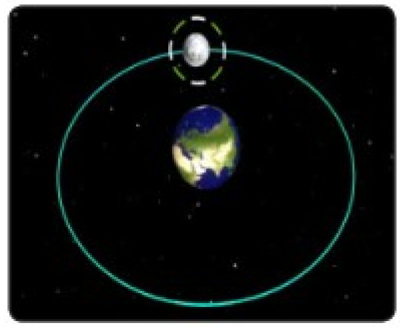
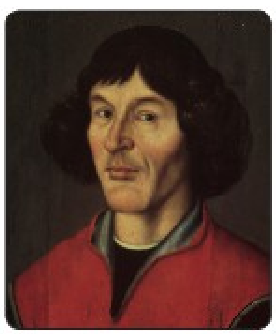
INTRODUCTION:

Ptolemy in second century gave **geo-centric theory** of planetary motion in which the Earth is considered stationary at the centre of the universe and all the stars and the planets including the sun revolving round it.

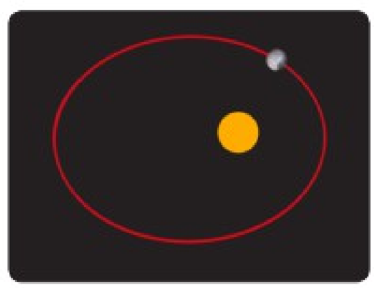
Ptolemy



Nicolas Copernicus in sixteenth century gave **Helio-centric theory** in which the Sun is fixed at the centre of the universe and all the planets move in perfect circles around it.



Copernicus



TychoBrahe had collected a lot of data on the motion of planets but died before analysing them.

Johannes Kepler analyzed Brahe's data and gave three laws of planetary motion known as Kepler's laws.

Tycho Brahe

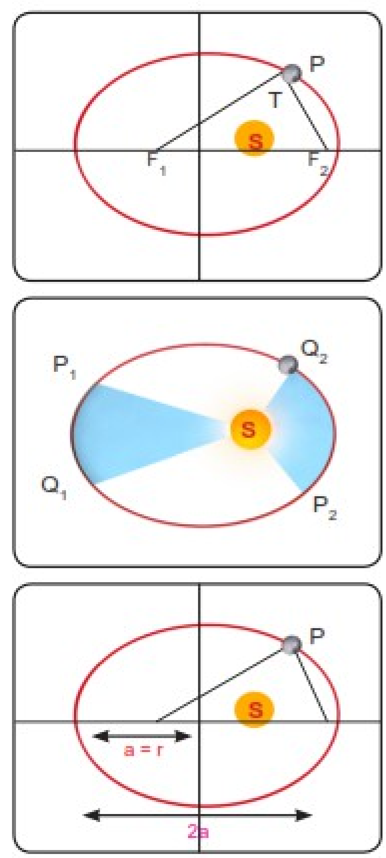
Johannes Kepler

KEPLER'S LAWS:

Johannes Kepler was born in south- west Germany. While working as a professor of mathematics in Graz, he was invited by Tycho Brahe to analyze his observations of Mars. Tycho's data was accurate and was good enough for Kepler to put forth his three laws of planetary motion.

Johannes Kepler's laws of planetary motion are:

1. The path of each planet around the sun is an ellipse with the sun at one focus.
2. A line joining the sun to the planet sweeps out equal areas in equal intervals of time.
3. The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.



Kepler's Laws

First law:

The path of each planet around the sun is an ellipse with the sun at one focus.

If F_1 and F_2 are the two foci, P is the planet and S is the sun:

$F_1P + F_2P$ = constant.

Second law: A line joining the sun to the planet sweeps out equal areas in equal intervals of time.

If the time taken by a planet to travel from P_1 to Q_1 is equal to the time taken to travel from P_2 to Q_2, the areas covered are equal (shaded region).

Third law:

A planet moves around the sun in such a way that the square of its time period is proportional to the cube of the semi-major axis of its elliptical orbit.

If T is the time period of revolution and 'a' the semi-major axis then,

For circular orbits a = r (radius),

$\propto$

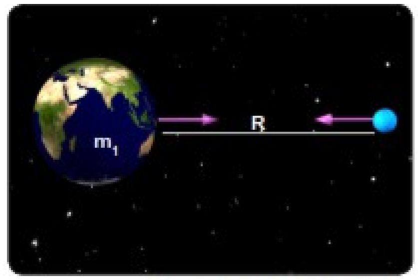
LAW OF UNIVERSAL GRAVITATION:

NATURE OF GRAVITY:

The force of attraction between the bodies having a certain mass is called gravitational force or simply gravity. The nature of gravity is as follows:

- Gravitational force between any two bodies depends upon the product of the masses and the distance between them.
- Gravitational force does not depend on the nature of attracting bodies and is always attractive.
- Gravitational force between a pair of bodies is a central force i.e., it acts the line joining their centre of mass.
- It is a long range force i.e., they act upon infinite distances.
- It is the weakest of all the four basic forces.
- Gravitational force is a conservative force i.e., it does not depend upon the path traversed.

