 1 = 0$ or $y = 1$

or $4^{x-2} = 1$ or $x = 2$

**llustration 4** (a) Solve $\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$.

(b) Solve $\log_5(\sqrt[x]{5} + 125) = \log_5 6 + 1 + \frac{1}{2x}$.

**Solution** (a) $\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$

or $\log_5(5^{1/x} + 125) - \log_5 6 = 1 + \frac{1}{2x}$

or $\log_5\left(\frac{5^{\frac{1}{x}} + 125}{6}\right) = 1 + \frac{1}{2x}$

or $\dfrac{5^{\frac{1}{x}}+125}{6}=5^{1+\frac{1}{2x}}$

or $\dfrac{5^{\frac{1}{x}}+125}{6}=5\times 5^{\frac{1}{2x}}$

or $\dfrac{y^2+125}{6}=5y$ (where $y=5^{\frac{1}{2x}}$)

or $y^2-30y+125=0$ or $y=5$ or $25$

$\Rightarrow 5^{\frac{1}{2x}}=5$ or $25$

or $x=1/2$ or $1/4$

(b) **No solution** because $x^{th}$ root defined only when $x\in N$ & $x\geq 2$

**llustration 5** Solve $\log_4(x-1)=\log_2(x-3)$.

**Solution** The given equality is meaningful if

$x-1>0, x-3>0 \Rightarrow x>3$.

The given equality can be written as

$$\frac{\log(x-1)}{\log 4}=\frac{\log(x-3)}{\log 2}$$

or $\log(x-1)=2\log(x-3)$ ($\log 4=2\log 2$)

or $(x-1)=(x-3)^2$ or $x^2-7x+10=0$

or $(x-5)(x-2)=0$ or $x=5$ or $2$

But $x>3$, so $x=5$

**llustration 6** Solve $\log_6 9-\log_9 27+\log_8 x=\log_{64}x-\log_6 4$.

**Solution** $(\log_6 9+\log_6 4)-\dfrac{\log_3 27}{\log_3 9}=\dfrac{\log_8 x}{2}-\log_8 x$

$\Rightarrow 2-\dfrac{3}{2}=-\dfrac{1}{2}\log_8 x$

or $\dfrac{1}{2}=-\dfrac{1}{2}\log_8 x$ or $x=\dfrac{1}{8}$

**llustration 7** Solve: (a) $\log(\log x)+\log(\log x^3-2)=0$ ; where base of log is 10 everywhere.

(b) $\log_x 2.\log_{2x}2=\log_{4x}2$.

(c) $5^{\log x}+5x^{\log 5}=3$ $(a>0)$ ; where base of log is a.

(d) $x^{\log x+4}=32$, where base of logarithm is 2.

**Solution :** $\log_{10}(\log_{10}x)+\log_{10}(\log_{10}x^3-2)=0$ ....(1)

$=\log_{10}((3\cdot\log_{10}x-2)\cdot\log_{10}x)=0$

$\Rightarrow (3\cdot\log_{10}x-2)\cdot\log_{10}x=1$

$\Rightarrow 3\cdot(\log_{10}x)^2-2\cdot(\log_{10}x)-1=0$

$\therefore$ Either $\log_{10}x=\dfrac{-1}{3}\Rightarrow x=10^{-1/3}$ (rejected)

Because $x=10^{-1/3}$ does not satisfy equation (i) or

$\log_{10}x=1\Rightarrow x-10$ **Ans.**

(b) $\log_x 2\cdot\log_{2x}2=\log_{4x}$

$\Rightarrow \left(\dfrac{1}{\log_2 x}\right)\left(\dfrac{1}{1+\log_2 x}\right)=\dfrac{1}{(2+\log_2 x)}$

$\Rightarrow (2+\log_2 x)=\log_2 x+(\log_2 x)^2$

$\Rightarrow (\log_2 x)^2=2\Rightarrow \log_2 x=\pm\sqrt{2}$

$\Rightarrow x=x=2^{\sqrt{2}}$ or $x=2^{-\sqrt{2}}$

(c) $5^{\log_a x}+5\cdot x^{\log_a 5}=3$

$\Rightarrow 5^{\log_a x}+5\cdot 5^{\log_a 5}=3$

$\Rightarrow 6\cdot 5^{\log_a x}=3\Rightarrow 5^{\log_a x}=\dfrac{1}{2}$

Taking log to the base 5 on both sides.

$\Rightarrow \log_a x=\log_5\left(\dfrac{1}{2}\right)\Rightarrow x=a^{\log_5\left(\frac{1}{2}\right)}=\left(\dfrac{1}{2}\right)^{\log_5 a}$

$\Rightarrow x=2^{-\log_5 a}$

(d) $x^{\log_2 x+4}=32$

$\Rightarrow x^4\cdot x^{\log_2 x}=32$

Put $\log_2 x=t\Rightarrow x=2^t$

Now, equation (1) becomes

$(2^t)^4\cdot(2^t)^t=32$

$\Rightarrow 2^{t^2+4t}=2^5$ $\Rightarrow t^2+4t=5$

$\Rightarrow t^2+4t-5=0$ $\Rightarrow (t+5)(t-1)=0$

$\therefore t=-5,1$

Hence, $x=2^t\Rightarrow x=2^{-5},2^1$ or $x=1/32, 2$

**llustration 8** Solve : $x+\log_{10}(1+2^x)=x\cdot\log_{10}5+\log_{10}6$

**Solution** We have $\log_{10}(10^x)+\log_{10}(1+2^x)$

$=\log_{10}(5^x)+\log_{10}6$

$\Rightarrow \log_{10}((10^x)\cdot(1+2^x))=\log_{10}(6.5^x)$

$\Rightarrow 10^x(1+2^x)=6.5^x$

$\Rightarrow 2^x\cdot 5^x(1+2^x)=6.5^x$

$\Rightarrow 5^x((2^x)^2+2^x-6)=0\Rightarrow 5^x(2^x+3)(2^x-2)=0$

$\therefore 2^x=2 \Rightarrow x=1$.

**llustration 9 Solve :** $5\log_{10}x-3^{\log_{10}x-1}=3^{\log_{10}x+1}-5^{\log_{10}x-1}$

**Solution** We have $5^{\log_{10}x}\left(1+\dfrac{1}{5}\right)=3^{\log_{10}x}\cdot\left(3+\dfrac{1}{3}\right)$

$$\Rightarrow \quad \frac{6}{5}\cdot 5^{\log_{10} x} = \frac{10}{3}\cdot 3^{\log_{10} x} \Rightarrow \frac{1\cdot 5^{\log_{10} x}}{5^2} = \frac{1\cdot 3^{\log_{10} x}}{3^2}$$

$$\Rightarrow \quad 5^{(\log_{10} x - 2)} = 3^{(\log_{10} x - 2)}$$

Which is possible when $(\log_{10} x - 2) = 0$

$\Rightarrow \quad \log_{10} x = 2 \Rightarrow x = 100$

**llustration 10** Solve $\log_2\left(2\sqrt{17-2x}\right) = 1 + \log_{1/2}(x-1)$.

**Solution** $\log_2\left(2\sqrt{17-2x}\right) = 1 + \log_2(x-1)$

or $\quad \log_2\left(\dfrac{2\sqrt{17-2x}}{x-1}\right) = 1$

or $\quad \left(\dfrac{2\sqrt{17-2x}}{x-1}\right) = 2$

or $\quad 2\sqrt{17-2x} = 2(x-1)$

or $\quad x^2 - 2x + 1 = 17 - 2x$

or $\quad x^2 = 16$

$\Rightarrow \quad x = 4$ or $-4$ (rejected)

**llustration 11** If $\log_k x \log_5 k = \log_x 5, k \neq 1, k > 0$, then sum of all values of x is :

**Solution**

$\log_k x \log_5 k = \log_x 5, k \neq 1, k > 0$

$$\frac{\log x}{\log k}\times\frac{\log k}{\log 5} = \log_x 5 \Rightarrow \log_5 x = \log_x 5$$

$$(\log_5 x)^2 = \frac{1}{\log_5 x} \quad \Rightarrow (\log_5 x) = 1 \Rightarrow x = 5$$

**llustration 12** The product of all values of x satisfying the equation $|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$, is :

**Solution** $|x-1|^{\log_3 x^2 - 2\log_3 9} = (x-1)^7$

Since, L.H.S. > 0, x > 1, we have

$$(x-1)^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$$

$\Rightarrow x - 1 = 1$ or $\log_3 x^2 - 2\log_x 9 = 7$ or

$\Rightarrow x = 2$ or $2\log_3 x - 4\dfrac{1}{\log_3 x} - 7 = 0$

$\Rightarrow x = 2$ or $2(\log_3 x)^2 - 7\log_3 x - 4 = 0$

$\Rightarrow x = 2$ or $\log_3 x = -1/2, 4$

$\Rightarrow x = 2$ or $x = 3^{-1/2}, 3^4$

$\Rightarrow x = 2, 81$

Hence product of $2 \times 81 = 162$

**llustration 13** The number of values of x satisfying the equation,

$\log_2\left(9^{x-1}+7\right) = 2 + \log_2\left(3^{x-1}+1\right)$ is :

**Solution** $\log_2\left(9^{x-1}+7\right) = 2 + \log_2\left(3^{x-1}+1\right)$

$\log_2(9^{x-1}+7) = \log_2 4\,(3^{x-1}+1)$

$9^{x-1} + 7 = 4(3^{x-1}+1)$

$$\Rightarrow \frac{9^x}{9} + 7 = 4\left(\frac{3^x}{3}+1\right)$$

$$\Rightarrow \frac{(3^x)^2}{9} + 7 = \frac{4}{3}\cdot(3^x) + 4 \qquad \Rightarrow 3^x = t \text{ (say)}$$

$$\Rightarrow \frac{t^2}{9} + 7 = \frac{4t}{3} + 4 \qquad \Rightarrow \frac{t^2+63}{9} = \frac{4t+12}{3}$$

$\Rightarrow t^2 - 12t + 63 - 36 = 0 \qquad \Rightarrow t^2 - 12t + 27 = 0$

$\Rightarrow t^2 - 9t - 3t + 27 = 0 \qquad \Rightarrow t(t-9) - 3(t-9) = 0$

$\Rightarrow (t-9)(t-3) = 0 \qquad \Rightarrow 3^x = 9, 3$

$\therefore x = 2, 1$

$\therefore$ The number of values of $x = 2$

**llustration 14** The product of the roots of equation,

$x^{\log_{10} x} = \dfrac{10^4}{x^3}$, is :

**Solution** Taking logarithm to the base 10 on both the sides of given equation, we get

$(\log_{10}x)^2 = 4 - 3\log_{10}x$

$\Rightarrow \quad (\log_{10}x)^2 + 3\log_{10}x - 4 = 0$

$\Rightarrow \quad \log_{10}x_1 + \log_{10}x_2 = -3$

$\therefore \quad$ Product of roots $= \log_{10}(x_1x_2) = -3$

Hence $x_1x_2 = 10^{-3} = \dfrac{1}{1000}$

**llustration 15** Solve: $\log_{x^2} 16 + \log_{2x} 64 = 3$

**Solution** $\dfrac{\log_2 16}{2\log_2 x} + \dfrac{\log_2 64}{\log_2 2x} = 3$

$$\frac{2}{\log_2 x} + \frac{6}{1+\log_2 x} = 3$$

let $\log_2 x = y$

$2 + 8y = 3y(1+y) \Rightarrow 3y^2 - 5y - 2 = 0$

$3y^2 - 6y + y - 2 = 0$

$3y(y-2) + 1(y-2) = 0$

$\Rightarrow \quad \log_2 x = y = 2\ ;\ \dfrac{-1}{3}$

$\Rightarrow \quad x = 4\ ;\ x = 2^{-1/3}$ (which is irrational)

Hence two real solution

**llustration 16** The number of prime solutions of the equation $5^{x-1} + 5(0.2)^{x-2} = 26$ is/are

**Solution** $5^{x-1} + 5^{3-x} = 26$

Let $5^{x-1} = t$ ; $t + \frac{25}{t} = 26 \Rightarrow (t-1)(t-25) = 0$

$\therefore$ t = 1 or t = 25

$5^{x-1} = 1 \quad \Rightarrow$ x =1 is not prime

$5^{x-1} = 25 \quad \Rightarrow$ x = 3 is prime

**llustration 17** Number of values of x in the interval (0,5) satisfying the equation

$$\frac{\ell n\left(\sqrt{\sqrt{x^2+1}+x}\right)+\ell n\sqrt{\left(\sqrt{x^2+1}-x\right)}}{\ell nx} = x, a \quad \text{is}$$

**Solution** We have, $\frac{\ell n\left(\sqrt{x^2+1-x^2}\right)}{\ell nx} = x$

$$\Rightarrow \frac{\ell n1}{\ell nx} = x \Rightarrow 0 = x\,(\ell n\, x)$$

$\therefore$ x = 0 or x = 1 but both are unacceptable. So no solution exists.

