Class 1

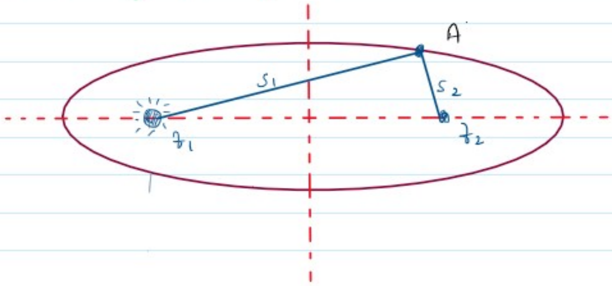
Friday, January 10, 2020 9:52 PM

Gravitation

Keplars law of planetory motion.

① 1st law [law of orbit]

it states that all planet revolve around the sun in an elliptical orbit with sun being situated at one of the focus.

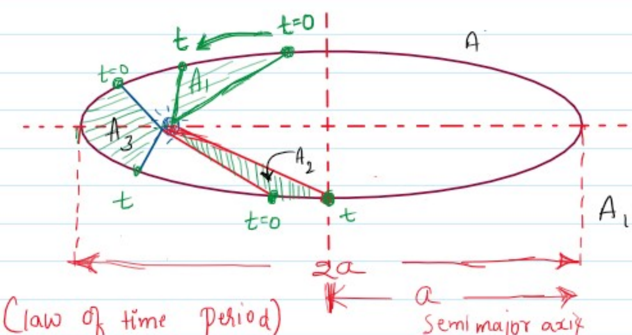


$s_1+s_2 = s_1'+s_2' = \text{const}$

② 2nd law (law of area)

An imaginary line drawn from Sun to any planet sweeps equal area in equal interval of time. This is based on conservation of angular momentum.

i.e $mvr = \text{constant}$



$A_1 = A_2 = A_3$

③ 3rd law (law of time period)

The square of time period is directly proportional to cube of semi major axis.

$T^2 \propto a^3$

Newtons law of gravity

Every body in the universe attracts every other body with a force that is directly proportional to product of their mass & inversely proportional to square of the distance b/w them.

$$F \propto \frac{m_1 m_2}{d^2}$$

$$\boxed{F = G\frac{m_1 m_2}{d^2}}$$

→ universal gravitational constant $= 6.67 \times 10^{-11}\ Nm^2/kg^2$

NOTE

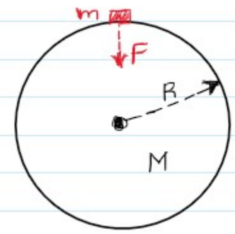
* $F_{12} = F_{21}$ [always attractive force]
* In vector form.

$$\vec{F_{12}} = \frac{Gm_1m_2}{d^2}\hat{r}$$ where $\hat{r}$ is a unit vector from m_1 to m_2

$$\vec{F_{21}} = -\frac{Gm_1m_2}{d^2}\hat{r}$$

$$\boxed{\vec{F_{12}} = -\vec{F_{21}}}$$

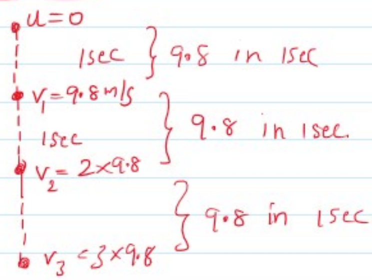
Expression for acceleration due to gravity on the surface of a planet



$$F = \frac{GMm}{R^2}$$

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\boxed{g_{on} = \frac{GM}{R^2}}$$

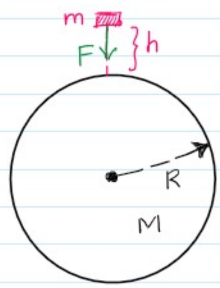


For earth, $M = 6\times10^{24}$ kg, $R = 6400$ km

$$g = \frac{6.67\times10^{-11}\times 6\times10^{24}}{(6400\times10^3)^2}$$

$$\boxed{g_{earth} = 9.81\ m/s^2}$$

Expression for acceleration due to gravity above the surface of Planet-



$$F = \frac{GMm}{(R+h)^2}$$

$$\cancel{m}g' = \frac{GM\cancel{m}}{(R+h)^2}$$

$$\boxed{g' = \frac{GM}{(R+h)^2}}$$ ✓

But WKT $g_{on} = \frac{GM}{R^2}$

$$\therefore \frac{g'}{g} = \frac{\cancel{GM}/(R+h)^2}{\cancel{GM}/R^2} = \boxed{\frac{R^2}{(R+h)^2} = \frac{g'}{g}}$$ ✓

$$\frac{g'}{g} = \frac{1}{\left(\frac{R+h}{R}\right)^2} = \frac{1}{(1+h/R)^2}$$

$$\left(\frac{R}{R}+\frac{h}{R} = 1+\frac{h}{R}\right)$$

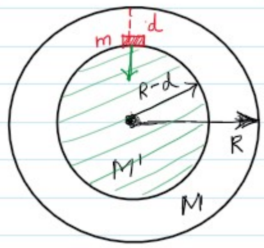
$$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

if $h << R$, then from binomial expansion we can write.

