INTEGERS

Definition :

Integers are numbers that can be positive, negative or zero, but cannot be a fraction.

1, 2, 5,8, -9, -12, 0, 123 etc. The symbol of integers is "I" or "Z".

Common Number Sets

$\mathbb{N}$ = Natural Numbers = {1, 2, 3, ...}

$\mathbb{W}$ or $\mathbb{N}_0$ = Whole Numbers = {0, 1, 2, 3, ...}

$\mathbb{Z}$ = Integers = {..., -1, -2, -3, 0, 1, 2, 3, ...}

$\mathbb{Q}$ = Rational Numbers = {p/q ; p and q are integers}

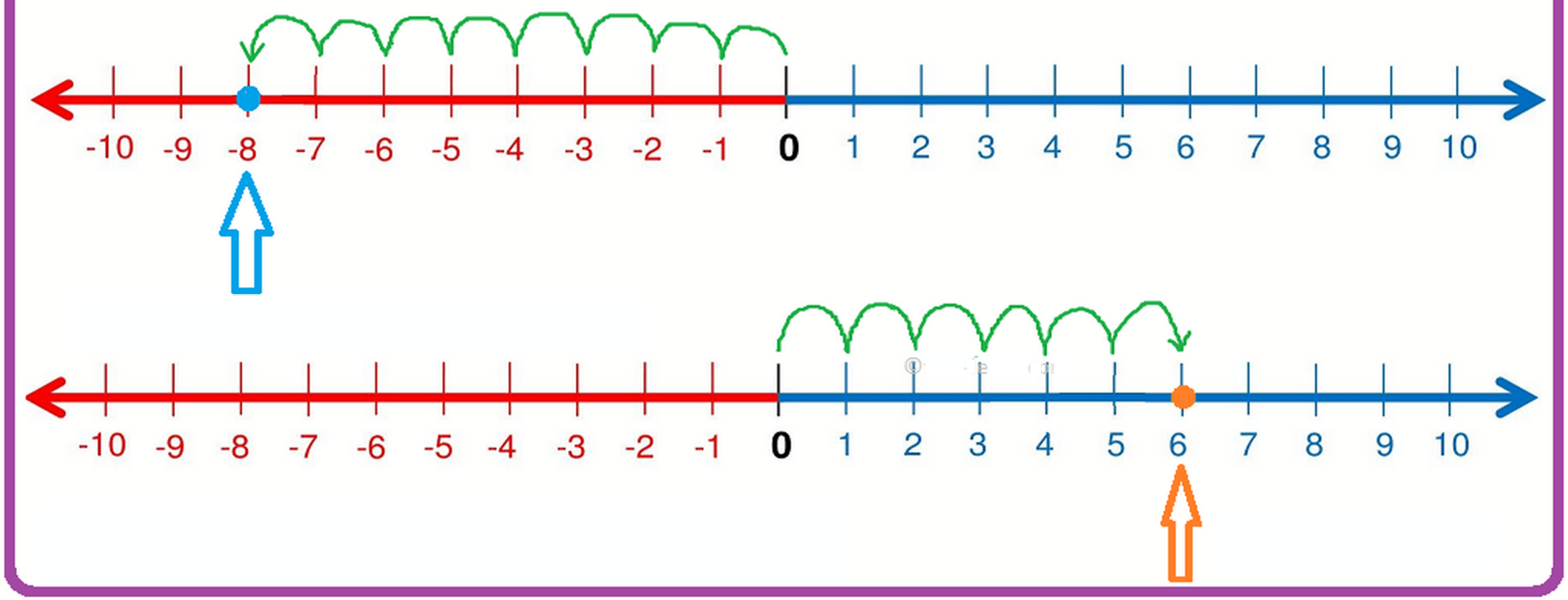
$\mathbb{I}$ = Irrational Numbers = {non-rational number}