**llustration 18** The absolute integral value of the solution of the equation $\sqrt{7^{2x^2-5x-6}} = \left(\sqrt{2}\right)^{3\log_2 49}$

**Solution** $\sqrt{7^{2x^2-5x-6}} = \left(\sqrt{2}\right)^{3\log_2 49}$

**L.H.S.**

$$(7)^{\frac{2x^2-5x-6}{2}}$$

**R.H.S.**

$$(2)^{\frac{3}{2}\log_2 49} \Rightarrow (2)^{\log_2 (49)^{3/2}}$$

$$\left(7^2\right)^{\frac{3}{2}} \Rightarrow 7^3$$

$$\frac{2x^2-5x-6}{2} = 3$$

$\Rightarrow \quad 2x^2 - 5x - 6 = 6$

$\Rightarrow \quad 2x^2 - 5x - 12 = 0$

$\Rightarrow \quad 2x^2 - 8x + 3x - 12 = 0$

$\quad (x-4)(2x+3) = 0$

$\therefore \quad x = 4, \frac{-3}{2}$

Hence absolute integral value of the solution 4.

**llustration 19** If $x_1$ and $x_2$ are the roots of the equation $e^2x^{\ln x} = x^3$ and with $x_1 > x_2$, then prove $x_2^2 = x_1$

**Solution** $e^2 . x^{\ln x} = x^3$

Taking log on both sides, we get

$\ln (e^2 . x^{\ln x}) = \ln (x^3)$

$\Rightarrow \quad (\ln x)^2 - 3 \ln x + 2 = 0$

$\Rightarrow \quad (\ln x - 2)(\ln x - 1) = 0$

If $\ln x = 2 \Rightarrow x = e^2$

If $\ln x = 1 \Rightarrow x = e$

Since $x_1 > x_2$, we get $x_1 = e^2$ and $x_2 = e$

$\Rightarrow \quad x_2^2 = x_1$

**llustration 20** The sum of all the roots of the equation $\log_2(x-1) + \log_2(x+2) - \log_2(3x-1) = \log_2 4$

**Solution** $\log_2 |x-1| + \log_2 1\, x+2) - \log_2 (3x-1) = \log_2 4$

$$\log_2 \frac{(x-1)(x+2)}{3x-1} = \log_2 4$$

$(x-1)(x+2) = (3x-1)4$

$x^2 + x - 2 = 12x - 4$

$x^2 - 11x + 6 = 0$

sum of all roots of of equation = 11

**llustration 21.**

Solve $\log_{(2x+3)}(6x^2 + 23x + 21) + \log_{(3x+7)}(4x^2 + 12x + 9) = 4$

**Solution** The given equation can be written as

$\log_{(2x+3)}(2x+3)(3x+7) + \log_{(3x+7)}(2x+3)^2 = 4$

or $\quad \log_{(2x+3)}(2x+3) + \log_{(2x+3)}(3x+7)$

$$+ \frac{2}{\log_{(2x+3)}(3x+7)} = 4$$

Let $\log_{(2x+3)}(3x+7) = t$. Then,

$$1 + t + \frac{2}{t} = 4$$

or $\quad t^2 - 3t + 2 = 0$

or $\quad (t-1)(t-2) = 0$

$\Rightarrow \quad t = 1, t = 2$

(i) If t = 1, then

$\log_{(2x+3)}(3x+7) = 1$

or $\quad 3x + 7 = 2x + 3$

Hence, x = –4, which is not possible as 2x + 3 > 0 and 3x + 7 > 0.

(ii) If t = 2, then

$\log_{(2x+3)}(3x+7) = 2$

or $\quad 3x + 7 = (2x+3)^2$

or $\quad 4x^2 + 9x + 2 = 0$

or $\quad (x+2)(4x+1) = 0$

$\Rightarrow \quad x = -2$ and $x = -\frac{1}{4}$

Since, x = –2 is not possible, there is only one solution, $x = -1/4$.

**llustration 22** Sum of all values of x satisfying the system of equation $5(\log_y x + \log_x y) = 26$, $xy = 64$ is :

**Solution** $5(\log_y x + \log_x y) = 26$

$\Rightarrow \log_y x + \log_x y = \frac{26}{5}$ $\quad\quad \Rightarrow \log_y x = t$

$\Rightarrow t + \frac{1}{t} = \frac{26}{5}$ $\quad\quad \Rightarrow \frac{t^2+1}{t} = \frac{26}{5}$

$\Rightarrow 5t^2 - 26t + 5 = 0$

$\Rightarrow 5t^2 - 25t + 5 = 0$

$\Rightarrow 5t(t-5) - 1(t-5) = 0$

$\Rightarrow t = 5, 1/5$

| When $\log_y x = 5$ | When $\log_y x = \frac{1}{5}$ |
|---|---|
| $x = y^5$ | $\Rightarrow x = (y)^{1/5}$ |
| Given, $xy = 64$ | $\Rightarrow x = (y)^{1/5}$ |
| $\Rightarrow y^5 \cdot y = 64 = 2^6$ | Given, $xy = 64$ |
| $\therefore y = 2$ | $x \cdot x^5 = 64$ |
| So, $x = 32$ | $\therefore x = 2$ |
| | $y = 32$ |

Finally, we set (x, y) = (32, 2), (2, 32)

Hence sum of the value of x = 34

**llustration 23** Prove that $\log_2 7$ is irrational.

**Solution** Let $\log_2 7 = \frac{p}{q}$ $\quad p, q \in N$

$7 = 2^{p/q}$ $\quad$ HCF = 1

$7^q = 2^p$

$7 \times 7 \times 7$ .... q times $= 2 \times 2 \times 2$ ..... P times

Odd = even not possible

So $\log_2 7 \neq \frac{p}{q}$ it is irrational.

## DPP 5

**Total Marks 28** **Time 45 Minute**

Question Number 1 to 14. **Marking Scheme** : +2 for correct answer 0 in all other cases.

1. Solve : $4^{\log_2 \log x} = \log x - (\log x)^2 + 1$ (base is e).
2. Solve $4\log_{x/2}(\sqrt{x}) + 2\log_{4x}(x^2) = 3\log_{2x}(x^3)$
3. Solve $4^{\log_9 x} - 6x^{\log_9 2} + 2^{\log_3 27} = 0$.
4. Solve $3\log_x 4 + 2\log_{4x} 4 + 3\log_{16x} 4 = 0$
5. Solve $(\log_3 x)(\log_5 9) - \log_x 25 + \log_3 2 = \log_3 54$
6. Solve $\left(x^{\log_{10} 3}\right)^2 - \left(3^{\log_{10} x}\right) - 2 = 0$
7. Solve $x^{\log_4 x} = 2^{3(\log_4 x + 3)}$
8. $x^2 + 7^{\log_7 x} - 2 = 0$
9. Solve : $\frac{\log_2(9 - 2^x)}{x - 2} = 1$
10. $(x+1)^{\log_{10}(x+1)} = 100(x+1)$
11. $\log_{x-1}(4) = 1 + \log_2(x-1)$
12. $3^{\log_3^2 x} + x^{\log_3 x} = 162$
13. $5^{1+(\log_4 x)} + 5^{(\log_{1/4} x)^{-1}} = \frac{26}{5}$
14. $\log_4(2\log_3(1 + \log_2(1 + 3\log_2 x))) = \frac{1}{2}$

**Result Analysis**

1. 22 to 28 Marks : **Advanced Level.**
2. 18 to 22 Marks : **Main Level**
3. < 18 Marks : **Below Averaege**
   **(Please go through this articles again)**

**LOGARITHMIC INEQUALITIES**

**1. Constant Base**

(i) $\log_a x > \log_a y \Leftrightarrow \begin{cases} x > y > 0, \text{if} & a > 1 \\ 0 < x < y, \text{if} & 0 < a < 1 \end{cases}$

(ii) $\log_a x < \log_a y \Leftrightarrow \begin{cases} 0 < x < y, \text{if} & a > 1 \\ x > y > 0, \text{if} & 0 < a < 1 \end{cases}$

(iii) $\log_a x > p \Leftrightarrow \begin{cases} x > a^p, \text{ if} & a > 1 \\ 0 < x < a^p, \text{ if} & 0 < a < 1 \end{cases}$

(iv) $\log_a x < p \Leftrightarrow \begin{cases} 0 < x < a^p, \text{ if} & a > 1 \\ x > a^p, \text{ if} & 0 < a < 1 \end{cases}$

**2. Variable Base**

(i) $\log_x a$ is defined, if $a > 0, x > 0, x \neq 1$.

(ii) If $a > 1$, then $\log_x a$ is monotonically decreasing in $(0,1) \cup (1,\infty)$

**VERY IMPORTANT CONCEPTS**

(i) If $a > 1, p > 1$, then $\log_a p > 0$

(ii) If $0 < a < 1, p > 1$, then $\log_a p < 0$

(iii) If $a > 1, 0 < p < 1$, then $\log_a p < 0$

(iv) If $p > a > 1$, then $\log_a p > 1$

(v) If $a > p > 1$, then $0 < \log_a p < 1$

(vi) If $0 < a < p < 1$, then $0 < \log_a p < 1$

(vii) If $0 < p < a < 1$, then $\log_a p > 1$

**llustration 1** Solve $\log_2 (x - 1) > 4$.

**Solution** $\log_2(x - 1) > 4$ or $x - 1 > 2^4$ or $x > 17$

**llustration 2** Solve $\log_3 (x - 2) \le 2$

**Solution** $\log_3 (x - 2)$ or $0 < x - 2 \le 3^2$ or $2 < x \le 11$

**llustration 3** Solve $\log_{0.3} (x^2 - x + 1) > 0$.

**Solution** $\log_{0.3} (x^2 - x + 1) > 0$ or $0 < x^2 - x + 1 < (0.3)^0$

or $(x^2 - x + 1) > 0$ or $0 < x^2 - x + 1 < 1$

and $x^2 - x < 0$ or $x(x - 1) < 0$

$\Rightarrow 0 < x < 1$ (as $x^2 - x + 1) = (x - 1/2)^2\ 3/4 > 0$,

for all real x).

**llustration 4** Solve $1 < \log_2 (x - 2) \le 2$.

**Solution** $1 < \log_2(x - 2) \le 2 \Rightarrow 2^1 < x - 2 \le 2^2$

$\Rightarrow \quad 4 < x < 6$

**llustration 5** Solve $\log_2 \dfrac{x-1}{x-2} > 0$.

**Solution** $\dfrac{x-1}{x-2} > 2^o$ (Automatically greater then 0)

or $\dfrac{x-1}{x-2} > 1$ or $\dfrac{x-1}{x-2} - 1 > 0$

or $\dfrac{x-1-x+2}{x-2} > 0$ or $\dfrac{1}{x-2} > 0$ or $x > 2$

**llustration 6** Solve $\log_{0.5} \dfrac{3-x}{x+2} < 0$.

**Solution** $\log_{0.5} \dfrac{3-x}{x+2} < 0$ or $\dfrac{3-x}{x+2} > (0.5)^0$

or $\dfrac{3-x}{x+2} > 1$ (Automatically greater then 0)

or $\dfrac{3-x}{x+2} - 1 > 0$ or $\dfrac{3-x-x-2}{x+2} > 0$

or $\dfrac{2x-1}{x+2} < 0$ or $-2 < x < 1/2$

**llustration 7** Solve $\log_3(2x^2 + 6x - 5) > 1$.

**Solution** $\log_3(2x^2 + 6x - 5) > 1$ or $2x^2 + 6x - 5 > 3^1$

or $2x^2 + 6x - 8 > 0$ or $x^2 + 3x - 4 > 0$

or $(x - 1)(x + 4) > 0 \Leftrightarrow x < -4$ or $x > 1$

**llustration 8** Solve $\log_{0.04}(x - 1) \ge \log_{0.2} (x - 1)$.

**Solution** $\log_{0.04} (x - 1) \ge \log_{0.2} (x - 1)$

or $\log_{(0.2)^2} (x-1) \ge \log_{0.2} (x-1)$

or $\dfrac{1}{2}\log_{0.2} (x-1) \ge \log_{0.2} (x-1)$

or $\log_{0.2} (x - 1) \ge 2 \log_{0.2} (x-1)$

or $\log_{0.2} (x - 1) \ge \log_{0.2} (x - 1)^2$

or $(x - 1) \le (x - 1)^2$

or $(x - 1)^2 - (x - 1) \ge 0$

or $(x - 1)(x - 1 - 1) \ge 0$

or $(x - 1)(x - 2) \ge 0$

or $x \le 1$ or $x \ge 2$, but $x > 1$ (for log to be defined)

So, $x \ge 2$

**llustration 9** Arrange in ascending order, $\log_2(x)$, $\log_3(x)$, $\log_{10}(x)$, $\log_e (x)$ if

(i) $x > 1$ (ii) $0 < x < 1$

**Solution** (i) For $x > 1$,

$\log_x 2 < \log_x e < \log_x 3 < \log_x 10$

$\Rightarrow \dfrac{1}{\log_2(x)} < \dfrac{1}{\log_e(x)} < \dfrac{1}{\log_3(x)} < \dfrac{1}{\log_{10}(x)}$

$\Rightarrow \log_2(x) > \log_e(x) > \log_3(x) > \log_{10}(x)$

(Inequality reverse when reciprocal of +ve no. taken on both sides)

Hence, ascending order is

$\log_{10}(x) < \log_3(x) < \log_e (x) < \log_2(x)$

(ii) For $0 < x < 1$,

$\log_x 2 > \log_x e > \log_x 3 > \log_x 10$

$\Rightarrow \dfrac{1}{\log_2(x)} > \dfrac{1}{\log_e(x)} > \dfrac{1}{\log_3(x)} > \dfrac{1}{\log_{10}(x)}$

$\Rightarrow \log_2(x) < \log_e(x) < \log_3 (x) < \log_{10} (x)$

which is in ascending order.