$$\rightarrow \boxed{\frac{g'}{g} = \left(1-\frac{2h}{R}\right)}$$ only if $h << R$

Expression for acceleration due to gravity below the surface of Planet

Expression for acceleration due to gravity below the surface of Planet



NOTE : The force on a body which is inside a shell is zero due to the shell.

← F (m) → will experience force

in this case 'm' will not experience any force. ($F_{net} = 0$)

$$\boxed{F} = \frac{GM'm}{(R-d)^2}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore M' = \rho \times \frac{4}{3}\pi (R-d)^3$$

$$\cancel{m}g' = \frac{G\left(\frac{M}{\frac{4}{3}\pi R^3}\right) \times \frac{4}{3}\pi (R-d)^3 \times \cancel{m}}{\cancel{(R-d)^2}}$$

$$g' = \frac{GM}{R^3}(R-d)$$

$$g' = \frac{GM}{R^2}\left(\frac{R-d}{R}\right)$$

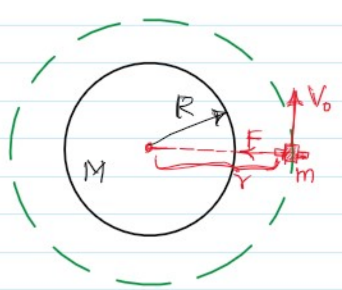
$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \boxed{\frac{g'}{g} = \left(1 - \frac{d}{R}\right)}$$

To summarise

* $g = GM/R^2$
* $g'/g = \left(R/_{R+h}\right)^2$ } for any height h — Above the planet

  $g'/g = \left(1 - \frac{2h}{R}\right)$ } for small value of h (i.e $h << R$) — Above the planet
* $g'/g = \left(1 - \frac{d}{R}\right)$ for any depth below the surface of the planet.

Expression for orbital velocity of satellite (or) any celestial body.



The necessary centrepetal force required for the satellite is provided the gravitational force.

$$\boxed{F} = \frac{mv_0^2}{r}$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v_0^2}{r}$$

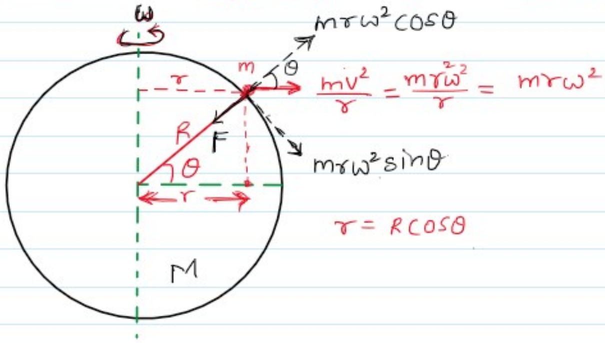
$$v_0^2 = \frac{GM}{r^2} \times r$$

$$V_0^2 = g'r$$

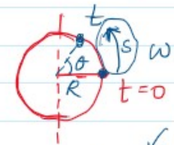
$$\boxed{V_0 = \sqrt{g'r}}$$

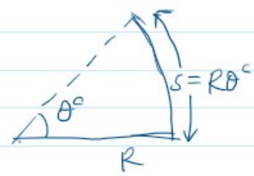
Expression for acceleration due to gravity along the lattitude.

$F = \dfrac{GMm}{R^2}$



$\omega = \dfrac{\theta}{t}$





Arc length for

$\boxed{S = R\theta}$

$\dfrac{s}{t} = R\dfrac{\theta}{t}$

$\boxed{V = R\omega}$

$$F_{inward} = F - mr\omega^2\cos\theta$$

$$mg' = \frac{GMm}{R^2} - m(R\cos\theta)\omega^2\cos\theta$$

$$\boxed{g' = g - R\omega^2\cos^2\theta}$$

At equator $\theta = 0 \Rightarrow \boxed{g' = g - R\omega^2}$

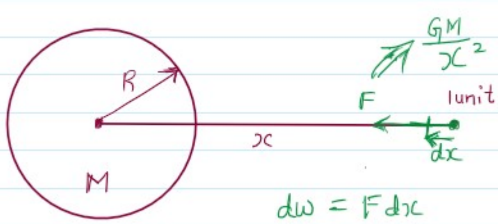
At poles $\theta = 90° \Rightarrow \boxed{g' = g}$

Acc due to gravity is max at poles & min equator.