Gravity of the Earth



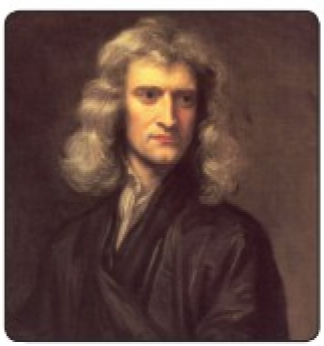
Gravitional Force

THE LAW OF UNIVERSAL GRAVITATION:

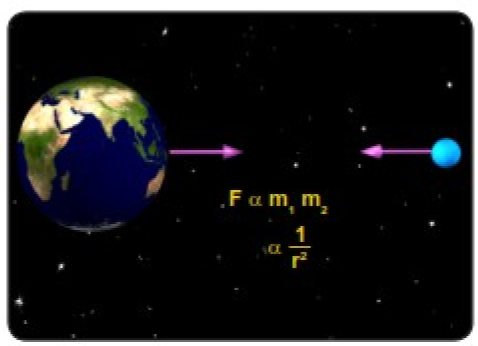
Isaac Newton has formulated the law of universal gravitation in the year 1666, using his laws of motion and Kepler's laws of planetary motion.

Newton's law of universal gravitation:

'Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them and this force acts along the line joining the two particles.

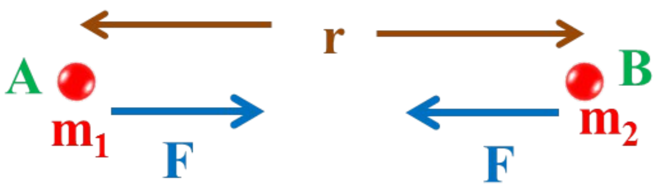


Isaac Newton



Law of universal Gravitation

Universal Law of Gravitation:



Let us consider…

Mass of body A

Mass of body B

r = Distance between A and B

- There exists a force of attraction
- This force is called gravitational force (F)
- This force is directly proportional to the product of masses

  F∝
- This force is inversely proportional to the square of the distance between the masses,

  F ∝

  ∴F ∝

Statement:

Every particle of matter attracts every other particle of matter with aforce which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

If we introduce a constant (G)

F = G.

- G is the constant of proportionality known as universal gravitational constant**.**

GRAVITATIONAL CONSTANT 'G':

Universal Gravitational Constant (G):

Let us consider the gravitational force formula to understand 'G'

F = G. ⇒ G =

Whenm$_1$ = m$_2$ = 1 and r = 1, then G = F

i.e., universal gravitational constant is numerically equal to the force of attraction between two unit masses placed at unit distance apart.

SI unit of 'G' is

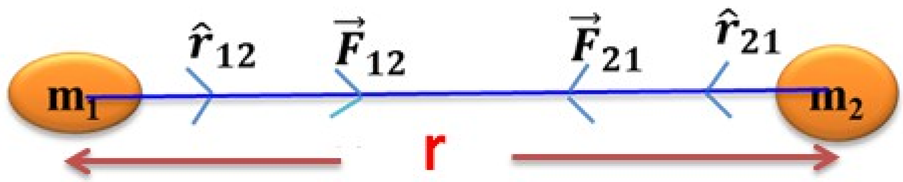
CGS unit of 'G' is

Dimensions of 'G'

So, Dimensions of**G=**

Value of G = 6.67×10^{-11} Nm^2/kg^2 = 6.67×10^{-8} dyne cm^2/

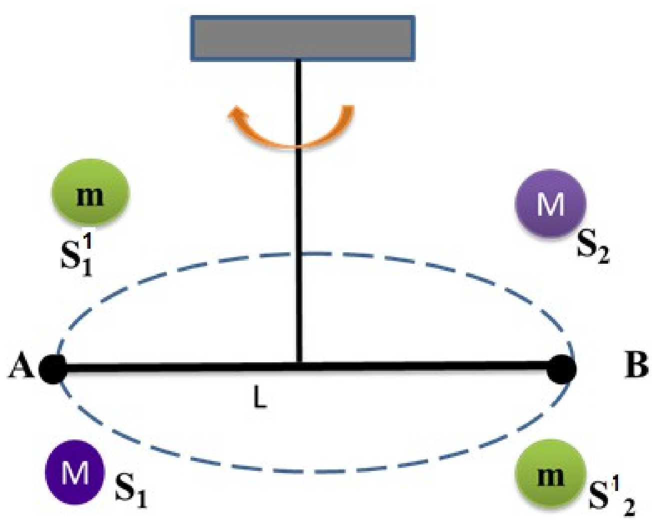
Vector form of Newton's law of gravitation:



= ⟶ (1) ,   = ⟶ (2)

But

Determination of Universal Gravitational constant (G):



Cavendish's experiment

- A bar AB suspended from a ceiling.
- Two small and identical lead spheres attached at their ends.