**llustration 10** If $\log 11 = 1.0414$, then prove that $10^{11} > 11^{10}$.

**Solution** $\log 10^{11} = 11 \log 10 = 11$

and $\log 11^{10} = 10 \log 11 = 10 \times 1.0414 = 10.414$

It is clear that,

$11 > 10.414$

$\Rightarrow \log 10^{11} > \log 11^{10}$

$\Rightarrow 10^{11} > 11^{10}$ [$\because$ here, base = 10]

**llustration 11** If $\log_2(x - 2) < \log_4 (x - 2)$, then find the interval in which x lies.

**Solution** Here, $x - 2 > 0$

and $\log_2(x-2) < \log_{2^2}(x-2) = \frac{1}{2}\log_2(x-2)$

$\Rightarrow \log_2(x-2) < \frac{1}{2}\log_2(x-2)$

$\Rightarrow \log_2(x-2) < 0$

$\Rightarrow x-2 < 2^o$

$\Rightarrow x < 3$

From Eqs. (i) and (ii), we get

$2 < x < 3$ or $x \in (2, 3)$

**llustration 12** Prove that $\log_n(n+1) > \log_{(n+1)}(n+2)$ for any natural number $n > 1$.

**Solution** Since, $\frac{n+1}{n} = 1 + \frac{1}{n} > 1 + \frac{1}{n+1}$

$\frac{n+1}{n} > \frac{n+2}{n+1}$

For $n > 1$,

$\log_n\left(\frac{n+1}{n}\right) > \log_{n+1}\left(\frac{n+1}{n}\right) > \log_{n+1}\left(\frac{n+2}{n+1}\right)$

$\Rightarrow \log_n(n+1) - \log_n n > \log_{(n+1)}(n+2) - \log_{(n+1)}(n+1)$

$\Rightarrow \log_n(n+1) - 1 > \log_{(n+1)}(n+2) - 1$

$\Rightarrow \log_n(n+1) > \log_{(n+1)}(n+2)$

**llustration 13** The true solution set of inequality $\log_{(2x-3)}(3x-4) > 0$ is equal to :

(A) $\left(\frac{4}{3}, \frac{5}{3}\right) \cup (2, \infty)$ (B) $\left(\frac{3}{2}, \frac{5}{3}\right) \cup (2, \infty)$

(C) $\left(\frac{4}{3}, \frac{3}{2}\right) \cup (2, \infty)$ (D) $\left(\frac{2}{3}, \frac{4}{3}\right) \cup (2, \infty)$

**Solution (B)**

$\log_{(2x-3)}(3x-4) > 0$

**Case I** When $2x-3 > 1 \quad \Rightarrow \quad x > 2$

$(3x-4) > (2x-3)^0$

$3x-4 > 1$

$x > 5/3 \quad \Rightarrow \quad x > 2$

**Case II** When $0 < 2x-3 < 1$

$3/2 < x < 2$

$0 < (3x-4) < 1$

$4/3 < x < 5/3$

$\Rightarrow x \in (3/2, 5/3)$

union of the cases is $\left(\frac{3}{2}, \frac{5}{3}\right) \cup (2, \infty)$

**llustration 14** Solve $\log_{x+\frac{1}{x}}\left(\log_2 \frac{x-1}{x+2}\right) > 0$.

**Solution** $\log_{x+\frac{1}{x}}\left(\log_2 \frac{x-1}{x+2}\right) > 0 \quad [x + \frac{1}{x} \geq 2 \text{ always}]$

$\Rightarrow \log_2 \frac{x-1}{x-2} > 1 \quad$ [x should be +ve]

or $\frac{x-1}{x+2} > 2 \quad$ or $\frac{x+5}{x+2} < 0$

Hence, $x \in (-5, -2)$, which is not possible as $x > 1$.

No solution

**llustration 15** Solve $\log_{\log_2\left(\frac{x}{2}\right)}\left(x^2 - 10x + 22\right) > 0$.

**Solution** $\log_{\log_2\left(\frac{x}{2}\right)}\left(x^2 - 10x + 22\right) > 0$

We must have

**(i)** $x^2 - 10x + 22 > 0$

$\Rightarrow x \in (-\infty, 5-\sqrt{3}) \cup (5+\sqrt{3}, \infty)$

**(ii)** $\frac{x}{2} > 0 \quad \Rightarrow x > 0$

**Case (i) :** $0 < \log_2\left(\frac{x}{2}\right) < 1$

$\Rightarrow 1 < \frac{x}{2} < 2$

or $2 < x < 4$

Therefore, from Eq. (i), $x^2 - 10x + 22 < 1$

or $x^2 - 10x + 21 < 0$

$\Rightarrow 3 < x < 7$

From Eqs. (ii), (iii), (iv) and (v), the common solution is

$3 < x < 5 - \sqrt{3}$

**Case (ii) :** $\log_2\left(\frac{x}{2}\right) > 1$ or $\frac{x}{2} > 2$ or $x > 4$

Therefore, from Eqs. (i), $x^2 - 10x + 22 > 1$

or $x^2 - 10x + 21 > 0$

or $x < 3$ or $x > 7$

From Eqs. (ii), (iii), (vi) and (vii), the common solutions is $x \in (7, \infty)$.

Hence, $x \in \left(3, 5-\sqrt{3}\right) \cup (7, \infty)$.

## DPP 6

**Total Marks 18** **Time 25 Minute**

Question Number 1 to 9. **Marking Scheme** : +2 for correct answer 0 in all other cases.

**1.** If $\log_{0.16}(a+1) < \log_{0.4}(a+1)$, then a satisfies

(A) $a > 0$ (B) $0 < a < 1$

(C) $-1 < a < 0$ (D) None of these

2. The value of x satisfying the inequation,

$x^{\frac{1}{\log_{10} x}} \cdot \log_{10} x < 1$, is

(A) $0 < x < 10$ (B) $0 < x < 10^{10}$

(C) $0 < x < 10^{1/10}$ (D) None of these

3. The value of $\log_{10}3$, lies in the interval.

(A) $\left(\frac{2}{5}, \frac{1}{2}\right)$ (B) $\left(0, \frac{1}{2}\right)$

(D) $\left(0, \frac{2}{5}\right)$ (D) None of these

4. Solve $\log_2 \frac{x-4}{2x+5} < 1$

5. Solve $\log_{10}(x^2 - 2x - 2) \le 0$.

6. Let $f(x) = \sqrt{\log_{10} x^2}$. Find the set of all values of x for which f(x) is real.

7. Solve $\log_{1/2}(x^2 - 6x + 12) \ge -2$.

8. Solve $\log_3 (x+2)(x+4) + \log_{1/3} (x+2) \le \frac{1}{2}\log_{\sqrt{3}} 7$.

9. Solve $\log_x (x^2 - 1) \le 0$.

**Result Analysis**

1. 14 to 18 Marks : **Advanced Level.**
2. 10 to 14 Marks : **Main Level**
3. < 10 Marks : **Below Averaege**
   **(Please go through this articles again)**

## FINDING LOGARITHM

To calculate the logarithm of any positive nummber in decimal form, we always express the given positive number in the decimal form as the product of an integral power of 10 and a number between 1 and 10, i.e.,

$K = m \times 10^p$

where p is an integer and $1 \le m < 10$. This is called the standard form of k.

## CHARACTERISTIC AND MANTISSA

The integral part of a logarithm is called the **characteristic** and the fractional pat (decimal part) is called **mantissa.**

i.e. log N = Integer + Fractional or decimal part (+ve)

↓ ↓

(p as above) Characteristic Mantissa (log of m as above)

The mantissa of log of a number is always kept non negative.

i.e., if log 564 = 2.751279, then 2 is the characteristic and 0.751279 is the mantissa of the given number 564.

And if log 0.00895 = –2.481769

$= -2 - 0.0481769$

$= (-2 - 1) + (1 - 0.0481769)$

Hence, –3 is the characteristic and 0.9518231 (not 0.0481769) is mantissa of log 0.00895.

In short, –3 + 0.9518231 is written as $\bar{3}.9518231$.

**Points to Consider**

1. If N > 1, then the characteristic of log N will be one less than the number of digits in the integral part of N.

   For example, If log 235.68 = 2.3723227

   Here, N = 235.68

   ∴ Number of digits in the integral part of N = 3

   ⇒ Characteristic of log 235.68 = N – 1 = 3 – 1 = 2

2. If 0 < N <1, the characteristic of log N is negative and numerically it is one greater than the number of zeroes immediately after the decimal part in N.

   For example, If log 0.0000279 = $\bar{5}.4456042$

   Here, four zeroes immediately after the decimal point in the number 0.0000279 is (4 + 1), i.e. $\bar{5}$.

3. If the characteristics of log N be n, then number of digits in N is (n + 1) (Here, N > 1).

4. If the characteristics of log N be – n, then there exists (n –1) number of zeroes after decimal part of N (Here, 0 < N < 1).

**Illustration 1** Write the characteristic of each of the following numbers by using their standard forms :

(i) 346.41 (ii) 62.723

(iii) 7.12345 (iv) 0.35792

(v) 0.034239 (vi) 0.0009468

**Solution**

| Number | Standard form | Characteristic |
|---|---|---|
| 346.41 | $3.4641 \times 10^2$ | 2 |
| 62.723 | $6.2723 \times 10^1$ | 1 |
| 7.12345 | $7.12345 \times 10^0$ | 0 |
| 0.35792 | $3.5792 \times 10^{-1}$ | –1 |
| 0.034239 | $3.4239 \times 10^{-2}$ | –2 |
| 0.0009468 | $9.468 \times 10^{-4}$ | –4 |

Mantissa is log of part other than exponential form

**FOR EXAMPLE**

Mantissa of number 346.41 is $\log_{10}(3.4641)$

Mantissa of number 62.723 is $\log_{10}(6.2723)$

Mantissa of number 0.0009468 is $\log_{10}(9.468)$

**Notice that all these numbers (Mantissa) ∈ [0, 1)**

### Significant Digits

The digits to compute the mantissa of a given number are called its significant digits.

### Negative Characteristics

When a characteristic is negative, such as –2, we do not perform the subtraction, because this would involve a negative mantissa. There are several ways to indicating a negative characteristic. Mantissas as presented in the table in the appendix are always positive, and the sign of the characteristic is indicated separately. For example, consider $\log 0.023 = \bar{2}.36173$. Here the bar over 2 indicates that only the characteristic is negative, i.e., the logarithm is $-2 + 0.36173$.

## ANTILOGARITHM

The positive number n is called the antilogarithm of a number m if log n = m. If n is antilogarithm of m, we write n = antilog m. For example.

