Forced SHM

Forced Damped SHM

$ma = -kx - bv + F_0 \sin(\omega_F t + \phi_F)$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin(\omega_F t + \phi_F)$$

General Solution

$x(t) = x_{\text{transient}} + x_{\text{steady state}}$

Under-Damped: $b^2 < 4mk$

$$x = \underbrace{A_T e^{-\gamma t}\sin(\omega_T t + \phi_T)}_{\text{Transient}} + \underbrace{A_s \sin(\omega_F t + \phi_s)}_{\text{Steady State}}$$

A_T and ϕ_T from boundary conditions, γ and ω_T as defined in Damped SHM. ω_F is the frequency of external periodic force

$$A_s = \frac{F_0/m}{\sqrt{(\omega_F^2 - \omega_N^2)^2 + 4\gamma^2\omega_F^2}} \qquad \tan(\phi_s - \phi_F) = \frac{2\gamma\omega_F}{\omega_F^2 - \omega_N^2}$$

Resonance

$$A_{s,max} = \frac{F_0/m}{2\gamma\sqrt{\omega_N^2 - \gamma^2}} = \frac{F_0}{b\sqrt{\omega_N^2 - \gamma^2}}$$

$$\omega_{F,max} = \sqrt{\omega_N^2 - 2\gamma^2}$$

Quality Factor

If $\gamma \ll \omega_N$, $\Delta\omega = |\omega_{F,max} - \omega_{F,hp}|$

$\omega_{F,hp}$ *is a* ω_F *s.t.* $A_s = A_{s,max}/\sqrt{2}$

$\Delta\omega \cong \gamma$

$$Q \cong \frac{\omega_N}{2\Delta\omega} \cong \frac{\omega_N}{2\gamma} = \frac{\sqrt{mk}}{b}$$

Run Away Resonance

For $b = 0$, above equations don't work. Steady state is never reached. Amplitude increases indefinitely