Basic Integration

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1 Introduction

Integration is the reverse process of differentiation.

$$\frac{d[f(x)]}{dx} = \int f(x)dx \tag{1}$$

1.1 Types of Integration

(a) Indefinite Integration

$$\int f(x)dx$$

(b) Definite Integration

$$\int_a^b f(x)dx$$

2 Basic Formula about Indefinite Integration

(a). $\int \cos x \; dx = \sin x + \text{c}$

(b). $\int \sin x \; dx = \text{-}\cos x + \text{c}$

(c). $\int \sec^2 x \; dx = \tan x + \text{c}$

(d). $\int cosec^2 x \; dx = \text{-}\cot x + \text{c}$

(e). $\int \sec x . \tan x \; dx = \sec x + \text{c}$

(f). $\int \cot x . cosecx \; dx = \text{-}cosecx + \text{c}$

(g). $\int e^x \; dx = e^x + \text{c}$

(h). $\int a^2 . \log_e a \; dx = a^x x + \text{c}$

(i). $\int a^x \; dx = \frac{a^x}{log_e a} + \text{c}$

(j). $\int x^n \; dx = \frac{x^{n+1}}{n+1} + \text{c}$

(k). $\int \frac{1}{x}\, dx = \log_e x + c$

3 Integration by substitution method

Rule:-

(a). $[f(x)]^2 + a^2$ Put $f(x) = a\tan\theta$ or $a\cot\theta$
(b). $[f(x)]^2 - a^2$ Put $f(x) = a\sec\theta$ or $a\csc\theta$
(c). $a^2 - [f(x)]^2$ Put $f(x) = a\sin\theta$ or $a\cos\theta$
(d). $\frac{a^2 - [f(x)]^2}{(a^2 + [f(x)]^2)}$ Put $f(x) = a\cos\theta$

4 Some important formula

(a)

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x$$

(b)

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x$$

(c)

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x$$

(d)

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x$$

(e)

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x$$

(f)

$$\int \frac{-1}{x\sqrt{x^2-1}} dx = \csc^{-1} x$$

5 Important formula for rational and irrational function

(a)

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log \mid x + \sqrt{x^2+a^2} \mid + c$$

(b)

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \log \mid x + \sqrt{x^2-a^2} \mid + c$$

(c)

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

(d)

$$\int \sqrt{x^2+a^2} dx = \frac{\mid x\sqrt{x^2+a^2} \mid}{2} + \frac{a^2}{2}\log \mid x + \sqrt{x^2+a^2} \mid + c$$

(e)

$$\int \sqrt{x^2 - a^2} dx = \frac{|\, x\sqrt{x^2 - a^2}\,|}{2} - \frac{a^2}{2} \log |\, x + \sqrt{x^2 - a^2}\,| + c$$

(f)

$$\int \sqrt{a^2 - x^2} dx = \frac{|\, x\sqrt{a^2 - x^2}\,|}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$$

(g)

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} tan^{-1}\left(\frac{x}{a}\right) + c$$

(h)

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log |\, \frac{x - a}{x + a}\,| + c$$

(h)

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log |\, \frac{a + x}{a - x}\,| + c$$

6 Integration by Parts

Basic Formula

$$\int U.V dx = U \int V dx - \int \left(\frac{dU}{dx}. \int V dx\right) dx$$