(i) $\log 100 = 2 \Leftrightarrow \text{antilog } 2 = 100$

(ii) $\log 431.5 = 2.6350 \Leftrightarrow \text{antilog }(2.6350) = 431.5$

(iii) $\log 0.1257 = 1.993 \Leftrightarrow \text{antilog }(1.993) = 0.1257$

**Illustration 1** Evaluate $\sqrt[3]{72.3}$, if $\log 0.723 = \bar{1}.8591$

**Solution** Let $x = \sqrt[3]{72.3}$. Then,

$$\log x = \log(72.3)^{1/3} \text{ or } \log x = \frac{1}{3}\log 72.3$$

$$\text{or} \quad \log x = \frac{1}{3} \times 1.8591 \Rightarrow \log x = 0.6197$$

$10^{.6197}$

or $x = \text{antilog}(0.6197)$

**llustration 2** If log 2 = 0.301 and log 3 = 0.477, then find the number of digits in $6^{20}$.

**Solution** Let $P = 6^{20} = (2 \times 3)^{20}$

$\Rightarrow \log P = 20 \log(2 \times 3) = 20\{\log 2 + \log 3\}$

$= 20\{0.301 + 0.477\} = 20 \times 0.778 = 15.560$

Since, the characteristics of log P is 15, therefore the number of digits in P will be 15 + 1, i.e. 16.

**llustration 3** Find the number of zeroes between the decimal point and first significant digit $(0.036)^{16}$, where log 2 = 0.301 and log 3 = 0.477.

**Solution** Let $P = (0.036)^{16}$

$$\Rightarrow \log P = 16\log(0.036) = 16\log\left(\frac{36}{1000}\right)$$

$$= 16\log\left(\frac{2^2 \cdot 3^2}{1000}\right)$$

$$= 16\{\log 2^2 + \log 3^2 - \log 10^3\}$$

$$= 16\{2\log 2 + 2\log 3 - 3\}$$

$$= 16\{2 \times 0.301 + 2 \times 0.477 - 3\}$$

$$= 16\{1.556 - 3\} = 24.896 - 48$$

$$= -48 + 24 + 0.896$$

$$= -24 + 0.896 = \overline{24} + 0.896$$

$\therefore$ The requires number of zeroes = 24 – 1 = 23.

**llustration 4** Let $x = (0.15)^{20}$. Find the characteristic and mantissa of the logarithm of x to the base 10.

Assume $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$.

**Solution** $\log x = \log(0.15)^{20} = 20\log\left(\frac{15}{100}\right)$

$= 20[\log 15 - 2]$ $\quad [\because \log_{10} 5 = \log_{10} 10/2]$

$= 20[\log 3 + \log 5 - 2]$

$= 20[\log 3 + 1 - \log 2 - 2]$

$= 20[-1 + \log 3 - \log 2]$

$= 20[-1 + 0.477 - 0.301]$

$= -20 \times 0.824 = -16.48 = \overline{17}.52$.

Hence, Characteristic = –17 and Mantissa = 0.52

**llustration 5** If P is the number of natural numbers whose logarithm to the base 10 have the characteristics p and Q is the number of natural numbers logarithm of whose reciprocals to the base 10 have the characteristics –q then $\log_{10}P - \log_{10}C$ has the value equal to :

**Solution** $10^p \leq$ number of natural numbers whose logarithm to the base 10 have the characteristics $p < 10^{p+1}$

[very simple]

$\Rightarrow$ Number of natural numbers whose logarithm to the base 10 have the characteristic $p \in \{10^p, 10^p + 1, 10^p + 2, \ldots, 10^{p+1} - 1\}$

$P = (10^{p+1} - 1) - (10^p) + 1$ (last – first + 1)

$\Rightarrow P = 9 \times 10^P$

Similarly, $10^{q-1} <$ the number of natural numbers logarithm of whose reciprocals to the base 10 have the characteristics $-q \leq 10^q$ [very simple]

$\Rightarrow$ The number of natural numbers logarithm of whose reciprocals to the base 10 have the

characteristic $-q \in \{10^{q-1} + 1, 10^{q-1} + 2, \ldots, 10^q\}$

$Q = 10^q - (10^{q-1} + 1) + 1$ (last – first + 1)

$= 10^{q-1}(10 - 1) = 9 \times 10^{q-1}$

$\therefore \log_{10}P - \log_{10}Q = \log_{10}(P/Q) = \log_{10} 10^{p-q+1}$

$= p - q + 1$

**llustration 6** If $2^{2010} = a_n 10^n + a_{n-1} 10^{n-1} + ..... + a_2 10^2 + a_1 \cdot 10 + a_0$, where $a_i \in \{0,1,2,...,9\}$ for all i = 0,1,2,3,....,n, find n.

Given $\log_{10} 2 = 0.301$,

**Solution** log of L.H.S.

$\log_{10} 2^{2010}$

$= 2010 \times \log_{10} 2 = 2010 \times .301$

$= 605.01$

Characteristic is 605

Number of digits of the number on L.H.S. is 606

R.H.S. is clearly a n + 1 digit number.

So, n + 1 = 606, n = 605

## ILLUSTRATIONS

**llustration 1** Let $a = \log_{81}(3)$, $b = \log_7(9)$ and $\log_9\left(\frac{\sqrt{3}}{7}\right) = c$, then the value of (ab – bc) equals.

**Solution** Given

$$a = \frac{1}{4}; b = \log_7 9$$

$$c = \log_9 \sqrt{3} - \log_9 7 = \frac{1}{4} - \frac{1}{b} = a - \frac{1}{b}$$

$$\Rightarrow \frac{ab-1}{b} = c \qquad \therefore ab - bc = 1$$

**llustration 2** Let a denotes the logarithm of $0.\overline{3}$ to the base $0.\overline{1}$, b denotes the logarithm of 243 to the base 81 and c denotes the number whose logarithm to the base 0.64 is minus $\frac{1}{2}$. Then the value of $\frac{c}{ab}$, is :

**Solution** $a = \log_{0.\overline{1}}\left(0.\overline{3}\right) = \log_{\frac{1}{9}}\left(\frac{1}{3}\right) = \frac{1}{2}$

$$b = \log_{81} 243 = \log_{3^4}(3)^5 = \frac{5}{4}$$

Let $\log_{0.64} N = \frac{-1}{2}$

$$\Rightarrow N = (0.64)^{\frac{-1}{2}} = \left(\frac{8}{10}\right)^{-1} = \frac{5}{4}$$

$$\therefore c = \frac{5}{4} \qquad \text{Hence, } \frac{c}{ab} = \frac{\left(\frac{5}{4}\right)}{\left(\frac{1}{2}\right)\left(\frac{5}{4}\right)} = 2$$

**llustration 3** The number of value (s) of x satisfying the equation

$$4^{\log_2(\ell n x)} - 1 + \ell n^3 x - 3\ell n^2 x - 5\ell n x + 7 = 0 \cdot$$

**Solution**

$(\ell n x)^2 - 1 + (\ell n x - 1)(\ell n^2 x - 2\ell n x - 7) = 0$

$(\ell n x - 1)[(\ell n x + 1) + (\ell n^2 - 2\ell n x - 7)] = 0$

$\therefore \ell n x = 1 ; \ell n x = -2 ; \ell n x = 3$

But $\ell n x = 1$ and $\ell n x = 3$ are acceptable only

$\therefore x = e$ and $x = e^3$.

Number of values of f(x) satisfying is 2.

**llustration 4** If $60^a = 3$ and $60^b = 5$, then $12^{\frac{(1-a-b)}{2(1-b)}}$ is equal to :

**Solution** $(60)^a\ 3 \Rightarrow a = \frac{\log 3}{\log 60}; (60)^b = 5 \Rightarrow b = \frac{\log 5}{\log 60}$

$$\text{Now, } \frac{1-a-b}{2(1-b)} = \frac{\left(1 - \frac{\log 3}{\log 60} - \frac{\log 5}{\log 60}\right)}{2\left(1 - \frac{\log 5}{\log 60}\right)}$$

$$= \frac{(\log 60 - \log 15)}{2(\log 60 - \log 5)} = \frac{\log 4}{2\log 12} = \log_{12}(2)$$

$$\therefore 12^{\frac{(1-a-b)}{2(a-b)}} = 12^{\log_{12}(2)} = 2$$

**llustration 5** If $3 \le a \le 2015$, $3 \le b \le 2015$ such that $\log_a b + 6 \log_b a = 5$, then the number of ordered pairs (a,b) of integers, is :

**Solution** Let $x = \log_a b$

$$\Rightarrow x + \frac{6}{x} = 5 \qquad \Rightarrow \qquad x^2 - 5x + 6 = 0$$

$\Rightarrow x = 2, 3$

From Eq. (i), we get $\log_a b = 2,3$

$\Rightarrow b = a^2$ or $a^3$

The pairs (a,b) are

$(3,3^2), (4.4^2), (5,5^2), (6,6^2),....(44,44^2)$ and $(3,3^3), (4,4^3), (5,5^3), .... (12,12^3)$.

There are 42 + 10 = 52 pairs.

**llustration 6** The length of the sides of a triangle are $\log_{10}12$, $\log_{10}75$ and $\log_{10}n$, where $n \in N$. If a and b are the least and greatest values of n respectively, the value of b – a is divisible by :

(A) 8 (B) 16 (C) 32 (D) 223

**Solution** $\log_{10}12 + \log_{10}15 > \log_{10} n$

$\Rightarrow n\ n < 12 \times 75 = 900$

$\therefore n < 900$ ....(i)

and $\log_{10} 12 + \log_{10} n > \log_{10} 75 \Rightarrow n > \frac{75}{12} = \frac{25}{4}$

$\therefore n > \frac{25}{4}$ ....(ii)

From Eqs. (i) and (ii), we get $\frac{25}{4} < n < 900$

$\therefore \quad n = 7, 8, 9, 10, ...899$

Hence, $\quad a = 7, b = 899$

$\therefore \quad b - a = 892 = 4 \times 223$

Hence, b – a is divisible by 223.

**llustration 7** If $5\log_{abc}(a^3+b^3+c^3) = 3\lambda\left(\frac{1+\log_3(abc)}{\log_3(abc)}\right)$ and

$(abc)^{a+b+c} = 1$ and $\lambda = \frac{m}{n}$, whose m and n are relative primes, then the value of |m + n| + |m – n|, is :

**Solution** $\because \; (abc)^{a+b+c} = 1$

$\therefore \; a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

Now, LHS $= 5\log_{abc}(a^3 + b^3 + c^3) = 5\log_{abc}(3abc)$

and RHS $3\lambda\left(\frac{1+\log_3(abc)}{\log_3(abc)}\right) = 3\lambda\left(\frac{\log_3(3abc)}{\log_3(abc)}\right)$

$= 3\lambda \log_{abc}(3abc)$

From Eqs. (i) and (ii), we get

$5\log_{abc}(3abc) = 3\lambda \log_{abc}(3abc)$

$\therefore \quad \lambda = \frac{5}{3} = \frac{m}{n}$ [given]

$\Rightarrow \quad m = 5, n = 3$

Hence, $|m + n| + |m - n| = 8 + 2 = 10$

**llustration 8** If $\log_6 a + \log_6 b + \log_6 c = 6$, where a ,b, c $\in$ N and a, b,c are in GP and b – a is a square of an integer, then the value of a + b – c, is :

**Solution** $\because \; \log_6 a + \log_6 b + \log_6 c = 6$

$\Rightarrow \; \log_6(abc) = 6 \Rightarrow abc = 6^6$

$\Rightarrow \quad b^3 = 6^6 \quad [\because b^2 = ac]$

Also, b – a = 36 – a is a square for a = 35,32,27, 20,11

Now, $c = \frac{b^2}{a} = \frac{36^2}{a}$ is an integer for a = 27.

$a = 27, b = 36, c = 48$

Hence, $\quad a + b - c = 27 + 36 - 48 = 15$

**llustration 9** If

$x = \log_{2a}\left(\frac{bcd}{2}\right), y = \log_{3b}\left(\frac{acd}{3}\right), z = \log_{4c}\left(\frac{abd}{4}\right)$

and $w = \log_{5d}\left(\frac{abc}{5}\right)$ and $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} + \frac{1}{w+1}$

$= \log_{abcd} N + 1$, then the value of N, is

**Solution** $\because \; x = \log_{2a}\left(\frac{bcd}{2}\right)$

$\Rightarrow \; x + 1 = \log_{2a}\left(\frac{2abcd}{2}\right) = \log_{2a}(abcd)$

$\therefore \; \frac{1}{x+1} = \log_{abcd} 2a$

Similarly, $\frac{1}{y+1} = \log_{abcd} 3b, \frac{1}{z+1} = \log_{abcd} 4c$ and

$\frac{1}{w+1} = \log_{abcd} 5d$

$\therefore \; \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} + \frac{1}{w+1} = \log_{abcd}(2a \cdot 3b \cdot 4c \cdot 5d)$

$= \log_{abcd}(120\,abcd)$

$= \log_{abcd} 120 + 1 = \log_{abcd} N + 1$ [given]

Hence, $\quad N = 120$

**llustration 10** Solve $\log_2(4 \times 3^x - 6) - \log_2(9^x - 6) = 1$

**Solution** $\log_2(4 \times 3^x - 6) - \log_2(9^x - 6) = 1$

or $\quad \log_2 \frac{4 \times 3^x - 6}{9^x - 6} = 1$

or $\quad \frac{4 \times 3^x - 6}{9^x - 6} = 2 \quad$ or $\quad 4y - 6 = 2y^2 - 12$

or $\quad y^2 - 2y - 3 = 0 \quad$ or $\quad y = -1, 3$

or $\quad 3^x = 3 \quad$ or $\quad x = 1$

**llustration 11**

Solve $4\log_{x/2}\left(\sqrt{x}\right) + 2\log_{4x}\left(x^2\right) = 3\log_{2x}\left(x^3\right)$.