- Two large lead spheres **(S_1 and S_2)** are brought close to the small ones but on opposite sides.
- The big spheres attract the nearby small ones by equal and opposite force as shown in the figure.
- There is no net force on the bar but only a torque which is clearly equal to F times the length of the bar.
- Where F is force of attraction between a big sphere & neighboring small sphere.
- Due to this torque, the suspended wire gets twisted until the restoring torque of the wire equals the gravitational torque.

  Restoring torque = $C\theta$ ---------(1)

  Where C is restoring couple per unit twist and θ is the angle of twist.

- If d is the separation between big and small balls having masses M & m.

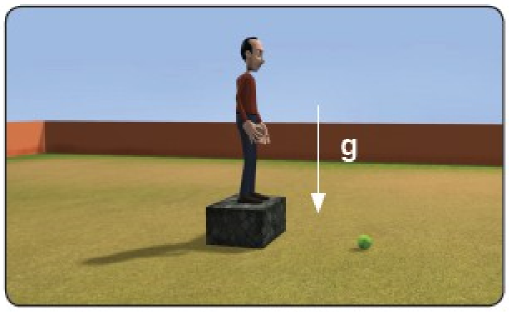
  Gravitational force (F) =------ (2)

- If L is the length of the bar AB, then torque arising out of F is multiplied by L,
- Then the deflecting gravitational torque **= F×L**

  = ×L

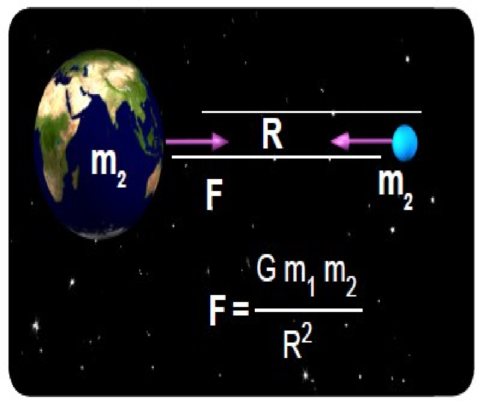
- At equilibrium, this is equal to the restoring torque.
- And hence **------(3)**
- Observing the value of θ and knowing the values of other quantities, G can be calculated.

- **<u>RELATION BETWEEN g AND G:</u>**
- **ACCELARATION DUE TO GRAVITY (g):**
- The uniform acceleration of a freely falling body due to the gravitational force of a planet is called as acceleration due to gravity (g) on that planet.
-
-



Accelaration of Gravity

- **THE GRAVITATIONAL CONSTANT (G)**
- Numerically universal gravitation constant (G) is equal to the value of the force between two bodies of unit mass separated by a distance of 1 meter.



Gravitational Constant

- The expression for Newton's law of universal gravitation F =
-
- Where 'G' is constant, called universal gravitational constant.
- Bodies allowed to fall freely, found that they reach the ground at the same time (Irrespective of their masses and by neglecting air resistance). The velocity of a freely falling body increases at a steady rate. This acceleration is called acceleration due to gravity **'g'.**
- If m, g and F represent the mass of the body, acceleration due to gravity, the force acting on the body due to gravitationalpull
- **F = mg** $\rightarrow$ **(1)**
- According to Newton's law of gravitation, the gravitational force on the body is,

- **F =** → (2)
- From equations (1) and (2), mg =
- **g =**
- From the above expression we note that the value of acceleration due to gravity is independent of mass, shape and size of the body but depends upon mass and radius of the planet.
- This relation is valid for any planet including the earth.

ACCELERATION DUE TO GRAVITY BELOW AND ABOVE THE SURFACE OF EARTH:

Acceleration due to gravity 'g' varies with

a. altitude (Height)

b. depth

c. latitude

Variation of 'g' with altitude:

Let a body of mass 'm' be placed on the surface of the earth, whose mass is 'M' and radius is R.

