Chapter-1
Electrostatics

Content: Coulomb's law (scalar and vector forms), electric field, electric field due to point charge, electric dipole and its moment, electric fields along the axial and equatorial lines, concept of dielectric and dielectric constant, Gauss's theorem and its application to find electric field due to an infinite wire and plane sheet of charge, conductors and insulators, force and torque experienced by a dipole(in uniform electric field), capacitance, parallel plate capacitor with air/dielectric medium between the plates, series and parallel combinations of capacitors, energy stored of a capacitor, numerical problems.

Introduction: Electrostatics, as the name implies, is the study of stationary electric charges.
A rod of plastic rubbed with fur or a rod of glass rubbed with silk will attract small pieces of paper and is said to be electrically charged. The charge on plastic rubbed with fur is defined as negative, and the charge on glass rubbed with silk is defined as positive.

Electric charge

Charge is that property of an object by virtue of which it applies electrostatic force of interaction on other charged objects.

Charges are of two kinds
(i) Positive charge
(ii) Negative charge
SI unit of electric charge is coulomb (C). CGS unit of charge is stat coulomb and ab coulomb.

1 coulomb= 3×10^{9} stat coulomb

1 ab coulomb= 10 coulombs

Electrically charged objects have several important characteristics:

- Like charges repel one another; that is, positive repels positive and negative repels negative.
- Unlike charges attract each another; that is, positive attracts negative.

Characteristics of Charge

- **Charge is conserved:** A neutral object has no net charge. If the plastic rod and fur are initially neutral, when the rod becomes charged by the fur, a negative charge is transferred from the fur to the rod. The net negative charge on the rod is equal to the net positive charge on the fur.
- **The additive nature of charge** means that the entire electric charge of a system is equal to the algebraic sum of electric charges located in the system. This is the law of superimposition of electric charge.
- **The quantization of electric charge** means that the total charge of the body is always an integral multiple of a basic quantum of charge (e) i.e

$$q = \pm ne \qquad \text{where,} \qquad n = 1, 2, 3, \ldots$$
$$e = \text{charge on an electron} = 1.6 \times 10^{-19}\text{C}$$

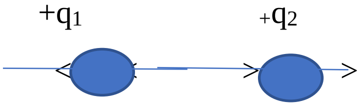
The basic cause of quantization of electric charge is that during rubbing only an integral number of electrons can be transferred from one body to another.

1.1 <u>Coulomb's law (scalar and vector forms):</u>

1.1.1 Coulomb's law in electrostatics (scalar form):

According to Coulomb's law, the force of attraction **or** repulsion between the two-point charges is

i) directly proportional to the product of the charges and
ii) inversely proportional to the square of distance between their (charges) centres.



Suppose two-point charges q_1 and q_2 are separated in vacuum by distance r.

$F \propto q_1 q_2$

$F \propto \frac{1}{r^2}$

$\therefore F \propto \frac{q_1 q_2}{r^2}$ ➔ $F = k\frac{q_1 q_2}{r^2}$

$k = F r^2 / q_1 q_2$

Where k is electrostatics force constant. The value of k depends upon the nature of medium separating the charges and on the system of units.

When the charges are situated in free space (air/vacuum)

In CGS system, $k = 1$

In SI system, $k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \mathrm{Nm^2C^{-2}}$

$\varepsilon_o = \frac{1}{4\pi k} = 8.854\times10^{-12}\ \mathrm{N^{-1}m^{-2}\ C^2}$

$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$ (Air/Vacuum)

$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$ (Medium)

$k = \varepsilon\ /\varepsilon_o$

Where ε_o is the absolute electrical permittivity of the free space and it is equal to $8.854\times10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}$. It is the measure of the property of medium surrounding electric charges which determine the force between the charges. More is the permittivity of medium, less is the Coulomb’s force.