**Solution** $\frac{4\log_2\sqrt{x}}{\log_2(x/2)} + \frac{2\log_2(x^2)}{\log_2(4x)} = \frac{3\log_2(x^3)}{\log_2(2x)}$

$\Rightarrow \quad \frac{4 \times \frac{1}{2}\log_2(x)}{\log_2 x - 1} + \frac{4\log_2(x)}{2 + \log_2(x)} = \frac{9\log_2(x)}{1 + \log_2(x)}$

Let $\log_2 x = t$. The given equation reduces to

$$\frac{2t}{t-1} + \frac{4t}{t+2} = \frac{9t}{t+1}$$

or $\quad t = 0$ or $\frac{2t}{t-1} + \frac{4t}{t+2} = \frac{9t}{t+1}$

or $\dfrac{2t+4+4t-4}{(t-1)(t+2)} = \dfrac{9}{t+1}$

or $t^2+1-6=0$ or $(t+3)(t-2)=0$

or $t=0$, or $-3$ $\Rightarrow$ $x=1, 4, 1/8$

**llustration 12** Solve $\log_{(x+3)}(x^2-x) < 1$.

**Solution** $\log_{x+3}(x^2-x) < 1$

$x(x-1) > 0 \Rightarrow x > 1$ or $x < 0$

**Case-I**

If $x+3 > 1 \Rightarrow x > -2$

then $x^2 - x < x + 3$

or $x^2 - 2x - 3 < 0$

or $(x-3)(x+1) < 0$

Hence, $x \in (-1, 0) \cup (1, 3)$

**Case-II**

If $0 < x+3 < 1$, $-3 < x < -2$,

then $x^2 - x > x + 3$

or $x^2 - 2x - 3 > 0$

or $(x-3)(x+1) > 0$

$\Rightarrow$ $x \in (-3, -2)$

$x \in (-3, -2) \cup (-1, 0) \cup (1, 3)$ **Ans.**

**Ilustration 13** Given that log(2) = 0.30103 the number of digits in the number $2000^{2000}$ is

(A) 6601 (B) 6602

(C) 6603 (D) 6604

**Solution** Let $= 2000^{2000}$

$\log x = 2000 \log_{10}(2000)$

$= 2000 (\log_{10} 2 + 3)$

$= 2000 (3.30103) = 6602.06$

Therefore, the number of digits is 6603.

# EXERCISES

## EXERCISE –1

**[Single Correct Answer Type]**

**1.** If $\log_2 x + \log_2 y \geq 6$, then the least value of x + y is
(A) 4 (B) 8 (C) 16 (D) 32

**2.** If $x = \log_5(1000)$ and $y = \log_7(2058)$, then
(A) x > y (B) x < y (C) x = y (D) none of these

**3.** If $A = \log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2$, then A is equal to
(A) 2 (B) 3 (C) 5 (D) 7

**4.** The number $\log_2 7$ is
(A) an integer (B) a rational number
(C) an irrational number (D) a prime number

**5.** $\log_7 \log_7 \sqrt{7\sqrt{(7\sqrt{7})}}$ is equal to
(A) $3 \log_2 7$ (B) $3 \log_7 2$
(C) $1 - 3 \log_7 2$ (D) $1 - 3 \log_2 7$

**6.** If $y = a^{\frac{1}{1-\log_a x}}$ and $z = a^{\frac{1}{1-\log_a y}}$, then x is equal to
(A) $a^{\frac{1}{1+\log_a z}}$ (B) $a^{\frac{1}{2+\log_a z}}$
(C) $a^{\frac{1}{1-\log_a z}}$ (D) $a^{\frac{1}{2-\log_a z}}$

**7.** If $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x$, then x be
(A) 2 (B) 3 (C) $\pi$ (D) none of these

**8.** Let $N = 10^{3\log 2 - 2\log\left(\log 10^3\right) + \log\left(\left(\log 10^6\right)^2\right)}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to :
(A) 2 (B) 3
(C) 4 (D) 5

**9.** The true solution set of the inequality $\log_7\left(\frac{2x-6}{2x-1}\right) > 0$, is :
(A) $\left(-\infty, \frac{1}{2}\right)$ (B) $(4, \infty)$
(C) $\left(-\infty, \frac{1}{3}\right)$ (D) $(7, \infty)$

**10.** Let $\ell = 5^{\frac{1}{\log_7 5}}$ and $m = \frac{1}{\sqrt{-\log_{10}(0.1)}}$ then the value of $\log_2(\ell + m)$ equals
(A) 1 (B) 2
(C) 3 (D) 8

**11.** The value of x satisfying the equation $\log_{\sqrt{8}}(x) = 3\frac{1}{3}$ is equal to :
(A) 16 (B) 24
(C) 32 (D) 64

**12.** The value of x satisfying the equation, $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ is equal to :
(A) 9 (B) 4
(C) 10 (D) 100

**13.** If $x = \alpha$ satisfies the equation $x^{\log_x (x-2)^2} = 16$, then $\alpha$ is :
(A) even composite
(B) odd composite
(C) rational but not integer
(D) irrational

**14.** The value of a for which $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$, is :
(A) 9 (B) 12
(C) 18 (D) 24

**15.** Suppose that x is real number such that $\frac{(27)(9^x)}{4^x} = \frac{3^x}{8^x}$, then the value of $(2)^{-(1+\log_2 3)x}$, is :
(A) 6 (B) 9
(C) 18 (D) 27

**16.** If $4^{\log_9 3} + 7^{\log_2 4} = 10^{\log_x 51}$, then x equals.
(A) 2 (B) 3
(C) 5 (D) 10

**17.** The number $\log_2 3$ is :
(A) an integer (B) prime
(C) rational (D) irrational

**18.** If $\log_{105}7 = a$, $\log_7 5 = b$ then $\log_{35} 105$ is equal to :

(A) ab (B) $(b+1)a$

(C) $\frac{1}{ab}$ (D) $\frac{1}{a(b+1)}$

**19.** The number of solution of $\log_2 3 . \log_x 2 = \log_3 x$ is :

(A) 0 (B) 1

(C) 2 (D) 3

**20.** If $P = \log_5(\log_5 3)$ and $3^{C+5^{-P}} = 405$ then C is equal to :

(A) 3 (B) 4

(C) 81 (D) 5

**21.** If $\log_4 5 = a$ and $\log_5 6 = b$, then $\log_3 2$ is equal to :

(A) $\frac{1}{2a+1}$ (B) $\frac{1}{2b+1}$

(C) $2ab+1$ (D) $\frac{1}{2b-1}$

**22.** The value of x satisfying $\sqrt{3}^{-4+2\log_{\sqrt{5}} x} = 1/9$ is :

(A) 2 (B) 3

(C) 4 (D) None of these

**23.** The number $N = 6 \log_{10} 2 + \log_{10} 31$ lies between two successive integers whose sum is equal to :

(A) 5 (B) 7

(C) 9 (D) 10

**24.** $\log_y x + \log_x y = 2$, $x^2 + y = 12$, then the value of xy is :

(A) 9 (B) 12

(C) 15 (D) 21

**25.** If $\log_a 3 = 2$ and $\log_b 8 = 3$, then $\log_a b$ is :

(A) $\log_3 2$ (B) $\log_2 3$

(C) $\log_3 4$ (D) $\log_4 3$

**26.** If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to :

(A) 2 (B) 3

(C) 10 (D) 30

**27.** The value of $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2}$ is :

(A) 3 (B) 0

(C) 2 (D) 1

**28.** If $a^4 \cdot b^5 = 1$, then the value of $\log_a(a^5 b^4)$ equals.

(A) 9/5 (B) 4

(C) 5 (D) 8/5

**29.** If the equation $2^x + 4^y = 2^y + 4^x$ is solved for y in terms of x, where $x < 0$, then the sum of the solutions is :

(A) $x \log_2(1-2^x)$ (B) $x + \log_2(1-2^x)$

(C) $\log_2(1-2^x)$ (D) $x \log_2(2^x+1)$

**30.** The number of solution $x^{\log_x (x+3)^2} = 16$ is :

(A) 0 (B) 1

(C) 2 (D) $\infty$

**31.** If $\log_{10}\left[\frac{1}{2^x + x - 1}\right] = x\left[\log_{10} 5 - 1\right]$, then x =

(A) 4 (B) 3

(C) 2 (D) 1

**32.** Solution set of the inequality

$\log_3(x-2)(x+4) + \log_{1/3}(x+2) < (1/2) \log_{\sqrt{3}} 7$ is :

(A) (–2,–1) (B) (–2,3)

(C) (–1,3) (D) $(3, \infty)$

**33.** If $\log_3\{5 + 4\log_3(x-1)\} = 2$, then x is equal to :

(A) 2 (B) 4

(C) 8 (D) $\log_2 16$

**34.** The solution set of the ineuqality of $\log_{10}(x^2 - 16) \le \log_{10}(4x - 11)$ is:

(A) $(4, \infty)$ (B) (4,5]

(C) $(11/4, \infty)$ (D) (11/4, 5)

**35.** The number of roots of the equation,

$\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$ is :

(A) 1 (B) 2

(C) 3 (D) 0

**36.** The set of all x satisfying the equation,

$x^{\log_3 x^2 + (\log_3 x)^2 - 10} = 1/x^2$ is :

(A) {1,9} (B) {1,9,1/81)

(C) {1,4,1/81} (D) {9,1/81}

**37.** If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then

(A) $f(x_1).(f(x_2) = f(x_1 + x_2)$

(B) $f(x+2) - 2f(x+1) + f(x) = 0$

(C) $f(x) + f(x+1) = f(x^2 + x)$

(D) $f(x_1)+f(x_2)=f\left(\dfrac{x_1+x_2}{1+x_1x_2}\right)$

**38.** The number of real values of the parameter k for which $(\log_{16}x^2) - \log_{16}x + \log_{16}k = 0$ with real coefficients will have exactly one solution is :

(A) 2 (B) 1

(C) 4 (D) None of these

**39.** The number of roots of the equation $3x^{\log_5 2} + 2^{\log_5 x} = 64$ is :

(A) 2 (B) 4

(C) 1 (D) 0

**40.** The solution set of the equation $\ell n(2x-1) + \ell n(3x-2) = \ell n\ 7$, is :

(A) $x = \dfrac{-1}{2}$ or $\dfrac{5}{3}$ (B) $x = \dfrac{-1}{2}$ or $\dfrac{-5}{3}$

(C) $x = \dfrac{1}{2}$ or $\dfrac{-5}{3}$ (D) $x = \dfrac{5}{3}$ only

**41.** If a,b,c and d are positive integers, then the value of

$$\log_{10}\left(\frac{2a}{b}\right) - \log_{10}\left(\frac{c}{2b}\right) + \log_{10}\left(\frac{5c}{d}\right) - \log_{10}\left(\frac{a}{5d}\right)$$

is equal to :

(A) 0 (B) 2 (C) 3 (D) 6

**42.** Let $S = \log_2\left(\sqrt{7}-\sqrt{5}\right)$, then the value of $\log_{12}\left(\sqrt{7}-\sqrt{5}\right)$ in terms of S, is :