From equation **g = ... (1)**

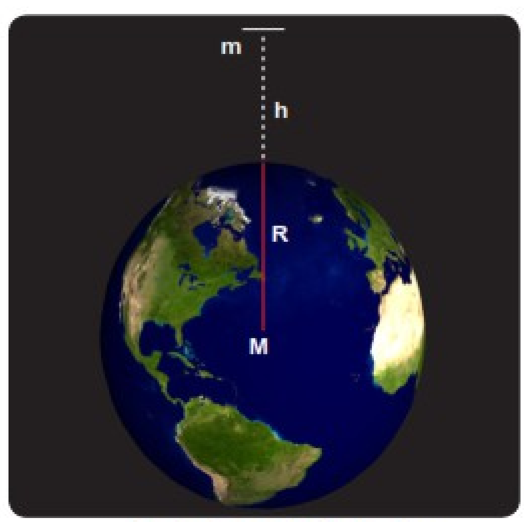
Let the body be now placed at a height **'h'** above the earth's surface. Let the acceleration due to gravity at that position be **g**'.

Then , =

For comparison, the ratio between g′and g is taken as

=

= g×



.Variation of alatitude

= g××

= g

= gfor , by using binomial expression

∴

This shows that acceleration due to gravity decreases with increase in altitude.

Loss in weight at height **'h' (h<<R).**

From equation (2)

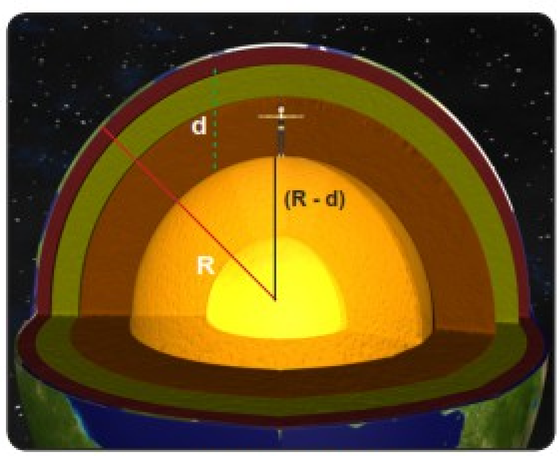
= g(1 - = g - = mg -

mg - =

∴loss in weight = ... (3)

Variation of 'g' with depth:

Consider earth to be a homogeneous sphere of radius R, density 'ρ' and mass M with centre at 'o'. Let 'g' be the acceleration due to gravity on the surface of earth.



Variation of depth

When a body is on the surface of the earth, its entire mass ‘M’ attracts the body towards its centre.

We know that g =

But mass of the earth = Volume of the earth × density of the earth

⇒ M =ρ

∴ g = ⇒ g = ρ → (1)

Let be the acceleration due to gravity at the depth ‘d’ below the surface of earth. The body will experience gravity pull due to earth whose radius is (R - d) and mass is

∴ = and =

∴ =

⇒ = πG(R-d)ρ   → (2)

On dividing equation (2) with equation (1), we can write

⇒

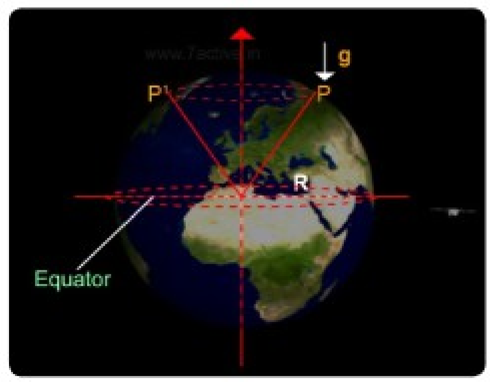
= g

⇒< g .

Therefore, the acceleration due to gravity decreases with increase in depth. If **d = R**, then **g' = 0**. Therefore, weight of a body at the centre of the earth is zero.

<u>Variation of 'g' with latitude:</u>

The value of g changes from place to place due to the elliptical shape of the earth and the rotation of the earth. Due to the shape of the earth,



Variation of latitude

From equation **g =** , **g ∝**

Hence, it is inversely proportional to the square of the radius.

∴ It is least at the equator and maximum at the poles, since the equatorial radius **(6378.2 km)** is more than the polar radius (6356.8 km)

<u>Variation of 'g' due to the rotation of the Earth:</u>

If **''** is the angular velocity of the earth and **'** is the latitude of the place,

$g' = g - R$

What is latitude?

Every point on the sphere lies on the same latitude, which lie on the base of the cone whose axis coincides with the polar axis and whose generators

makes an angle **'Ø'** with the horizontal or equatorial plane. The angle **'Ø'** is

called the latitude of the place. Latitude of equator = **0°** and latitude of North Pole = **90°** and latitude of South Pole = -**90°.**

We can observe that if the earth is considered to be a sphere of radius R, the radius of the smaller circle in the latitude **r = R cos**

A particle on the latitude f which is undergoing uniform circular motion with angular velocity **' '** experiences centripetal acceleration **'a'** directed towards the centre of the small circle. This acceleration **'a'** can be resolved into two components, tangential and radial.