$\therefore F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$ (scalar form of Coulomb’s law)

A (vector) = A (magnitude) x unit vector

$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^3}\vec{r}$

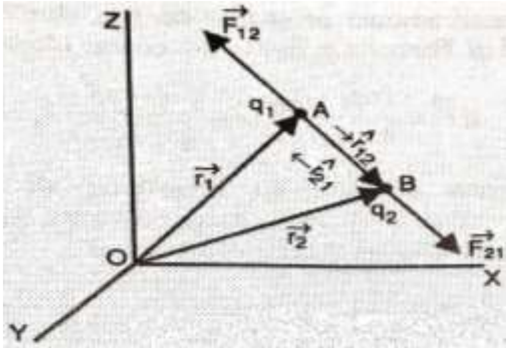
$\vec{F} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^3}\vec{r}$

Unit vector= Is a vector whose magnitude is unit (one) and direction is same as that of the vector

$\vec{A} = A\hat{A}$

$\vec{r}\ /r = \hat{r}$

1.1.2 Coulomb's law in vector form:



Let $\vec{r}_1 = \overrightarrow{OA}$= Position vector of charge q_1.

And $\vec{r}_2 = \overrightarrow{OB}$= Position vector of charge q_2.

$r_2 = r_1 + r_{12} \Rightarrow r_{12} = r_2 - r_1$

$\therefore$ The vector leading from q_1 to q_2 is $\overrightarrow{AB} = \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$-r_{21} + r_1 = r_2$

$-r_{21} + r_1 = r_2$

$-r_{21} = r_2 - r_1$

$r_{21} = -(r_2 - r_1)$

$r_{21} = r_1 - r_2$

Similarly, vector leading from q_2 to q_1 is $\overrightarrow{BA} = \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

So, unit vectors along AB is $\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$ and along BA is $\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}}$

If $\vec{F}_{12}$ =Force on q_2 due to q_1

$\vec{F}_{21}$ =Force on q_1 due to q_2

$\therefore \vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{AB^2}$ along AB

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{{r_{12}}^2}\hat{r}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{{r_{12}}^3}\vec{r}_{12}$$

$\vec{F}_{21}$ =Force on q_1 due to q_2

$\therefore \vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{BA^2}$ along BA

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{{r_{21}}^3}\vec{r}_{21}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Similarly,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

$\mathbf{F_{12} = - F_{21}}$

$\mathbf{F_{21} = - F_{12}}$

1.2 <u>Electric field</u>

Electric lines of forces

Properties/characteristics

1. Outward from +ve charge and inward from negative charge
2. They never cancel each other
3. They are always perpendicular at the surface

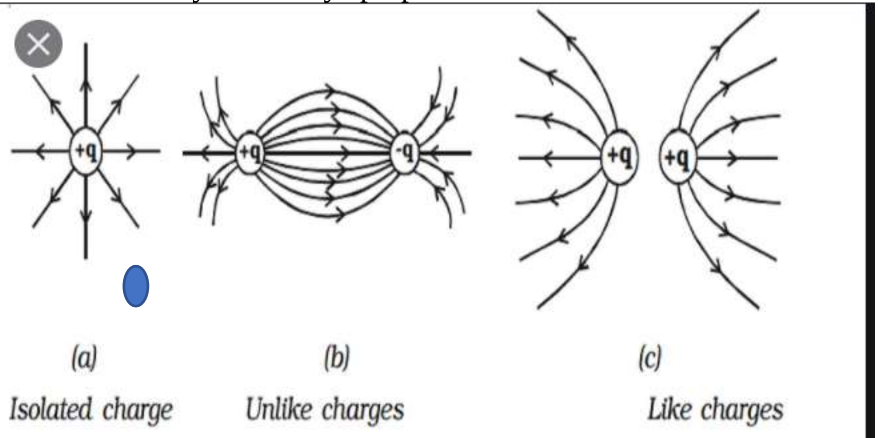
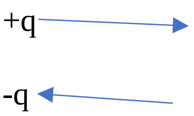


Fig1.5 Lines of Forces



Test charge

When a small **positive test charge** is brought near a large positive charge, it experiences a force directed away from the large charge. This force is due to electric field set up by the source charge.
We can define electric field due to a given charge as the space around the charge in which the electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.