$\mathbb{R}$ = Real Numbers = {All of the above number sets}

Imaginary Numbers = {Numbers containing $i = \sqrt{-1}$ }

$\mathbb{C}$ = Complex Numbers = {$a + bi$; a and b are real, $i = \sqrt{-1}$ }

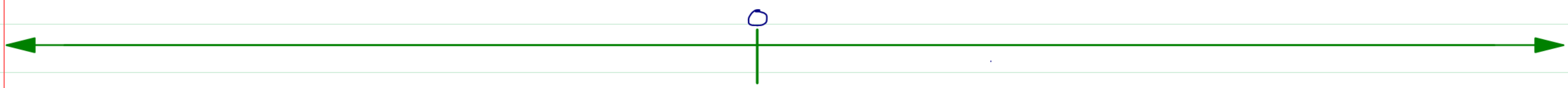
Representation of Integers on a Number Line



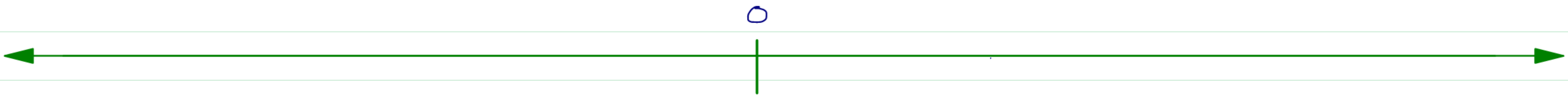
★ **Value of Integers : Increases to the right and decreases to the left.**

1. Arrange in ascending order :

   a) -4, 12, 32, -59, 0, 18, -1



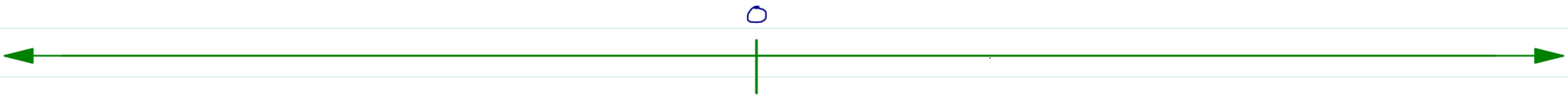
b) 456, -125, 124, -525, 399, -400



NOTE : For Ascending order arrange numbers from ________ to ________ on the Number Line.

2. Arrange in Descending order

a) +124, -1, -14, 0, 123, -123



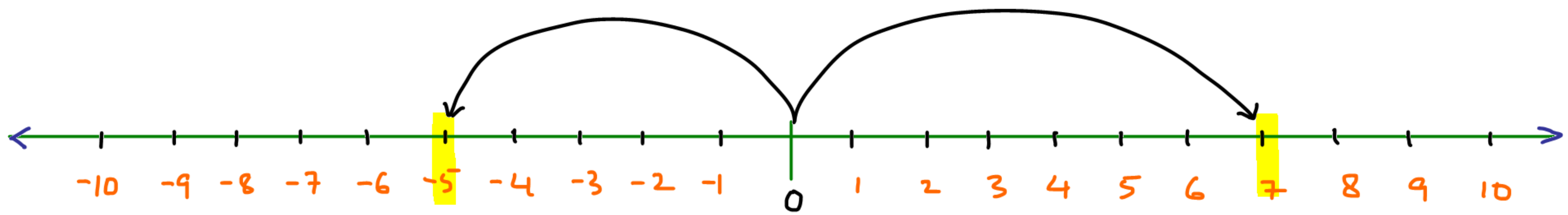
Absolute Value of Integers : It is the numerical value of the Integer.

Only consider the number, neglect its sign (+ or -)

Symbol : | |

Example : The absolute value of -125 is 125 and this is expressed as :

$$|-125| = \_\_\_\_\_\_\_\_$$



Absolute value refers to the distance of the integer from zero.
Distance is a positive quantity.