$a_T = R$ **and** $a_R = R\,Ø$

Since all the forces acting on the body at the latitude f result in uniform circular motion, the net of all forces should be equal to centripetal force.

i.e., **mg - mg'= mR**

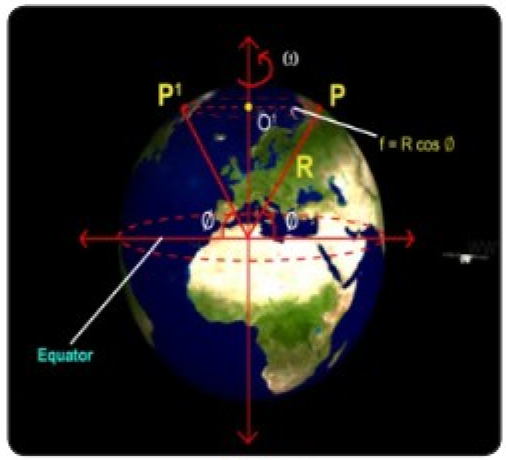
or**mg’ = mg - mR**

or**g′= g - R**

At the equator: **=**

= 1

g′= g - R = g -



latitude

At the pole:= ±,

= 0.

g’= g

Hence, the acceleration due to gravity is maximum at the poles and minimum at the equator.

We should observe that the net of all the forces acting on the body results in uniform circular motion, which means that uniform circular motion, is the result of all the forces acting on the body.

g- g’= R

g’= g- R

At the equator **’’ = 0,**

g’= g- , g’< g

At the poles, **= 90°, cos = 0, g’= g**

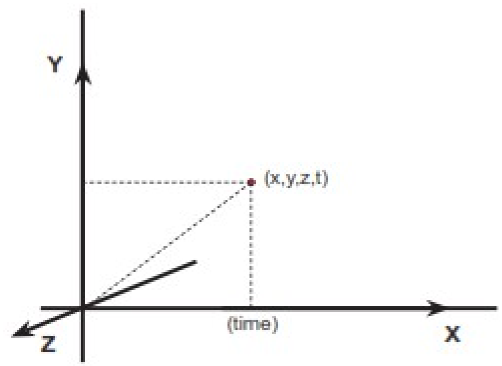
- It is less at the equator and maximum at the poles.

FRAMES OF REFERENCE:

A system of coordinate axes in which the position or motion of anobject can be described is called a frame of reference.

The Cartesian coordinate system named after Rene Descartes is the simplest form of reference frame.

In Cartesian coordinate system we consider three mutually perpendicular axes X, Y, Z and measure thedistances from the origin to specify the position of an object. In this case the position of the object can bespecified as (x,y,z). If we wish to specify the position or an event, also in terms of time then the object is described by the coordinates (x,y,z,t). There are two types of reference frames,



1) Inertial reference frame.2) Non- inertial reference frame.

Inertial and Non-inertial frames of reference:

Inertial frame of reference	inertial frame of reference
It is coordinate system in which Newton's laws of motion hold good.	It is coordinate system in which Newton`s laws of motion does not hold good.
The frame that moves with a uniform velocity with respect to another fixed inertial frame of reference.	A non-inertial frame is that which is accelerating linearly with respect to an inertial frame or which is rotating uniformly with respect to a fixed inertial frame.
The acceleration of a body is caused by real forces.	In this frame the acceleration is caused by fictitious or pseudo forces.

Distinguish between inertial and gravitational masses.

Inertial mass	Gravitational mass
The ratio of force to the acceleration of the body	The ratio of weight of the body to acceleration due to gravity
It is difficult to measure.	It is relatively easy to measure.
It is measured by physical balance.	It is measured by common balance.
The body must be under acceleration.	The body must be under the influence of gravitational force.

Weight & Mass:

Weight and mass are not the same terms. Weight is a force. Mass is a measure of the amount of matter an object contains.

The greater the weight, the greater the attraction between the object and earth.

GRAVITATIONAL POTENTIAL AND GRAVITATIONAL POTENTIAL ENERGY:

Gravitational intensity:

Suppose we keep a point mass **'m_0'** in the gravitational field.

The mass **'m_0'** experiences a gravitational force.

F = G

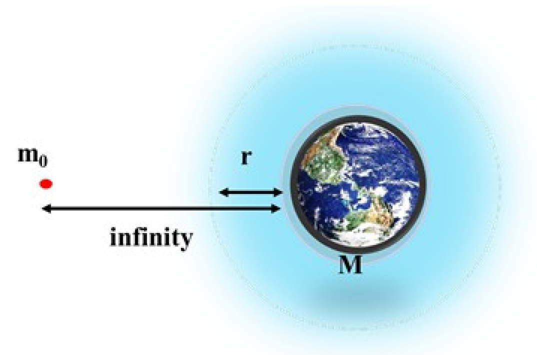
Gravitational intensity is given by E= =

The gravitational intensity at any point in a gravitational field is defined as the force acting on a unit mass placed at that point.

