L-5

22 April 2021 21:56

D.D.P (Range)

Find Range of the function.

(a) $y = \dfrac{e^x - 1}{e^x + 1}$ (b) $y = \dfrac{|x| - 1}{2|x| + 1}$

(c) $y = \dfrac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ (d) $y = \dfrac{\sin x}{\sin x + 2}$

(e) $y = \dfrac{\sin x - 1}{2\sin x - 1}$ (f) $y = \dfrac{e^{|x|} - 1}{e^{|x|} + 1}$

(g) $y = 3\sin^2 x - 2\sin x + 1$ (h) $y = \dfrac{x^2 + 1}{x^2 + 2}$

(i) $y = \cos^2 x + \sin x \cos x + 1$

(j) $y = \cos^2 x - 2\sin^2 x + 3$

(k) $y = \dfrac{1 - \tan^2 x}{3 - \tan^2 x}$

(l) Hint $\boxed{-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}}$

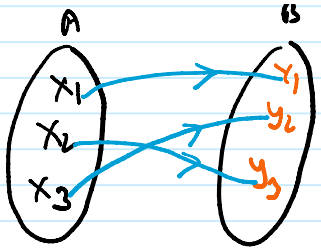
Types of function. (12th)

[1] one-one function or Injective function:

A function $f: A \to B$ is called one-one function if each element of set A has different image in set B.

OR Different elements of set A have different images in set B.

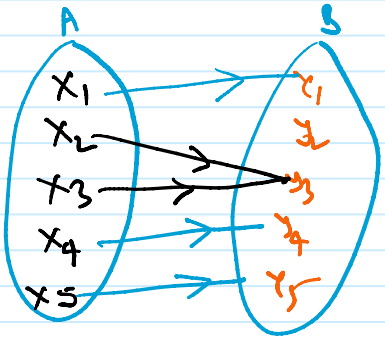
$f: A \to B$



$f: A \to B$

one-one function.

[2] Many-one function: A function $f: A \to B$ is called many-one function. If there exists at least two different elements in set A such that they have same image in set B.



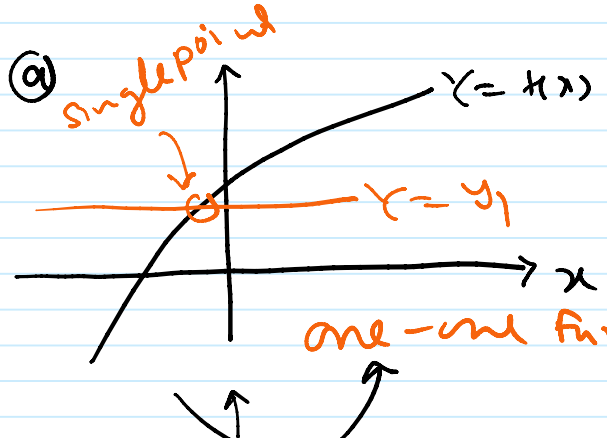
Many-one function.

Graphical testing:

Step I: Draw a horizontal line i.e. $\boxed{Y = y_1}$

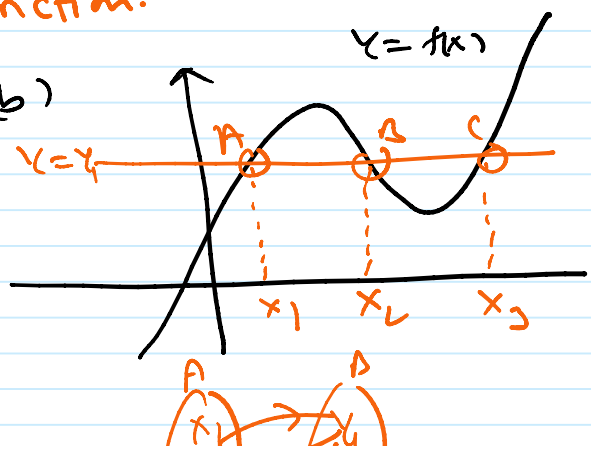
Step II: If line $Y = y_1$ intersects the function $Y = f(x)$ at a single point then given function is one-one otherwise $f(x)$ will be many-one function.

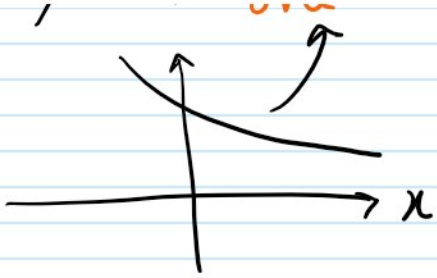
(a)

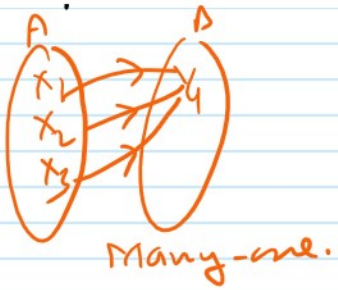


one-one function.