The electric field intensity **(E) is defined as the force per unit charge exerted on a small positive test charge (q_o) placed at that point**. Mathematically,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q\, q_0}{r^2}$$

q- source charge

q_0 – test charge

$$E = F/q_0 = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

$$\vec{E} = \lim_{q_o \to 0} \frac{\vec{F}}{q_o}$$

E= N/C = NC^{-1} (SI units of E)

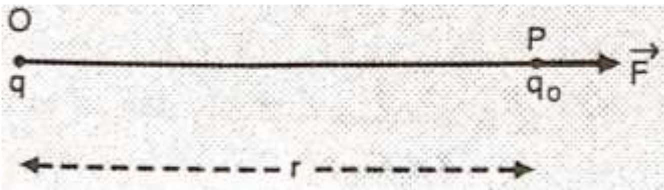
Note that both the force and electric field are vector quantities.

The test charge is required to be small so that the field of the test charge does not affect the field of the set charges being examined.

The SI unit for electric field is Newtons per coulomb (N/C)

1.2.1 Electric field intensity at a point due to point charge:

Suppose P is the point where electric field has to be calculated due to charge q at O. Let $\overrightarrow{OP} = \vec{r}$.



Imagine a small positive test charge q_o at P. According to coulomb's law, force experienced by test charge q_o is

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{qq_0}{r^2}\hat{r}$$

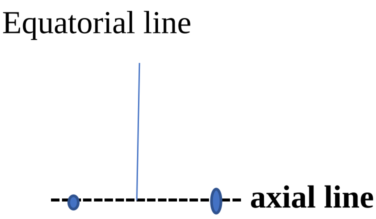
As $\vec{E} = \frac{\vec{F}}{q_o}$

∴ $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$

$E \, \alpha \, 1/r^2$

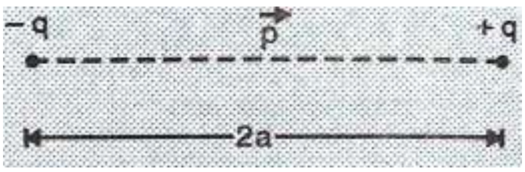


1.3 <u>Electric Dipole and its dipole moment</u>

Electric dipole: Is a combination of two **equal and opposite** charges separated by a small distance

Ideal dipole = when the distance between charges is minimum

Electric dipole moment($\vec{p}$) : the magnitude of electric dipole moment is defined as the product of magnitude of either charge and the distance of separation between the two charges.



$$|\vec{p}| = q \times 2a$$

$$p = q \times 2a = (q\,2a)$$

It is vector quantity and directed from –ve charge to +ve charge (-q to +q).

Its SI unit is coulomb metre (C-m)

1.4 <u>Electric field due to an electric dipole</u>

i) along the axial line

ii) along equatorial line

1.4.1 Electric field intensity due to dipole at a point lying on axial line:

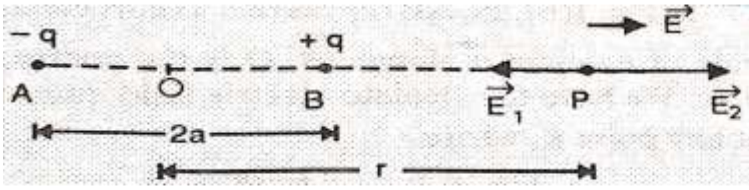
Consider an electric dipole consisting of two-point charges -q and +q separated by some distance 2a. Let P be an observation point on axial line such that its distance from centre of the dipole is r.

Electric field due to a point charge $E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$

$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{-q}{AP^2}$ Electric field due to –q charge (placed at A) at a point of observation P

If $\vec{E}_1$ is the electric field intensity at P due to charge -q at A then

$E_1 = |\vec{E}_1| = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+a)^2}$ directed along PA =r+a



Again, if $\vec{E}_2$ is the electric field intensity at P due to charge +q at B then

I------------------------ I

BP= OP –OB

BP= r-a

$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2}$

$E_2 = |\vec{E}_2| = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-a)^2}$ directed along BP = r-a

Clearly, $E_2 > E_1$

(r+a) > (r-a)