It is also called the strength of the gravitational field.

E =

Gravitational potential:



The gravitational potential of a body at any point in a gravitational field is defined as the work done to bring a unit mass from infinity to that point without acceleration.

V = -

Gravitational Potential Energy (P.E):

Gravitational P. E. = mass × gravitational potential

= mV

Gravitational potential V =**-**

V =

P.E. =

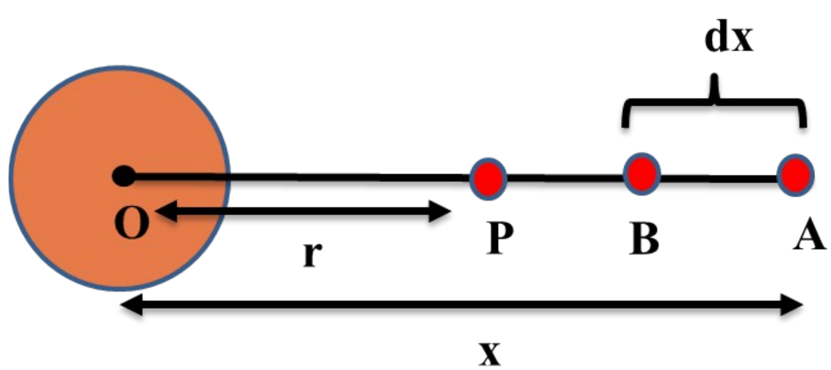
Gravitational potential energy at a height ‘h’ above the surface of the earth

Gravitational potential energy on the surface of the earth

P.E. =

The work done to displace a body from infinity to a point in the gravitational field against the gravitational field direction is called gravitational potential energy. It is denoted by U.

<u>Equation for gravitational potential energy:</u>



Consider a body of mass ‘m’ placed at a distance ‘x’ from the earth of mass ‘M’ & radius ‘R’.

∴ Gravitational force of attraction

F =

dw is the amount of work done to move the body from A to B at the distance dx against the gravitational force.

$\therefore$ dw = F.dx

$\therefore$ dw = .dx

The total work done in bringing the body from infinity to the point 'p' is given by,

W =

W =

(But $\int x^{-2}$ dx = -)

W = - GMmW = -GMm $\because$ = 0

W = -

$\therefore$ Gravitational potential energy**U = -**

$\therefore$ Here, **r = R + h**

$\therefore$ **U = -**

Workdone to lift the body from the height h_1 to h_2 is

W_1 = - ………….(1W_2 = - ………….(2)

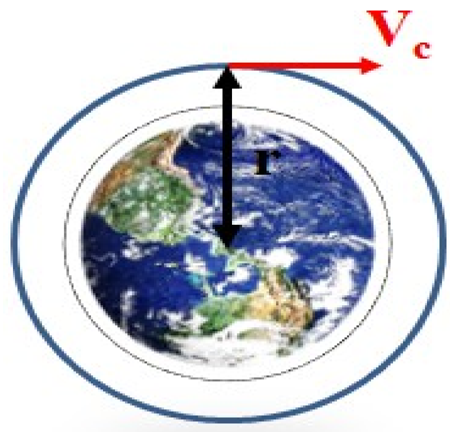
$W = W_2 – W_1$

W = GMm

<u>Kinetic energy of a satellite revolving around the earth:</u>

K.E. =

∴ K.E. = K.E. =



<u>ESCAPE VELOCITY AND ORBITAL VELOCITY</u>

<u>Escape velocity:</u>

The minimum vertical velocity required for an object to escape from the gravitational influence of a planet is known as escape velocity. It is denoted by $\mathbf{v_e}$.

<u>Expression for escape velocity:</u>

Consider a body (rocket) of mass ‘m’ on the surface of the planet (earth) of mass **‘m’** and radius ‘**R’**.

To find the minimum velocity, $\mathbf{v_e}$ which will cause the rocket to escape the earth's gravity, assume K E of distant rocket is also equal to zero.

As the body is moving away from the planet it is losing kinetic energy and gaining gravitational potential energy.

If the mass of the rocket is m, then the gravitational potential energy it possesses at the surface of the planet is

We know gravitational potential = **-**

If we bring the object from infinity to the surface of the planet. The work done is stored as gravitational potential.