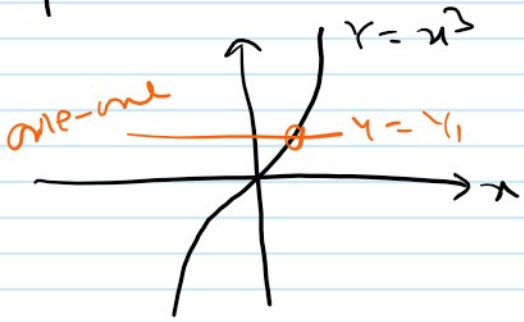
(b)







Ex.



$x = 0.1$

$x = -0.1$

$\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = 0$; $x = 0$

$x = 0$ neither maxima nor minima (Point of inflection)

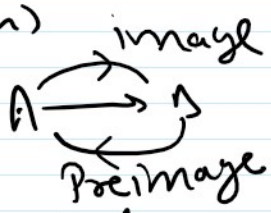
$\frac{d^2y}{dx} = 6x$; $\frac{d^2y}{dx^2} = 0$

$\Rightarrow$ $y = x^3$ is an one-one function.

[3] **onto function:** (Surjective function)

$f: A \to B$

domain, co-domain



If the function $f: A \to B$ is such that each element of set B is the preimage of at least one element of set A Then function f is called onto function.

in this case **co-domain = Range**

[4] **Into function:** If there exists at least one element in set B, which is not the preimage of any element of set A. Then f is called into function

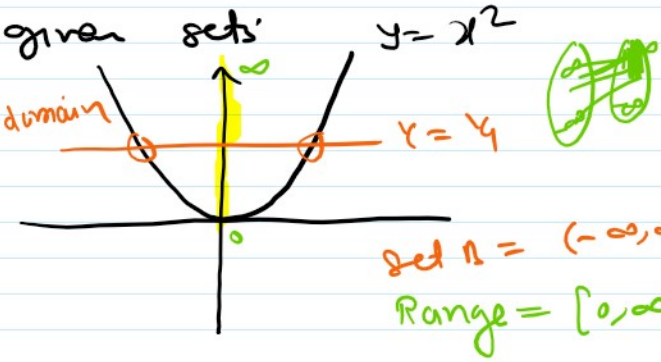
Then f is called into function

In this case **Co-domain ≠ Range**

Ex: Discuss the type of the function $f(x) = x^2$ under the given sets.

(a) $f: R \to R$ (R: co-domain)

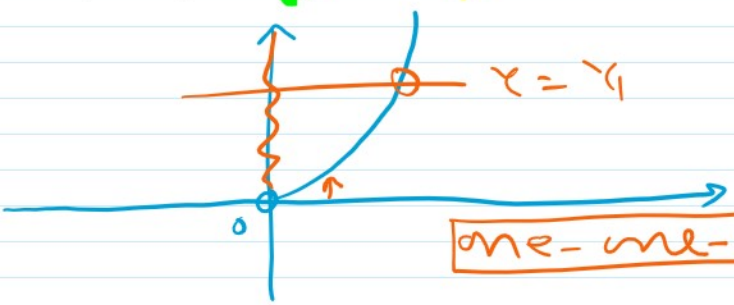
Many-one-Into



Set $R = (-\infty, \infty)$

Range $= [0, \infty)$

(b) $f: R^+ \to R^+$

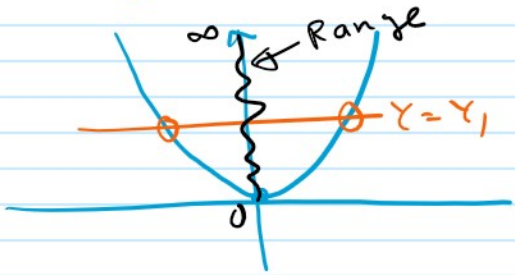


$R^+ = (0, \infty)$

one-one-onto

(Range $= (0, \infty)$, co-domain $= R^+$)

(c) $f: R \to [0, \infty)$



many-one

$y \in [0, \infty)$

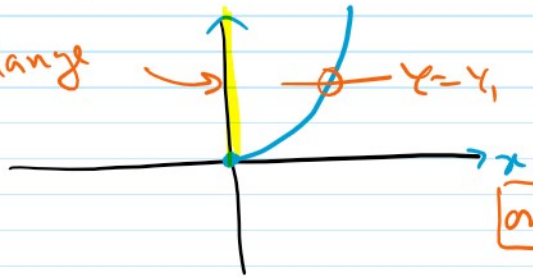
co-domain $= [0, \infty)$

$=$ Range

Many-one-onto

onto

(d) $f: [0, \infty) \to [0, \infty)$



Range = co-domain $= [0, \infty)$

one-one-onto

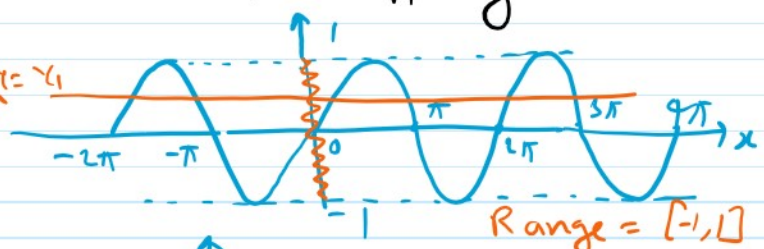
(Injective + Surjective)
(Bijective function)