So, $\mathbf{E_{axial}}$ (at P) $= \mathbf{E_2 - E_1}$

$$E_{axial} = \frac{q}{4\pi\varepsilon_o(r-a)^2} - \frac{q}{4\pi\varepsilon_o(r+a)^2} = \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q(4ar)}{4\pi\varepsilon_o(r^2-a^2)^2}$$

$= (r+a)^2 - (r-a)^2 / (r-a)^2 (r+a)^2$

$(r-a)^2 (r+a)^2 = (r^2 - a^2)^2$

$(r+a)^2 - (r-a)^2 =$

$\frac{q(4ar)}{4\pi\varepsilon_o(r^2-a^2)^2} = \frac{q2a\,2r)}{4\pi\varepsilon_o(r^2-a^2)^2} = \frac{p\,2r}{4\pi\varepsilon_o(r^2-a^2)^2}$

Since, $p = q(2a)$

So, E_{axial} (at P) $= \frac{1}{4\pi\varepsilon_o}\frac{2pr}{(r^2-a^2)^2}$

For short dipole, $r \gg a$, neglect 'a' as compared to r

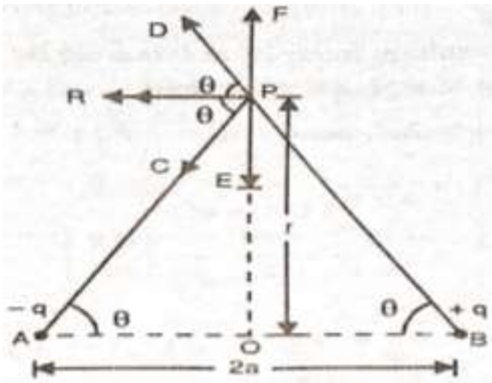
$So,\ E_{axial} = \frac{1}{4\pi\varepsilon_o}\frac{2pr}{(r^2)^2} = \frac{1}{4\pi\varepsilon_o}\frac{2pr}{r^4}$ ➔ $E_{axial} = \frac{2p}{4\pi\varepsilon_o r^3}$

Variation of E w.r.t. position

$E\ \alpha\ 1/r^2$ (point charge)

$E\ \alpha\ 1/r^3$ (dipole)

1.4.2 Electric field due to dipole at a point lying on the equatorial line:



Consider an electric dipole consisting of two-point charges -q and +q separated by distance 2a. Let P be an observation point on equatorial line such that its distance from mid-point O of the electric dipole is r. Let $\angle PBA = \theta$.

If $\vec{E}_1$ is the electric field intensity at P due to charge -q then

$E_1 = |\vec{E}_1| = \frac{1}{4\pi\varepsilon_o}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}$ directed along PA

Again, if $\vec{E}_2$ is the electric field intensity at P due to charge +q then

$E_2 = |\vec{E}_2| = \frac{1}{4\pi\varepsilon_o}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r^2+a^2)}$ directed along PD

Clearly, $E_1 = E_2$

AO= a

OP =r

$AP^2 = AO^2 + OP^2$

$AP^2 = r^2 + a^2$

$AP= (a^2 + r^2)^{1/2}$

$AP=BP = (a^2 + r^2)^{1/2}$

$PF= E_2 \sin\theta$

$PE= E_1 \sin\theta$

Because $E_1 = E_2$

PF=PE and opposite so they cancel each other

$PR_1 = E_1 \cos\theta$

$PR_2 = E_2 \cos\theta$

$E= E_1 \cos\theta + E_2 \cos\theta$

E (on equitorail line) = $2 E \cos\theta$

Let us resolve $\vec{E}_1$ and $\vec{E}_2$ into two components in two mutually perpendicular directions. Components of $\vec{E}_1$ and $\vec{E}_2$ along the equatorial line cancel each other but the components perpendicular to equatorial line get added up because they act in same direction. So, magnitude of resultant intensity $\vec{E}$ at P,

$$E = E_1\cos\theta + E_2\cos\theta = 2E_1\cos\theta = 2\frac{q}{4\pi\varepsilon_o(r^2+a^2)}\frac{a}{\sqrt{(r^2+a^2)}}$$