(A) $\dfrac{2}{S}$ (B) $\dfrac{S-1}{S}$

(C) $1-S$ (D) $2-S$

**43.** The value (s) of x that satisfying the equation $\log x + \log(x-2) = \log(x^2-2x)$ is:

(A) $x>0$ (B) $x>2$

(C) $0<x<2$ (D) all real numbers

**44.** If $60^a = 3$ and $60^b = 5$, then $12^{\frac{(1-a-b)}{2(1-b)}}$ is equal to :

(A) $\dfrac{1}{2}$ (B) 2

(C) 15 (D) $\dfrac{1}{15}$

**45.** Number of cyphers after decimal before a significant figure start in [**Use :** $\log_{10}2 = 0.3010$]

(A) 17 (B) 18

(C) 19 (D) 20

## EXERCISE –2

### [Multiple Correct Answers Type]

**1.** For $a > 0, \neq 1$, the roots of the equation $\log_{ax} a + \log_x a^2 + \log_{a^2x} a^3 = 0$ are given by

(A) $a^{-4/3}$ (B) $a^{-3/4}$

(C) a (D) $a^{-1/2}$

**2.** The real solutions of the equation $2^{x+2} . 5^{6-x} = 10^{x^2}$ is/are

(A) 1 (B) 2

(C) $-\log_{10}(250)$ (D) $\log_{10}4 - 3$

**3.** If $\log_k x \cdot \log_5 k = \log_x 5, k \neq 1, k > 0$, then x is equal to

(A) k (B) 1/5 (C) 5 (D) none of these

**4.** If $p, q \in N$ satisfies the equation $x^{\sqrt{x}} = \left(\sqrt{x}\right)^x$, then p and q are

(A) relatively prime (B) twin prime

(C) coprime

(D) if $\log_q p$ is defined, then $\log_p q$ is not and vice versa

**5.** If $x = a^b$ for permissible values of a and x, then identify the statement(s) which can be correct

(A) If a and b are two irrational numbers, then x can be rational

(B) If a is rational and b is irrational, then x can be rational

(C) If a is irrational and b is rational, then x can be rational

(D) If a and b are rational, then x can be rational

**6.** The equation $\sqrt{1+\log_x\sqrt{27}}\ \log_3 x + 1 = 0$ has

(A) no integral solution (B) one irrational solution

(C) two real solution (D) no prime solution

**7.** The values of x satisfying the equation,

$$|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7 \text{ is/are}$$

(A) $\dfrac{1}{\sqrt{3}}$ (B) 1

(C) 2 (D) 81

**8.** If $\log_{1/2}(4-x) \geq \log_{1/2}2 - \log_{1/2}(x-1)$, then x belongs to

(A) (1, 2] (B) [3, 4)

(C) (1, 3] (D) [1, 4)

**9.** If the equation $x^{\log_a x^2} = \dfrac{x^{k-2}}{a^k}$, $a \neq 0$, has exactly one solution for x, then the value of k is/are

(A) $6+4\sqrt{2}$ (B) $2+6\sqrt{3}$

(C) $6-4\sqrt{2}$ (D) $2-6\sqrt{3}$

**10.** The possible values of x, satisfying the equation

$\log_2(x^2-x)\cdot\log_2\left(\frac{x-1}{x}\right)+(\log_2 x)^2=4$, is (are)

(A) $\frac{5}{4}$ (B) 5 (C) $\frac{25}{4}$ (D) $\frac{15}{4}$

**11.** Assuming that all logarithmic terms are defined which of the following statement(s) is/are incorrect ?

(A) $\log_b(y\sqrt{x})=\log_b y\cdot\left(\frac{1}{2}\log_b x\right)$

(B) $\log_b x-\log_b y=\frac{\log_b x}{\log_b y}$

(C) $2(\log_b x+\log_b y)=\log_b(x^2y^2)$

(D) $4\log_b x-3\log_b y=\log\frac{x^4}{y^{-3}}$

**12.** The equation,

$(\log_{10}x+2)^3+(\log_{10}x-1)^3=(2\log_{10}x+1)^3$ has

(A) no natural solution (B) two rational solutions

(C) no prime solution (D) one irrational solution

**13.** Let $\log_M N=\alpha+\beta$, where $\alpha$ is an integer and $\beta$ is non negative fraction. If M and $\alpha$ are prime and $\alpha+M=7$ then $N\in[a,b]$. The absolute value of $(b-5a)$ can be

(A) 0 (B) 24 (C) 48 (D) 96

**14.** How many of the following statement(s) is/are not always true ?

(A) If log(ab) exists, then log(ab) = log a + log b for a and b in R (the set of real numbers)

(B) If $\log(a^2)$ exists, then $\log(a^2)=2\log a$ where a is in R.

(C) If $\ell n\,a$ exists, $e^{\ell n\,a}=a$

(D) If $\log\left(\frac{b}{c}\right)$ exists, then $\log\left(\frac{b}{c}\right)=\log b-\log c$.

**15.** If $\log_{\sqrt[n]{2}}\sqrt[n]{x}+\log_{\sqrt[n-1]{2}}\sqrt[n-1]{x}\ ....+\log_2 x=8$ then x can be equal to (x and n are both integrals)

(A) 2 (B) 4

(C) 16 (D) 256

**16.** Let $a=(\log_3\pi)(\log_2 3)(\log_\pi 2)$, $b=\frac{\log 576}{3\log 2+\log 3}$ the base of the logarithm being 10, c = 2 (sum of the solution of the equation $(3)^{4x}-(3)^{(2x+\log_3(12))}+27=0$

and $d=7^{(\log_7 2+\log_7 3)}$ then $(a+b+c\div d)$ simplifies to

(A) rational which is not natural

(B) natural but not prime

(C) irrational

(D) even but not composite

**17.** $\log_p\log_p\underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\sqrt[p]{p}}}}}_{n\text{ times}}$, p > 0 and $p\neq 1$ is equal to

(A) n (B) – n (C) $\frac{1}{n}$ (D) $\log_{1/p}(p^n)$

**18.** If $\log_a x=\alpha$, $\log_b x=\beta$, $\log_c x=\gamma$ and $\log_d x=\delta$, $x\neq 1$ and a, b, c, $d\neq 0$, > 1, then $\log_{abcd}x$ equals

(A) $\leq\frac{\alpha+\beta+\gamma+\delta}{16}$ (B) $\geq\frac{\alpha+\beta+\gamma+\delta}{16}$

(C) $\frac{1}{\alpha^{-1}+\beta^{-1}+\gamma^{-1}+\delta^{-1}}$ (D) $\frac{1}{\alpha\beta\gamma\delta}$

**19.** If $\log_{10}5=a$ and $\log_{10}3=b$, then

(A) $\log_{10}8=3(1-a)$ (B) $\log_{40}15=\frac{(a+b)}{(3-2a)}$

(C) $\log_{243}32=\left(\frac{1-a}{b}\right)$ (D) All of these

**20.** If x is a positive number different from 1, such that $\log_a x$, $\log_b x$ and $\log_c x$ are in AP, then

(A) $\log b=\frac{2(\log a)(\log c)}{(\log a+\log c)}$ (B) $b=\frac{a+c}{2}$

(C) $b=\sqrt{ac}$ (D) $c^2=(ac)^{\log_a b}$

**21.** If $|a|<|b|$, $b-a<1$ and a, b are the real roots of the equation $x^2-|\alpha|x-|\beta|=0$, then the equation,

$\log_{|b|}\left|\frac{x}{a}\right|-1=0$, has

(A) one root lying in interval $(-\infty, a)$

(B) one root lying in interval $(b,\infty)$

(C) one positive root

(D) one negative root

**22.** If $x^{\frac{3}{4}(\log_3 x)^2+(\log_3 x)-\frac{5}{4}}=\sqrt{3}$, then x has

(A) one integral solutions

(B) two rational solutions

(C) two irrational solutions

(D) no prime solution

**23.** Which of the following statement(s) is (are) correct :

(A) log (1 + 2 + 3) = log 1 + log 2 + log 3

(B) $\frac{\log_9 11}{\log_5 13}\div\frac{\log_3 11}{\log_{\sqrt{5}}13}$ equals 1.

(C) The value of $\log_2 7$ is a rational number which is not an integer.

(D) The equation $\log_2 x + \log_4 (x + 2) = 2$ has two real solutions.

**24.** The number $N = \sqrt{10^{2+\left(\frac{1}{2}\right)\log 16}}$ (where base of logarithm is 10)

(A) is coprime with 9 (B) is a simple surd

(C) is an odd composite

(D) forms the sides of a right triangle with length 99 and 101.

**25.** Which of the following numbers are non-positive ?

(A) $5^{\sqrt{\log_5 4}} - 4^{\sqrt{\log_4 5}}$ (B) $\log_{\cot^2 \frac{\pi}{6}}\left(\sqrt{27}-\sqrt{12}\right)$

(C) $\log_{2-\sqrt{3}}\left(\dfrac{2+\sqrt{3}}{2-\sqrt{3}}\right)$ (D) $\log_{\sec^2 \frac{\pi}{3}}\left(\text{cosec } \dfrac{\pi}{6}\right)$

**26.** The value of

$$\frac{\log_5 9 \cdot \log_7 5 \cdot \log_3 7}{\log_3 \sqrt{6}} + \frac{1}{\log_4 \sqrt{6}}$$ is co-prime with :

(A) 1 (B) 3

(C) 4 (D) 5

**27.** Which of the following quantities are irrational for the quadratic equation

$(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$?

(A) Sum of roots (B) Product of roots

(C) Sum of coefficients (D) Discriminant

**28.** The system of equations,

$\log_{10}(2000\,xy) - \log_{10} x \cdot \log_{10} y = 4$

$\log_{10}(2yz) - \log_{10} y \cdot \log_{10} z = 1$

and $\log_{10}(zx) - \log_{10} z \cdot \log_{10} x = 0$

has two solutions $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$, then

(A) $x_1 + x_2 = 101$ (B) $y_1 + y_2 = 25$

(C) $x_1 x_2 = 100$ (D) $z_1 z_2 = 100$

## EXERCISE - 3

### PART - I : ASSERTION-REASON TYPE

**Each question has four choices A, B, C and D, out of which only one is correct. Each question contains statement A and Statement R.**

**(A) Both A and R are individually true and R is the correct explanation of A**

**(B) Both A and R are individually true but R not the corrct explanation of A**

**(C) A is true but R is false**

**(D) A is false but R is true**

**1.** **Assertion (A) :** If $x > 1$, then $\log_{10} x < \log_3 x < \log_e x < \log_2 x$

**Reason (R) :** If $0 < x < 1$, then $\log_x a > \log_x b \Rightarrow a < b$

**2.** **Assertion (A) :** $\log_\pi 2 + \log_2 \pi$ is smaller than 2.

**Reason (R) :** AM $\geq$ GM

**3.** **Assertion (A) :** The least value of $\log_2 x - \log_x (0.125)$ is $2\sqrt{3}$ for $x > 1$.

**Reason (R) :** AM $\geq$ GM

**4.** **Assertion (A) :** If $\log_e \log_5 \left(\sqrt{(2x-2)} + 3\right) = 0$, then the value of x is 3.

**Reason (R) :** If $\log_b a = c$, then $a = c^b$ but $a \neq 0, b > 0, b \neq 1$

**5.** **Assertion (A) :** $a = y^2, b = z^2, c = x^2$, then $8 \log_a x^3 . \log_b y^3 . \log_c z^3 = 27$

**Reason (R) :** $\log_b a . \log_c b = \log_c a$, also $\log_b a = \dfrac{1}{\log_a b}$

**6.** **Assertion (A) :** If $x^{\log_x (1-x)^2} = 9$, then $x = -2$

**Reason (R) :** $a^{\log_a b} = b$, if $a > 0$ and $a \neq 1, b > 0$

**7.** **Assertion (A) :** If $\log_5 x = (5)^{1/2}$, then $x = 5^{\sqrt{5}}$

**Reason (R) :** $\log_x a = b, a > 0$, then $x = a^{1/b}$

**8.** **Assertion (A) :** The equation

$\log_{\frac{1}{2+|x|}}\left(5 + x^2\right) = \log_{(3+x^2)}\left(15 + \sqrt{x}\right)$ has no solution.

**Reason (R) :** $\log a^{2m} = 2m \log |a| \; \forall \; a > 0$ and $m \in N$

### PART - II COMPREHENSION TYPE

**Comprehension # 1**

Let $\log_3 N = \alpha_1 + \beta_1$

$\log_5 N = \alpha_2 + \beta_2$

$\log_7 N = \alpha_3 + \beta_3$

Where $\alpha_1$, $\alpha_2$ and $\alpha_3$ are integers and $\beta_1, \beta_2\, \beta_3 \in [0,1)$.

**1.** Number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$ :

(A) 46 (B) 45

(C) 44 (D) 47

**2.** Largest integral value of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$.

(A) 342 (B) 343

(C) 243 (D) 242

**3.** Difference of largest and smallest integral values of N if $\alpha_1 = 5$, $\alpha_2 = 3$ and $\alpha_3 = 2$

(A) 97 (B) 100

(C) 98 (D) 99

**Comprehension # 2**

If $\log_{10}|x^3 + y^3| - \log_{10}|x^2 - xy + y^2| + \log_{10}|x^3 - y^3| - \log_{10}|x^2 + xy + y^2| = \log_{10} = 221$. Where x,y are integers, then

**1.** If x = 111, then, y can be :

(A) ±111 (B) ±2

(C) ±110 (D) ±109

**2.** If y = 2, then value of x can be :

(A) ±111 (B) ±15

(C) ±2 (D) ±110

**Comprehension # 3**

Let $\alpha$ and $\beta$ are the solutions of the equation $\left(\sqrt{x}\right)^{-1+\log_5 x} = 5$ where $\alpha \in I$ and $\beta \in Q$.