Ex (d) $f(x) = \sin x$ then discuss type of mapping

(a) $f: R \to R$ (R = co-domain)

$f(x) = \sin x$

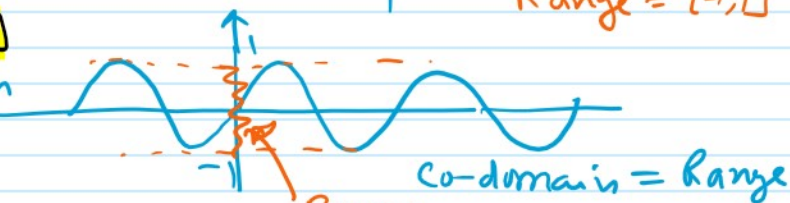
Many-one into



Range = [-1, 1]

(b) $f: R \to [-1,1]$ ([-1,1] = co-domain)

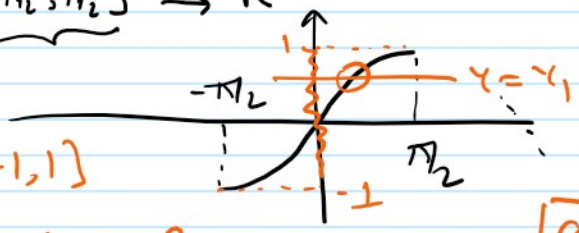
Many-one onto



Co-domain = Range

Range

(c) $f: [-\pi/2, \pi/2] \to R$



one-one

Range = [-1, 1]

co-domain = R

one-one into

(d) $f: [-\pi/2, \pi/2] \to [-1,1]$

one-one onto

Working method for one-one & Many-one function.

[1] If $f(-x) = f(x)$ (even function) $(f: R \to R)$

Then f is many-one.

[2] If $f(x)$ is a periodic function.

Then f is many-one.

Then f is many-one.

[3] Then discuss maxima, minima in f(x)
If there exists neither maxima nor minima in f(x) Then f is one-one.
otherwise f is many-one

$x \neq 1, 2$

Ex. JEE main 2019

Let $A = \{x \in R : x \text{ is not a positive integer}\}$

Define a function $f: A \to R$ as

$$f(x) = \frac{2x}{x-1}$$

Then f is

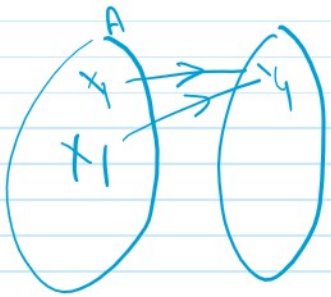
(a) Injective but not surjective
(b) not injective
(c) surjective but not injective
(d) neither injective nor surjective

Sol^n $\quad y = \frac{2x}{x-1}$, $\quad \frac{dy}{dx} = \frac{2(x-1) - 2x \cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$\frac{dy}{dx} < 0$

$\therefore$ f is one-one

Method:



$\therefore$ f is one-one

Let there exists two different elements x_1, x_2 such that

$f(x_1) = f(x_2)$

$\frac{2x_1}{x_1-1} = \frac{2x_2}{x_2-1}$

$x_1(x_2-1) = x_2(x_1-1)$

one-one

$x_1(x_2-1) = x_2(x_1-1)$

$-x_1 = -x_2$

$x_1 = x_2$

Q JEE·mains 2017

The function $f: R \to [-\frac{1}{2}, \frac{1}{2}]$ defined as:

$f(x) = \frac{x}{1+x^2}$ is

(a) one-one-onto (b) injective but not surjective

(c) surjective but not injective (d) neither injective nor surjective

Solution:

$$f(x) = \frac{(1+x^2)\cdot 1 - 2x\cdot x}{(1+x^2)^2}$$

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

$f'(x) = 0 \Rightarrow \boxed{x = \pm 1}$

$\therefore$ Many-one.

Range:

$\frac{y}{1} = \frac{x}{1+x^2}$ $\quad y + yx^2 = x$

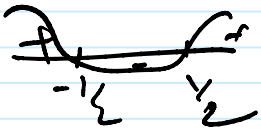
$yx^2 - x + y = 0$

$(-1)^2 - 4\cdot y\cdot y \geq 0$

$4y^2 - 1 \leq 0$

$y^2 - \frac{1}{4} \leq 0$

$(y+\frac{1}{2})(y-\frac{1}{2}) \leq 0$



$y \in [-\frac{1}{2}, \frac{1}{2}]$

co-domain $= [-\frac{1}{2}, \frac{1}{2}] =$ Range

onto