Gravitational potential.

it is the work done in order to bring unit mass from ∞ to a given point. is the gravitational potential at that point.

Expression for gravitational potential.



$$dW = F\,dx$$

$$dW = \frac{GM}{x^2}\,dx$$

$$W = GM\int_{\infty}^{x}\frac{dx}{x^2}$$

$$= GM\int_{\infty}^{x} x^{-2}\,dx$$

$$= GM\left.\frac{x^{-2+1}}{-2+1}\right|_{\infty}^{x}$$

differentiation.

$$\boxed{\frac{d\,x^n}{dx} = nx^{n-1}}$$

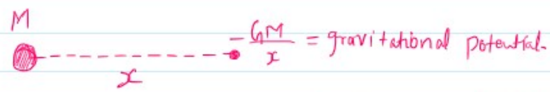
$$\therefore \boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

Integration

$$= GM \left. \frac{-1}{x} \right|_{\infty}^{x}$$

$$= GM \left[\frac{-1}{x} - \left(-\cancel{\frac{1}{\infty}}^{\,0} \right) \right]$$

$$\boxed{\text{Gravitational Potential} = -\frac{GM}{x}}$$



$\therefore$ PE = $U \times m$

=

gravitational Potential Energy (PE) of a body of mass m at a point = Potential × mass

$$= -\frac{GM}{x} \times m$$

$$\boxed{PE = -\frac{GMm}{x}}$$

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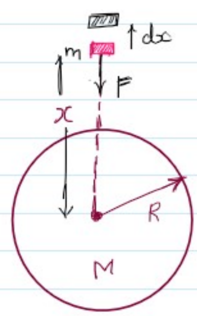
-ve sign indicates that the body is bound to the planet & $+\frac{GMm}{x}$ amount of work needs to be done in order to free the bo

Escape speed

The speed with which a body must be projected so that it escapes the influence of gravitation is called escape speed.

Expression for escape speed.



$$dw = F\,dx$$

$$dw = \frac{GMm}{x^2}\,dx$$

$$W = \int_x^{\infty} \frac{GMm}{x^2}\,dx$$

$$= GMm \int_x^{\infty} \frac{1}{x^2}\,dx$$

$$= GMm \int_x^{\infty} x^{-2}\,dx$$

$$= GMm \left. \frac{x^{-2+1}}{-2+1} \right|_x^{\infty}$$

$$= GMm \left. \frac{x^{-1}}{-1} \right|_x^{\infty}$$

$$= -GMm \left. \frac{1}{x} \right|_x^{\infty}$$

$$= -GMm \left[\cancel{\frac{1}{\infty}}^{\,0} - \frac{1}{x} \right]$$

$$= -GMm\left(-\frac{1}{x}\right) = \frac{GMm}{x}$$

This Energy is supplied to the body in the form of KE i.e $\frac{1}{2}\cancel{m}V_e^2 = \frac{GM\cancel{m}}{x}$

$$V_e^2 = \frac{2GM}{x}$$

$$V_e = \sqrt{2}\sqrt{\frac{GM}{x}} \rightarrow \sqrt{\frac{GM}{x^2}\times x} = \sqrt{g'x} = V_o$$

$$\boxed{V_e = \sqrt{2}\,V_o} \qquad \boxed{V_e = \sqrt{2g'x}}$$

NOTE

From the surface of planet $g' = g$ & $x = R$

$$V_e = \sqrt{2gR}$$

$$V_e = \sqrt{2\times 9.81\times 6400\times 10^3} \text{ m/s}$$

$$= \sqrt{2\times 9.81\times 640}\times 10^2 \text{ m/s}$$

$$\cong 112.05\times 10^2 \text{ m/s}$$

$$= 11.2\times 10^3 \text{ m/s}$$

$$\boxed{V_e = 11.2 \text{ Km/s}}$$

Satellites

Geostationary	polar satellite.
* placed in an equitorial plane which appears to be at rest wrt the observer on earth	* These satellites revolves from N-pole to S-pole.
* The are called high altitude satellile & placed at a height of 35,800 km from the surface of a planet	* They are called low altitu-de satellites & placed at a height of around 800 to 900 km. from the surface.
* its time period is 24 hrs	* its time period is around 1.2 hrs to 1.5 hrs
* used for communication purpose. & weather forecasting	* used to take pictures of earth