[**Use :** $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

**1.** The number of significant digits before decimal in $(\alpha)^{10}$ is :

(A) 13 (B) 14

(C) 15 (D) 16

**2.** Number of zeroes after decimal before a significant digit starts in $(\beta)^{10}$ is :

(A) 5 (B) 7

(C) 8 (D) 6

**3.** The value of $(\beta)^{\log_{25} 9}$ is :

(A) $\frac{1}{3}$ (B) 5

(C) $\frac{1}{5}$ (D) 9

**Comprehension # 4**

Let x = $\alpha$ be the value of x satisfying the equation

$\log_p(x+32) = 6$, where $p = \sqrt{\sqrt{9+4\sqrt{5}} - \sqrt{9-4\sqrt{5}}}$

**1.** The value of $(\log_2 \alpha)^2 + 3\log_2\left(\sqrt[3]{\alpha}\right) - \text{antilog}_8\left(\frac{1}{3}\right)$ is equal to :

(A) 25 (B) 26 (C) 27 (D) 28

**2.** Which of the following is **incorrect** ?

(A) The characteristic of logarithm of number $\alpha^2$ with respect to base 2 is 10.

(B) $2^{\log_7(\alpha)} = \alpha^{\log_7(2)}$

(C) $3^{\sqrt{\log_3 \alpha}} = \alpha^{\sqrt{\log_\alpha 3}}$

(D) $\log_6(\alpha+4) > \log_5(\alpha-7)$

**3.** Number of zeroes after decimal in $N = \frac{1}{\alpha^\alpha}$ is equal to:

[Use : $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$]

(A) 46 (B) 47

(C) 48 (D) 49

## PART - III MATCH THE COLUMN

**1.**

| **Column –I** | **Column –II** |
|---|---|
| (A) If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$, then the value of $\log_a b$ is equal to : | **(P) –1** |
| (B) If $a = \left(\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}\right)$, $b = \sqrt{11+6\sqrt{2}} - \sqrt{11-6\sqrt{2}}$ then the value of $\log_a b$ is equal to | **(Q) 1** |
| (C) If $a = \sqrt{3+2\sqrt{2}}$, $b = \sqrt{3-2\sqrt{2}}$, then the value of $\log_a b$ is equal to | **(R) 2** |
| (D) If $a = \sqrt{7+\sqrt{7^2-1}}$, $b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to | **(S)** $\frac{3}{2}$ |
| | **(T)** None |

**2.**

| **Column – I** | **Column – II** |
|---|---|
| (A) The number of solution (s) of the equation $\log_4(x-1) = \log_2(x-3)$ is | **(P) 0** |
| (B) The smallest integer greater than $\frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi}$ is coprime with | **(Q) 1** |
| (C) The number of solution (s) of the equation $x^{\log_3 x^2 + (\log_3 x)^2 - 10} = \frac{1}{x^2}$ is | **(R) 2** |
| (D) The number of solution(s) of the equation $\log_3 \log_5\left(\sqrt{x+5} - \sqrt{x}\right) = 0$ is | **(S) 3** |
| | **(T) 4** |

**3.** **Column – I** **Column – II**

(A) $\log_2 x = \log_{\frac{1}{2}} 7$, then the value of x is — (P) $\frac{1}{49}$

(B) $\log_8 y = \frac{-1}{\log_3 2}$, then the value of y is — (Q) $\frac{1}{36}$

(C) $\log_{\frac{1}{4}} z = \log_2 6$, then the vlaue of z is — (R) $\frac{1}{27}$

(D) $\log_{\frac{1}{9}} w = \log_3 7$, then the value of w is — (S) $\frac{1}{7}$

(T) $\frac{1}{6}$

**4.** **Column – I** **Column – I**

(A) The expression,

$$x = \log_2 \log_9 \sqrt{6+\sqrt{6+\sqrt{6+....\infty}}}$$

simplifies to — (P) an integer

(B) The number,

$$N = 2^{(\log_2 3 \cdot \log_3 4 \cdot \log_4 5 ..... \cdot \log_{99} 100)}$$

simplifies to — (Q) a prime

(C) The expression, $\frac{1}{\log_5 3} + \frac{1}{\log_6 3} - \frac{1}{\log_{10} 3}$

simplifies to — (R) a natural

(D) The number,

$$N = \sqrt{2+\sqrt{5}-\sqrt{6-2\sqrt{5}+\sqrt{14-6\sqrt{5}}}}$$

simplifies to — (S) a composite

**5.** **Column - I** **Column – II**

(A) The value of x, which does not satisfy the equation $\log_2^2(x^2 - x) - 4\log_2(x-1)\log_2 x = 1$, is (are) — (P) 2

(B) The value of x satisfying the equation

$$2^{\log_2 e^{\ell n 5^{\log_5 7^{\log_7 10^{\log_{10}(8x-3)}}}}} = 13, \text{is}$$

— (Q) 3

(C) The number

$$N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right) \text{is less than}$$

— (R) 4

(D) Let $\ell = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1+9^{\log_3 8}\right)^{\log_{65} 5}$ then $\ell$ + m is divisible by — (S) 5

(T) 6

## PART - IV INTEGER TYPE

**1.** Let x,y,z be positive real number such that $x^{\log_2 7} = 8$; $y^{\log_3 5} = 9$ and $z^{\log_5 216} = 5^{1/3}$ then find the vlaue of $x^{(\log_2 7)^2} + y^{(\log_3 5)^2} + z^{(\log_5 216)^2}$

**2.** If p denotes the product of the two values of x satisfying the equation $\log_{2x}(2) + \log_4(2x) = \frac{-3}{2}$ then find the value of $\log_2\left(\frac{1}{p}\right)$.

**3.** If $x_1$ and $x_2$ are the two solutions of the equation $3^{\log_2 x} - 12x^{\log_{16} 9} + 27 = 0$, then find the value of $x_1^2 + x_2^2$.

**4.** Find the value of $\frac{\log_5 9 \cdot \log_7 5 \cdot \log_3 7}{\log_2 \sqrt{6}} + \frac{1}{\log_9 \sqrt{6}}$

**5.** If p,q are relative prime number satisfying

$$\left(4+\sqrt{15}\right)^{\frac{1}{\log_p(4-\sqrt{15})} + \frac{1}{\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)}} = \frac{1}{10}$$

then find the value of $(p^2 + q^2)$ ?

**6.** Let $A = \log_{11}\left(11^{\log_{11}(1331)}\right)$, $B = \log_{385}(5) + \log_{385}(7) + \log_{385}(11)$, $C = \log_4(\log_2(\log_5 625))$, $D = 10^{\log_{100}(16)}$.

Find the value of $\frac{AD}{BC}$

**7.** If m, n are positive real numbers such that $m^{\log_3 5} = 9$ and $n^{\log_5 7} = 25$ then

$A = m^{(\log_3 5)^2} + n^{(\log_5 7)^2}$

$B = \left(2+49^{\log_7 5}\right)^{\log_{27} 7}$

$C = \text{anti } \log_{\sqrt{7}}\left(\log_7 2401\right)$

find the value of $(A + C \div B)$ ?

**8.** Let A denotes the value of

$$\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)+\log_{10}\left(\frac{ab-\left(\sqrt{(ab)^2-4(a+b)}\right)}{2}\right)$$

where a = 43 and b = 57 and B denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$. Find the value of (AB).

**9.** Find the square of the sum of the roots of the equation $\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x \cdot \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$.

**10.** Find 'x' satisfying the equation,

$$4^{\log_{10} x+1}-6^{\log_{10} x}-2.3^{\log_{10} x^2+2}=0$$

**11.** Integral value of x which satisfies the equation

$\log_6 54+\log_x 16=\log_{\sqrt{2}} x-\log_{36}(4/9)$ is ______

**12.** The reciprocal of $\dfrac{2}{\log_4(2000)^6}+\dfrac{3}{\log_5(2000)^6}$ is ____.

**13.** Number of integers satisfying the inequality $\log_{1/2}|x-3|>-1$ is ______.

**14.** The number of positive integers satisfying, $x+\log_{10}(2^x+1)=x\log_{10}5+\log_{10}6$ is _______.

**15.** If x,y,z are positive real numbers such that $\log_{2x}z=3$, $\log_{5y}z=6$, and $\log_{xy}z=2/3$, then the value of $(1/2z)$ is _____.

## PART - V SUBJECTIVE

**1.** Prove that $\log_3 5$ is an irrational.

**2.** Find the value of the expression, $(\log 2)^3+\log 8.\log 5+(\log 5)^3$.

**3.** Prove that $\log_7 11$ is greater than $\log_8 5$,

**4.** If $a^x=b$, $b^y=c$, $c^z=a$, $x=\log_b a^{k_1}$, $y=\log_c b^{k_2}$, $z=\log_a c^{k_3}$, then find the minimum value of $3k_1+6k_2+12k_3$.

**5.** If $x=1+\log_a bc$, $y=1+\log_b ca$, $z=1+\log_c ab$, then prove that $xyz=xy+yz+xz$.

**6.** If $\dfrac{\ln a}{(b-c)}=\dfrac{\ln b}{(c-a)}=\dfrac{\ln c}{(a-b)}$,

Prove that $a^{b+c}\cdot b^{c+a}\cdot c^{a+b}=1$

Also, prove that $a^{b+c}+b^{c+a}+c^{a+b}\geq 3$

**7.** Simplify :

$$5^{\log_{1/5}(1/2)}+\log_{\sqrt{2}}\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)+\log_{1/2}\left(\frac{1}{10+2\sqrt{21}}\right)$$

**8.** Find the square of the sum of the roots of the equation

$\log_2 x\cdot\log_3 x\cdot\log_5 x=\log_2 x\cdot\log_3 x+\log_3\cdot\log_5 x+\log_5\cdot\log_2 x$

**9.** Show that the sum of the roots of the equation $x+1=2\log_2(2^x+3)-2\log_4(1980-2^{-x})$ is $\log_2 11$.

**10.** Solve the following equations for x and y

$$\log_{100}|x+y|=\frac{1}{2},\log_{10}y-\log_{10}|x|=\log_{100}4$$

**11.** Solve the following equation for x :

$$\frac{6}{5}a^{\log_a x.\log_{10} a.\log_a 5}-3^{\log_{10}(x/10)}=9^{\log_{100} x+\log_4 2}$$

**12.** Find the value of x satisfying the equation

$$|x-1|^{\log_3 x^2-2\log_x 9}=(x-1)^7$$

**13.** Solve for x, $\log_{3/4}\log_8(x^2+7)+\log_{1/2}\log_{1/4}(x^2+7)^{-1}=-2$

**14.** Prove that :

$$2^{\left(\sqrt{\log_a\sqrt[4]{ab}+\log_b\sqrt[4]{ab}}-\sqrt{\log_a\sqrt[4]{b/a}+\log_b\sqrt[4]{a/b}}\right)\sqrt{\log_a b}}=\begin{cases}2 & b\geq a>1\\ 2^{\log_a b} & 1<b<a\end{cases}$$

## EXERCISE - 4

### [Previous Years' JEE Questions]]

### SUBJECTIVE

**1.** Solve for x : $4^x-3^{x-\frac{1}{2}}=3^{x+\frac{1}{2}}-2^{2x-1}$ **[IIT -JEE,1978]**

**2.** Solve the following equation for x : $2\log_x a+\log_{ax} a+3\log_{a^2x} a=0, a>0$, **[IIT -JEE,1978]**

**3.** Show that the equation $e^{\sin x}-e^{-\sin x}-4=0$ has no real solution. **[IIT -JEE,1982]**

**4.** If $\log_3 2, \log_3\left(2^x-5\right)$, and $\log_3\left(2^x-\dfrac{7}{2}\right)$ are in arithmetic progression, determine the value of x. **[IIT -JEE,1990]**

### FILL IN THE BLANKS

**1.** The solution of the equation $\log_7\log_5\left(\sqrt{x+5}+\sqrt{x}\right)=0$ is ___________. **[IIT -JEE,1986]**

### SINGLE CORRECT TYPE

**1.** The least value of the expression $2\log_{10}x - \log_x(0.01)$, for $x > 1$, is **[IIT -JEE,1980]**

(A) 10 (B) 2

(C) –0.01 (D) 4

**2.** If $\ln(a + c)$, $\ln(a - c)$, $\ln(a - 2b + c)$ are in A.P., then **[IIT -JEE,1994]**

(A) a,b,c are in A.P. (B) $a^2, b^2, c^2$ are in A.P.