$$\cos\theta = \text{base/hypotunuse}$$

$$E_{equitorial} = \frac{p}{4\pi\varepsilon_o(r^2+a^2)^{3/2}}$$

1+ ½ = 3/2

For short dipole, $r \gg a$

$$E_{equitorial} = \frac{p}{4\pi\varepsilon_o r^3}$$

$$E_{axial} = \frac{2p}{4\pi\varepsilon_o r^3}$$

1.5 <u>Concept of Dielectric and dielectric constant</u>

1.5.1 Dielectrics:

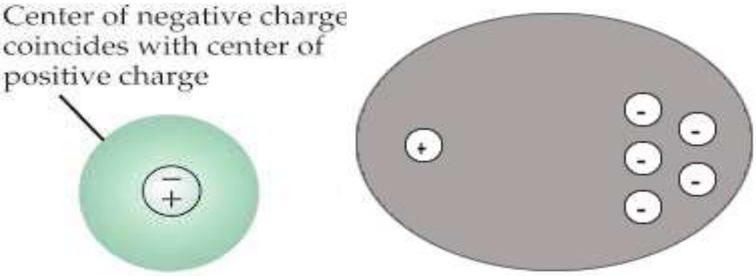
A dielectric is an insulating material in which all the electrons are tightly bound to the nuclei of the atoms and there are no free electrons available for the conduction of current. So, the electrical conductivity of a dielectric is very low. The conductivity of an ideal dielectric is zero. Materials such as glass, polymers, mica, oil and paper are examples of dielectrics. They prevent flow of current through them. So, they can be used for insulating purposes.

Types of dielectrics:

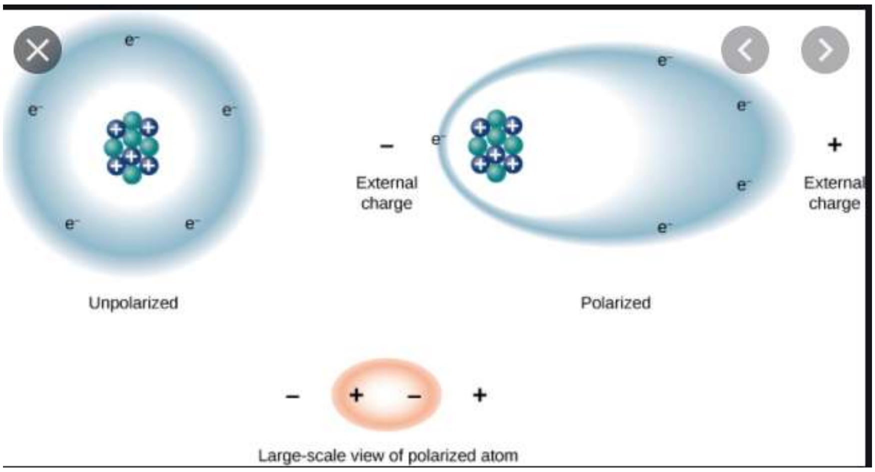
A molecule is a neutral system in which the algebraic sum of all the charges is zero. Based on the dipole moment, the molecules of dielectrics are termed as non-polar and polar molecules. Accordingly, these dielectrics are referred to as non-polar and polar dielectrics.

Non-polar dielectrics:

A non-polar molecule is one in which the centre of gravity of the positive charges (protons) and negative charges (electrons) coincide. So, such molecule does not have any permanent dipole moment, as shown in figure. Common examples are oxygen (O_2), nitrogen (N_2) and hydrogen (H_2). So, the dielectrics having non-polar molecules are known as non-polar dielectrics.



Polar dielectrics: A polar molecule is the one in which the centre of gravity of the positive charges is separated by finite distance from that of the negative charges. So, these molecules possess permanent electric dipole moment as shown in figure. Examples are H_2O, HCl and NH_3. The dielectrics having polar molecules are known as polar dielectrics.