Evaluate :

a) $|16-5| = |11| = 11$

b) $|5-16| = |-11| = 11$

c) $|6+4| - |-5|$

On a number line when we :

(i) Add a positive integer, we move to the right. ✓

$+2+2=4$

$-2+2=0$

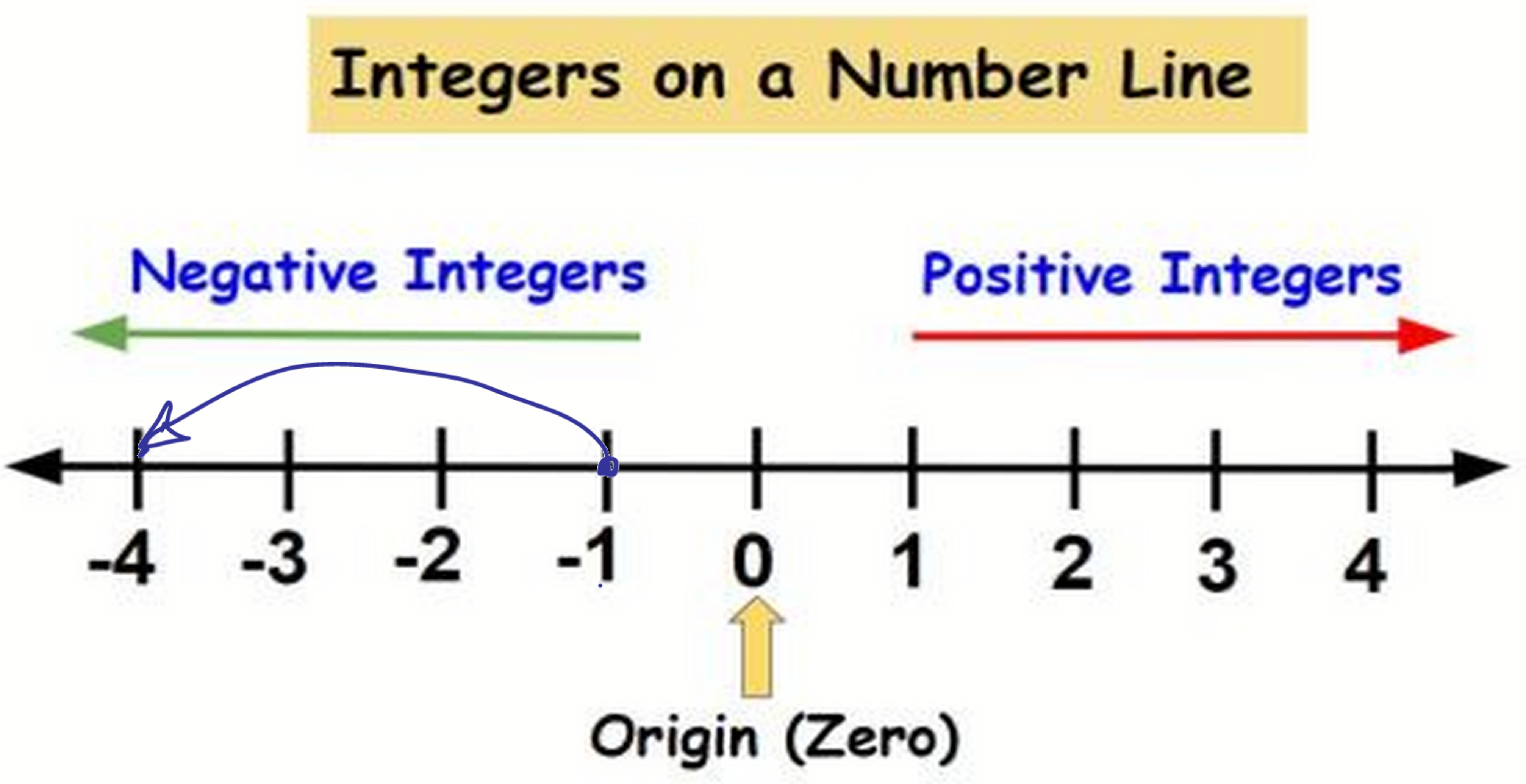
(ii) Add a negative integer, we move to the left.

$+1+(-3)=$

$-1+(-3)=$

(iii) Subtract a positive integer, we move to the left.