(C) a,b,c are in G.P. (D) a,b,c are in H.P.

**3.** Let $(x_0, y_0)$ be the solution of the following equations :

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then $x_0$ is : **[IIT -JEE,2011]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

### MULTIPLE CHOICE QUESTION

**1.** The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has **[IIT -JEE,1989]**

(A) at least one real solution

(B) exactly three solutions

(C) exactly one irrational solution

(D) complex roots

**2.** If $3^x = 4^{x-1}$, then, x = **[IIT -JEE,2013]**

(A) $\frac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\frac{2}{2 - \log_2 3}$

(C) $\frac{1}{1 - \log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3 - 1}$

### INTEGER TYPE

**1.** Let $a = \log_3 \log_3 2$. An integer k, satisfying $1 < 2^{\left(-k+3^{-a}\right)} < 2$, must be less than ____. **[IIT -JEE,2008]**

**2.** The value of $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}....}}\right)$ is_______. **[IIT -JEE,2012]**

**3.** The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

**[JEE Adv. 2018]**

## MOCK TEST

### SINGLE CORRECT CHOICE TYPE

**1.** The number of real values of the parameter $\lambda$ for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} \lambda = 0$ with real coefficients will have exactly one solution is

(A) 1 (B) 2 (C) 3 (D) 4

**2.** If $\log_{10} 3 = 0.477$, the number of digit in $3^{40}$ is

(A) 18 (B) 19 (C) 20 (D) 21

**3.** If x, y, z are in G.P. and $a^x = b^y = c^z$, then

(A) $\log_b a = \log_c b$ (B) $\log_c b = \log_a c$

(C) $\log_a c = \log_b a$ (D) $\log_a b = 2 \log_a c$

**4.** If $\ln\left(\frac{a+b}{3}\right) = \left(\frac{\ln a + \ln b}{2}\right)$, then $\frac{a}{b} + \frac{b}{a}$ is equal to

(A) 1 (B) 3 (C) 5 (D) 7

**5.** The solution of the equation $\log_7 \log_5 (\sqrt{x+5} + \sqrt{x}) = 0$ is

(A) 1 (B) 3 (C) 4 (D) 5

### MULTIPLE CORRECT CHOICE TYPE

**6.** If $\frac{\log_2 x}{4} = \frac{\log_2 y}{6} = \frac{\log_2 z}{3k}$ and $x^3y^2z = 1$, then k is equal to

(A) –8 (B) –4 (C) 0 (D) $\log_2\left(\frac{1}{256}\right)$

**7.** If $\frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)}$, then $a^{b+c} \cdot b^{c+a} \cdot c^{a+b}$ is equal to

(A) 0 (B) 1

(C) a + b + c (D) $\log_b a \cdot \log_c b \cdot \log_a c$

**8.** If $\log_a x = \alpha$, $\log_b x = \beta$, $\log_c x = \gamma$ and $\log_d x = \delta$, $x \neq 1$ and a, b, c, d $\neq 0$, > 1, then $\log_{abcd} x$ equals

(A) $\leq \frac{\alpha+\beta+\gamma+\delta}{16}$ (B) $\geq \frac{\alpha+\beta+\gamma+\delta}{16}$

(C) $\frac{1}{\alpha^{-1}+\beta^{-1}+\gamma^{-1}+\delta^{-1}}$ (D) $\frac{1}{\alpha\beta\gamma\delta}$

**9.** $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\sqrt[p]{p}}}}}_{n \text{ times}}$, p > 0 and p ≠ 1, is equal to

(A) n (B) –n (C) $\frac{1}{n}$ (D) $\log_{1/p}(p^n)$

**10.** If $\frac{\ln a}{y-z} = \frac{\ln b}{z-x} = \frac{\ln c}{x-y}$, then

(A) $a^x \cdot b^y \cdot c^z = 1$

(B) $a^{y^2+yz+z^2} \cdot b^{z^2+zx+x^2} \cdot c^{x^2+xy+y^2} = 1$

(C) $a^{y+z} \cdot b^{z+x} \cdot c^{x+y} = 1$

(D) abc = 1

### INTEGER

**11.** Find the solution of equation $\log_3(x^2 - 3x - 5) = \log_3(7 - 2x)$.

**12.** Find the solution of the equation $\log(x + 4) + \log(2x + 3) = \log(1 - 2x)$.

**13.** If $\alpha$, $\beta$ are the roots of equation $\log_5(5^{1/x} + 125) = \log_5 6 + 1 + 1/2x$ then evaluate the sum $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$.

**14.** Find the number of solutions of the equation

$$x^{\log_{10} x} + 6x^{\log_{10}\left(\frac{1}{x}\right)} = 5$$

**15.** If the values of x for which 1, $\log_3\sqrt{3^{1-x} + 2}$, $\log_3(4.3^x - 1)$ are in A.P. is given by $(1 - \log_3 K)$ then evaluate k.

### SUBJECTIVE

**16.** Prove that $(zx)^{\log\frac{z}{x}} (xy)^{\log\frac{x}{y}} (zy)^{\log\frac{y}{z}} = 1$

**17.** Solve the equation $\log_{x^2+6x+8} \log_{2x^2+2x+3}(x^2 - 2x) = 0$

**18.** Prove that $\log_4 18$ is an irrational number.

**19.** Solve the inequality : $\frac{(x-0.5)(3-x)}{\log_2|x-1|} > 0$

**20.** Solution of the equation $x^{0.25\log_{\sqrt[4]{x}}\left(x^2 - x\right)} = (3)^{\log_3 2}$

# ANSWERS KEY

## DPP 1

**1.** Given set is $\left\{ x : x = \frac{x}{n+1},\ \text{where n is a natural number and } 1 \le x \le 6 \right\}$

**2.** (i) $\rightarrow$ D  (ii) $\rightarrow$ C

(iii) $\rightarrow$ A  (iv) $\rightarrow$ B

**3.** C **4.** B **5.** A

**6.** (i) $A = B$ (ii) $A \ne B$

**7.** NO NO **8.** D

**9.** A **11.** $\{1, 3, 5\}, \{8\}$

## DPP 2

**1.** $(-2, \infty)$ **2.** $[8, \infty)$

**3.** $(-\infty, 3)$ **4.** $(2, 3)$

**5.** $(-\infty, 1) \cup (2, 3)$ **6.** $[-1, 2)$

**7.** $\left[-\frac{11}{3}, 5\right]$ **8.** $(-\infty, -1) \cup (0, \frac{1}{2}) \cup (1, \infty)$

**9.** $(-\infty, -3) \cup (-2, -1)$

**10.** $\boldsymbol{(-\infty, -9) \cup (-9, -3) \cup [-1, 0) \cup (0, 2) \cup [4, 6)}$

## DPP 3

**1.** (i) 1 (ii) 1
(iii) 0 (iv) 0
(v) –1 (vi) –1
(vii) –1

**2.** (i) 12 (ii) 2 (iii) 0

**3.** (i) 0 (ii) 0

**5.** (i) $\pm 2$ (ii) $\pm 1/2$ (iii) 3/2

## DPP 4

**1.** 1023 **2.** 0

**6.** (a) (b)

**10.** A **11.**

**12.** C **13.** C

**14.**

## DPP 5

**1.** $x = e$ **2.** $x = 1, 4, 1/8$

**3.** $x = 9$ **4.** $x = \frac{1}{2} \cdot \frac{1}{8}$

**5.** $x = 25$ **6.** $x = 10^{\log_3 2}$

**7.** $x = 64, 1/8$ **8.** $x = 1$

**9.** $x = \log_2\left(\frac{36}{5}\right)$ **10.** $x = \{-0.9, 99\}$

**11.** $x \in \left\{\frac{5}{4}, 3\right\}$ **12.** $x = 9$ or $1/9$

**13.** $x = \frac{1}{16}$ **14.** $x = 2$

## DPP 6

**1.** **(C)** **2.** **(C)**

**3.** **(A)** **4.** $x \in (-\infty, -14/3) \cup (4, \infty)$

**5.** $x \in [1-1, -\sqrt{3}) \cup (1+\sqrt{3}, 3]$

**6.** $x \ge 1$ or $x \le -1$

**7.** $x \in [2, 4]$

**8.** $x \in (-2, 3)$

**9.** $x \in (1, \sqrt{2}]$

# Exercise 1
## SINGLE CORRECT ANSWER TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | C | **2.** | A | **3.** | C | **4.** | C | **5.** | C |
| **6.** | C | **7.** | A | **8.** | B | **9.** | A | **10.** | C |
| **11.** | C | **12.** | C | **13.** | A | **14.** | D | **15.** | D |
| **16.** | D | **17.** | D | **18.** | D | **19.** | C | **20.** | B |
| **21.** | D | **22.** | D | **23.** | B | **24.** | A | **25.** | C |
| **26.** | C | **27.** | A | **28.** | A | **29.** | B | **30.** | A |
| **31.** | D | **32.** | B | **33.** | B | **34.** | B | **35.** | B |
| **36.** | B | **37.** | D | **38.** | A | **39.** | C | **40.** | D |
| **41.** | B | **42.** | C | **43.** | B | **44.** | B | **45.** | C |

# Exercise 2
## MULTIPLE CORRECT ANSWER TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | B,D | **2.** | B,C,D | **3.** | B,C | **4.** | A,C,D | **5.** | A,B,C,D |
| **6.** | A,D | **7.** | C,D | **8.** | A,B | **9.** | A,C | **10.** | A,B |
| **11.** | A,B,C,D | **12.** | B,C,D | **13.** | A,D | **14.** | A,B,D | **15.** | A,B,C,D |
| **16.** | A | **17.** | B,D | **18.** | A,C | **19.** | A,B,C,D | **20.** | A,D |
| **21.** | C,D | **22.** | A,B | **23.** | A,B | **24.** | A,D | **25.** | A,C |
| **26.** | A,B,D | **27.** | C,D | **28.** | A,B,C,D | | | | |

# Exercise 3
## PART - I ASSERTION - REASON TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | B | **2.** | D | **3.** | A | **4.** | C | **5.** | B |
| **6.** | D | **7.** | A | **8.** | B | | | | |

## PART - II COMPREHENSION

| | **Q.1** | **Q. 2** | **Q. 3** |
|---|---|---|---|
| **Comprehension 1** | C | A | D |
| **Comprehension 2** | C | B | |
| **Comprehension 3** | B | D | A |
| **Comprehension 4** | D | D | C |

## PART - III MATCH THE COLUMN

| | **A** | **B** | **C** | **D** |
|---|---|---|---|---|
| **1.** | R | S | P | P |
| **2.** | Q | Q, R, T | S | P |
| **3.** | S | R | Q | P |
| **4.** | P | P, R, S | P, R | P, Q, R |
| **5.** | Q, R, S, T | P | Q, R, S, T | P, R, S |

### PART - IV INTEGER TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | 0374 | **2.** | 5 | **3.** | 272 | **4.** | 4 | **5.** | 29 |
| **6.** | 0024 | **7.** | 0081 | **8.** | 12 | **9.** | 3721 | **10.** | |
| **11.** | 4 | **12.** | 6 | **13.** | 2 | **14.** | 1 | **15.** | 5 |

### PART - V SUBJECTIVE

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **2.** | 1 | **4.** | 18 | **7.** | 6 | **8.** | 961 |
| **10.** | $\left(\frac{10}{3},\frac{20}{3}\right),(-10,20)$ | **11.** | 100 | **12.** | 2, 18 | **13.** | ±3 |

## Exercise 4

### Previous Year Questions

### Subjective Type

**1.** $\frac{3}{2}$    **2.** Solution is $\begin{cases} x > 0, \neq 1, & \text{if} \quad a = 1 \\ x = a^{-\frac{1}{2}}, a^{-\frac{4}{3}}, & \text{if} \quad a > 0, \neq 1 \end{cases}$    **4.** 3

### FILL IN THE BLANKS

**1.** 4

### SINGLE CORRECT TYPE

**1.** D    **2.** D    **3.** C

### MULTIPLE CORRECT TYPE

**1.** A, B, C    **2.** A, B, C

### INTEGER TYPE

**1.** 1    **2.** 4    **3.** 8

## Mock Test

### SINGLE CORRECT CHOICE TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | A | **2.** | C | **3.** | A | **4.** | D | **5.** | C |

### MULTIPLE CORRECT CHOICE TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **6.** | A, D | **7.** | B, D | **8.** | A, C | **9.** | B, D | **10.** | A,B,C,D |

### INTEGER TYPE

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **11.** | –3 | **12.** | –1 | **13.** | 6 | **14.** | 4 | **15.** | 4 |