Gravitational potential energy of the system = gravitational

potential $\times$mass of the body

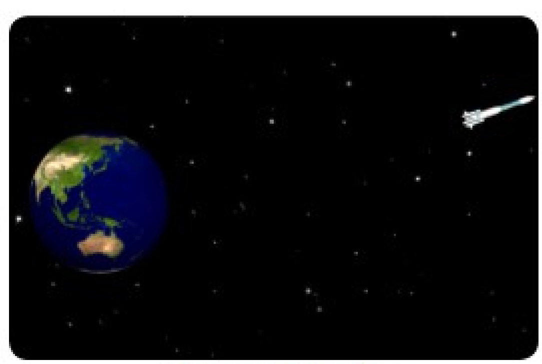
Gravitational potential energy = **-**

The negative sign indicates that the object is attracted by the planet.

Kinetic energy of the body K E = $\frac{1}{2}mv_e^2$

To escape from the gravitational field of the planet (earth) the object, which is projected, should have kinetic energy which is equal and opposite to the gravitational potential energy.

Therefore, $\frac{1}{2}mv_e^2$ **= -** (-)



Escape Velocity

Also, we know that, g = ,

⇒**g = GM**

Therefore**, V_e =**

The value of escape velocity is 11.2 Km/sec.

From the above expressions we can conclude that,

i) The value of escape velocity of an object does not depend upon the mass (m) of the object and its angle of projection.

ii) The value of escape velocity depends upon the mass and radius of the planet.

Orbital velocity:

The velocity required for an object to revolve around a planet in a circular orbit is called orbital velocity.

It is denoted by $\mathbf{V_o}$.

Expression for orbital velocity :

Let us consider an object of mass (satellite) to revolve around a planet (earth) of mass ‘M’ and radius ‘R’.

Let, m = Mass of the satellite

r = radius of circular orbit of the satellite

h = height of the satellite above surface of the earth.

R = radius of the earth

$\mathbf{v_o}$= orbital velocity

M = Mass of the earth.

The centripetal force on the satellite() =

The centripetalforce is supplied by the gravitational force of attraction between the earth and the satellite.

The gravitational force of attraction () =

In order that the satellite may revolve in its orbit,

Centripetal force = gravitational force of attraction.

=

Here r = R+h,

= = ,

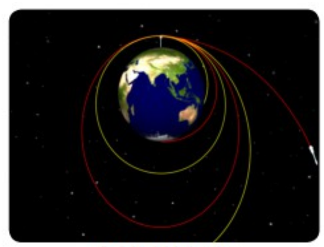
=

The expression gives us the orbital velocity of a satellite revolving around a planet at a height 'h'.

Here **(R+h)** is called the orbital radius. Orbital velocity does not depend upon the mass of the object. It depends on the mass of the planet, radius of the planet and height of the object from the surface.

When **'h'** is very small when compared to the radius of the planet R, then **(R+h)** becomes **'R'**.

In such case,



Orbital Velocity

We know that, **g =**

Therefore,**= ,**

Relation between escape velocity and orbital velocity:

When a body is projected from the surface of the planet it may revolve round the planet in a circular orbit when it is provided with orbital velocity. The body may escape from the gravitational influence of the planet if it is given escape velocity. The relation between escape velocity and orbital velocity can be found,

We know that, **V_0 = and v_e =**

v_e = v_e = v_o

Escape velocity **= ()**× (orbital velocity).

EARTH SATELITE:

- Earth satellites are objects which revolve around the earth.
- Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.
- In particular, their orbits around the earth are circular or elliptic.
- Moon is the only natural satellite of the earth with an ear circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.
- Advances in technology have enabled many countries including India to launch artificialearth satellites for practical use in fields like telecommunication, geophysics and meteorology.
- We will consider a satellite in a circular orbit of a distance (R_E + h) from the centre of the earth, where R = radiusof the earth. If m is the mass of the satellite and V its speed, the centripetal force required for this orbit is

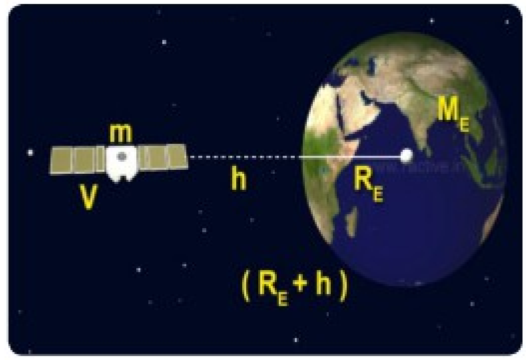
F(centripetal) = → (1)Directed towards the centre. This centripetal force is provided by the gravitational force, which is

F(Gravitation) = → (2)

Where M_E is the mass of the earth.

Equating R.H.S of Eqs. (1) and (2) and cancelling out m,we get

= → (3)



Earth sattilite

Thus V decreases as h increases.