(iv) Subtract a negative integer, we move to the right.



Addition & Subtraction of Integers

Math operator (+ or -)

Example : $25 + (-25) + (-20) - (+17) + (-1)$

Sign of the Number

Step 1 : Open the brackets and follow the rule given.

Step 2 : Add the + numbers & the - numbers seperately

Step 3 : Find the difference and apply sign of larger number.

Rules :

$+ \times + = +$

$+ \times - = -$

$- \times + = -$

$- \times - = +$

3. a) Add 25 to 13

b) Add -13 to 25

c) Add 22 to -3

d) Subtract 3 from 5

e) Subtract -3 from 5

f) Subtract 12 from 9

g) Subtract -8 from -9

4. Simplify the following :

a) $(-10) + (-12) + 8 + 4$

b) $45 + (-28) + (-10) + (20)$

c) $15 + (-15) - (-20) + (+17) - (-1)$

d) $90 - (-100) - (42)$

Multiplication of Integers

Positive Integer x Positive Integer = Positive Integer

$(6) \times (8) = +48$

Positive Integer x Negative Integer = Negative Integer

$(6) \times (-8) = -48$

Negative Integer x Positive Integer = Negative Integer

$(-6) \times (8) = -48$

Negative Integer x Negative Integer = Positive Integer

$(-6) \times (-8) = +48$

Rules :

$+ \times + = +$

$+ \times - = -$

$- \times + = -$

$- \times - = +$

5. Simplify :

a) $(-9) \times 8 =$

b) $-5 \times -7 =$

c) Find the product of 81 by (-2)

d) Find the product of $(-2) \times (-4) \times (-6) \times (+10) =$

NOTE :

If the number of negative integers multiplied is odd, then the product will be a negative integer.

Example : $(-1) \times (-1) \times (-1) \times (-1) \times (-1) =$

If the number of negative integers multiplied is even, then the product will be a positive integer.

Example : $(-1) \times (-1) \times (-1) \times (-1) =$

Multiplication by Zero

In general, for any integer a, : $a \times 0 = 0 \times a = 0$

Multiplicative Identity

In general, for any integer a, : $a \times 1 = 1 \times a = a$

Therefore 1 is called the Multiplicative Identity for Integers (also for Whole numbers and Natural Numbers)

Division of Integers

Division of a Positive & a Negative Integer results in a Negative Integer.

$(14) \div (-2) = -7$
$(-14) \div (2) = -7$

Division of two Negative Integers, results in a Positive Integer

$(-14) \div (-2) = 7$

6. Divide the following :

a) 424 by 8

b) (-192) by 6

c) (-891) by (-11)

Rules of Signs in Division

$+ \div - = -$

$+ \div + = +$

$- \div + = -$

$- \div - = +$

Properties of Integers:

1. Closure Property
2. Commutative Property
3. Associative Property
4. Distributive property of Multiplication over addition and subtraction

1a. Closure Property under Addition

$17 + 23 =$

$(-10) + 3 =$

$(-35) + (-10) =$

Since addition of Integers gives Integers, we say :

INTEGERS ARE CLOSED UNDER ADDITION.

In general, for any two integers, a and b, $(a + b)$ is an Integer

1b. Closure Property under Subtraction

$(-10) - 3 =$

$(-35) - (-10) =$

Since Subtraction of Integers gives Integers, we say :

INTEGERS ARE CLOSED UNDER SUBTRACTION

In general, for any two integers, a and b, $(a - b)$ is an Integer

1c. Closure Property under Multiplication

$17 \times 20 =$

$(-35) \times (-10) =$

Since Multiplication of Integers gives Integers, we say :

