51. Prove that the curves $2x^2 + 4y^2 = 1$ and $6x^2 - 12y^2 = 1$ cut orthogonally.

52. Expand $\log(1-x)$ by using Maclaurins series.

53. Evaluate $\lim_{x \to 0} x^{\sin x}$.

54. Find two positive numbers whose product is 100 and whose sum is minimum.

55. Solve by matrix inversion method $x + y = 3$, $2x + 3y = 8$. (OR)

Solve : $6x^4 - 28x^3 + 32x^2 + 3x - 10 = 0$ given that one of the roots is $2 - i$.

SECTION - C

10 X 10 = 100

Note : a) Answer any TEN questions.

b) Question No. 70 is Compulsory and choose any NINE questions from the remaining :

56. Discuss the solutions of the system of equations for all values of λ. $x + y + z = 2$, $2x + y - 2z = 2$, $3x + y + 4z = \lambda$.

57. Solve : $3x + 4y + 4z = 0$, $x + 2y + 3z = 0$, $7x + 10y + 11z = 0$

58. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ verify $A(\text{adj } A) = (\text{adj}A)\, A = |A|\, I_3$.

59. Find the vector and cartesian equation of the plane through the point (1,3,2) and parallel to the lines $\frac{x+1}{2} = \frac{y+2}{-1} = \frac{z+3}{3}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+2}{2}$.

60. Prove that $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

61. Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find their point of intersection.

62. Find all the values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1.

63. If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$, show that $\frac{(y+\alpha i)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}$.

64. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$ find the co-ordinates of the point of contact.

65. P represents the variable complex number z find the locus of P, if $\arg\left(\frac{z-1}{z+3}\right) = \frac{\pi}{2}$.

66. Assume that water issuing from the end of a horizontal pipe 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground ?

67. Find the eccentricity, centre, foci and vertices of the following hyperbola and draw the diagram $9x^2 - 16y^2 + 36x + 32y + 164 = 0$.

68. A water tank has the shape of an inverted right circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2m^3/min$. Find the rate at which the water level is rising when the water is 3m deep.

69. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$.

a) Find the point of inflection and determine the intervals of convexity and concavity of the Gaussian curve $y = e^{-x^2}$. (OR)