From equation (3),thespeed V for h = 0 is

→ (4)

Time period of satellite:

The time take by satellite to complete one revolution around the planet is calle time period of satellite. It is denoted by T.

In every orbit, the satellite traverses a distance $2(R_E + h)$ with speed V. It's time period T is

T =

From equation (3) **=**

By substituting, we get

T = → (5)

Squaring on both sides, we get

= k

Where**k =** → (6)

For a satellite very close to the surface of earth h can be neglected, then ∝

Which is Kepler's law of periods.

By substituting G = g in equation (5), we get

T = 2π →(7)

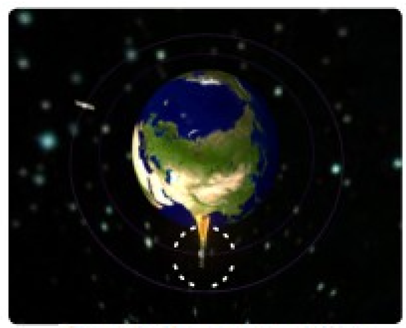
If we substitute the numerical values g = 9.8 m/and = 6400 km., we get the value of T is approximately 85 minutes.

GEOSTATIONARY AND POLARSATELLITES:

Geosatationary satellite:

If period of revolution of an artificial satellite is equal to the period of rotation of the earth then such a satellite is called Geostationary Satellite.

In other words a geostationary satellite takes 24 hours to complete one revolution around the earth.



Geostationary sattilite

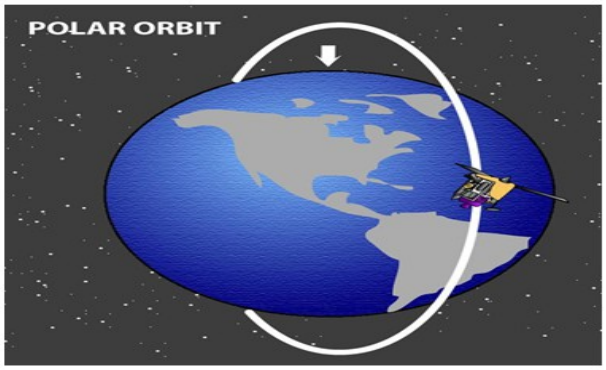
Uses of Geostationary Satellites:

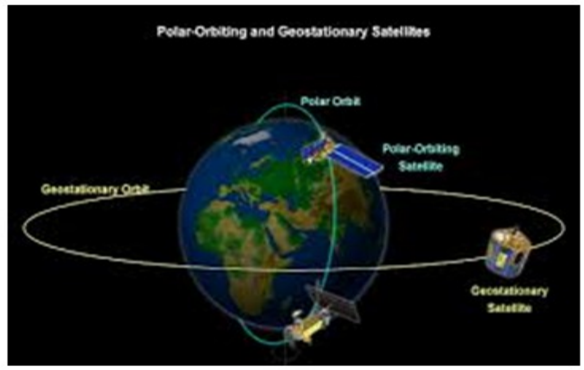
Geostationary satellites can be used

1. To study the upper layers of the atmosphere.
2. To forecast the changes in the atmosphere.
3. To know the shape and size of the earth.
4. To identify the minerals and natural resources present inside and on the surface of the earth.
5. To transmit the T.V. programs to distant places.
6. To study the properties of radio waves in the upper layers of the atmosphere.
7. To undertake space research i.e., to know about the planets, satellites and comets etc.

Polar satellites:

- The satellites which go around the poles of the earth in north-south direction are known as polar satellites.
- The orbit in which this satellite moves is known as polar orbit.
- The time period of the polar satellite is about 100 minutes.
- The height of the polar satellite from the surface of the earth is 500 to 800 km.
- The angle between the equatorial plane and the orbital plane of a polar satellite is 90^{0}.





Uses of polar satellites

- For ground water survey.
- For preparing wasteland maps.
- For identifying the sources of pollution.
- For detecting the potential fishing zones.
- To locate the position and movement of the troops of enemy.
- To locate the place and time for any nuclear explosion.

WEIGHTLESSNESS:

- When an object is in free fall, it is weightless and this phenomenon is usually called the phenomenon of weightlessness. In a satellite around the earth, every part and parcel of the satellite has acceleration towards the center of the earth which is exactly the value of earth's acceleration due to gravity at that position. Thus in the satellite everything inside it is in a state of free fall. This is just as if we were falling towards the earth from a height. Thus, in a manned satellite, people inside experience no gravity. Gravity for us defines the vertical direction and thus for them there are no horizontal or vertical directions, all directions are the same. Pictures of astronauts floating in a satellite show this fact.