INTEGERS ARE CLOSED UNDER MULTIPLICATION

In general, for any two integers, a and b, $(a \times b)$ is an Integer

1d. Closure Property under Division

$(-10) \div 2 =$

$(-35) \div (-6) =$

Since Division of Integers DOES NOT ALWAYS give Integers

INTEGERS ARE NOT CLOSED UNDER DIVISION

In general, for any two integers, a and b, $(a \div b)$ Need not be an Integer

2a Commutative Property under Addition

$+5 + (-8) =$ ______ ; $(-8) + 5 =$ ______

$-5 + (-8) =$ ______ ; $(-8) + (-5) =$ ______

Addition is Commutative for Integers

In general, for any two integers a & b, $(a+b) = (b+a)$

2b Commutative Property under Subtraction

$+5 - (-8) =$ ______ ; $(-8) - 5 =$ ______

$-5 - (-8) =$ ______ ; $(-8) - (-5) =$ ______

Subtraction is NOT Commutative for Integers

In general, for any two integers a & b, $(a-b) \neq (b-a)$

2c Commutative Property under Multiplication

$+5 \times (-8) =$ ______ ; $(-8) \times 5 =$ _____

$-5 \times (-8) =$ ______ ; $(-8) \times (-5) =$ _____

Multiplication is Commutative for Integers

In general, for any two integers a & b, $(a \times b) = (b \times a)$

2d Commutative Property under Division

$+5 \div (-8) =$ ______ ; $(-8) \div 5 =$ _____

$-5 \div (-8) =$ ______ ; $(-8) \div (-5) =$ _____

Division is NOT Commutative for Integers

In general, for any two integers a & b, $(a \div b) \neq (b \div a)$

3a. Associative property under Addition

$(-3) + [(-2) + (-5)] =$

$[(-3) + (-2)] + (-5) =$

Addition of Integers is Associative

In general, for any integers a, b & c, we can say $a+[b+c] = [a+b]+c$

3b. Associative property under Subtraction

$(-3) - [(-2) - (-5)] =$

$[(-3) - (-2)] - (-5) =$

Subtraction of Integers is NOT Associative

In general, for any integers a, b & c, we can say $a-[b-c] \neq [a-b]-c$

3c. Associative Property under Multiplication

$7 \times [(-10) \times (-6)] =$

$[7 \times (-10)] \times (-6) =$

Multiplication of Integers is Associative

In general, for any three integers, a, b & c; $a \times [b \times c] = [a \times b] \times c$

3d. Associative Property under Division

$18 \div [9 \div 3] =$

$[18 \div 9] \div 3 =$

Division of Integers is NOT Associative

In general, for any three integers, a, b & c; $a \div [b \div c] \neq [a \div b] \div c$

	Closure	Commutative	Associative
Addition	✓	✓	✓
Subtraction	✓	✗	✗
Multiplication	✓	✓	✓
Division	✗	✗	✗

Special Properties of Zero and 1

I) Under Addition

1. Addition of 1 to any integer gives its Successor

Example : $5+1 = 6$; $-7 + 1 = -6$

2. Additive Identity (Zero)

It is that number, which when added to any integer, gives the same integer.
0 is the Additive Identity.

$5 + \_\_\_\_ = 5$

3. Additive Inverse

It is that number, which when added to an integer, gives the result as zero

$17 + \_\_\_\_ = 0$

$-17 + \_\_\_\_ = 0$

II) Under Subtraction

1. Subtraction of 1 from any integer gives its Predecessor

Example : $5-1=4$; $-7-1=-8$

2. Property of Zero

It is that number, which when subtracted from any integer, gives the same integer.

$5 - \_\_\_\_ = 5$

III) Under Multiplication

1. Property of Zero

The product of any integer with zero is zero

In general , for any integer a; $a \times 0 = 0 \times a = 0$

2. Multiplicative Identity (1)

The product of any integer with 1, equals the integer.

Therefore, the Multiplicative Identity for integers is 1

In general, for any integer a ; $a \times 1 = 1 \times a = a$

III) Under Division

1. Property of 1

When any integer is divided by 1, the quotient is the same integer
In general , for any integer a; $a \div 1 = a$

2. Property of Zero

When zero is divided by any non-zero integer, the result is zero
In general, for any integer a ; $0 \div a = 0$

NOTE : Any integer divided by zero is UNDEFINED