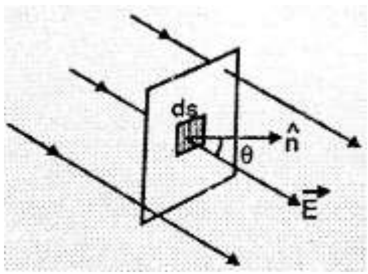
1.5.2 Dielectric constant: It is defined as the ratio of the permittivity of a substance to the permittivity of free space.

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$$

where ε is the absolute permittivity of the substance, ε_r is the relative permittivity and ε_o is the permittivity in free space. It is dimensionless quantity.

Question: Find the absolute permittivity of mica, if its relative permittivity is 8?
Solution: The expression for the absolute permittivity of the substance is as follows;
Here, ε is the absolute permittivity of the substance, ε_r is the relative permittivity and ε_o is the permittivity in free space.
It is given in the problem that the relative permittivity of mica is 8, Put $\varepsilon_r = 8$
$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$ ➔ $\varepsilon = \varepsilon_r\varepsilon_o = 8\varepsilon_o$
Therefore, the value of the absolute permittivity of mica is $8\varepsilon_o$.

1.6 <u>Gauss's theorem and its application to find electric field due to an infinite wire and plane sheet of charge:</u>



1.6.1 Electric Flux: Electric flux over an area in an electric field represents the total number of electric field lines crossing the area. It is represented by ϕ_E.
It is scalar quantity. The unit of ϕ_E is Nm^2C^{-1}.
If $\vec{E}$ is electric field intensity over a small area element $d\vec{S}$ and θ is angle between $\vec{E}$ and outdrawn normal to area element. So, electric flux through this element is

$$d\phi_E = \vec{E}.d\vec{S} = E\ dS\ cos\theta$$

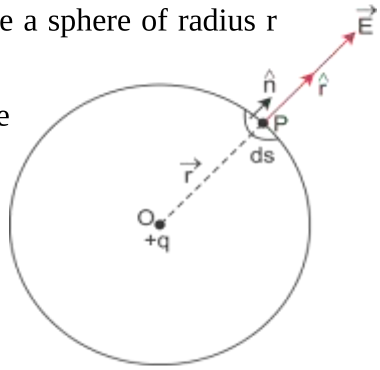
Value of total electric flux emerging out of a closed surface is given by $\phi_E = \oint_S d\phi_E = \oint_S \vec{E}.d\vec{S}$

1.6.2 Gauss's theorem in electrostatics: The surface integral of electrostatic field $\vec{E}$ over any closed surface S enclosing a volume V in vacuum i.e., total electric flux over the closed surface S in vacuum is $\frac{1}{\varepsilon_o}$ times the total charge (q) enclosed by closed surface S.

$$\phi_E = \oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_o}$$

Where q is the total charge enclosed by the surface S and ε_o is the permittivity of free surface.

Proof: Consider an isolated point charge +q placed at point O. Let surface S be a sphere of radius r around the charge +q.



Then electric field intensity due to charge +q at every point on the surface of the sphere is given by $\vec{E} = \frac{1}{4\pi\,\varepsilon_o}\frac{q}{r^2}\hat{r}$

Now consider a small element of area dS with area vector $d\vec{S}$ normal to the surface of the area element. So, electric flux through the area element is given by

$$d\phi = \vec{E}.d\vec{S} = \frac{1}{4\pi\,\varepsilon_o}\frac{q}{r^2}\hat{r}.dS\hat{n} = \frac{1}{4\pi\,\varepsilon_o r^2}qdS\hat{r}.\hat{n}$$

where $\hat{n}$ is the unit vector perpendicular to the surface of the area element. Since, $\hat{r}$ and $\hat{n}$ are along the same direction. So, $\hat{r}.\hat{n} = 1$

hence, $d\phi = \vec{E}.d\vec{S} = \frac{1}{4\pi\,\varepsilon_o}\frac{qdS}{r^2}$

Now electric flux over the entire closed surface S is given by

$$\phi = \oint_S d\phi = \oint_S \frac{1}{4\pi\,\varepsilon_o}\frac{qdS}{r^2} = \frac{1}{4\pi\,\varepsilon_o}\frac{q}{r^2}\oint_S dS$$

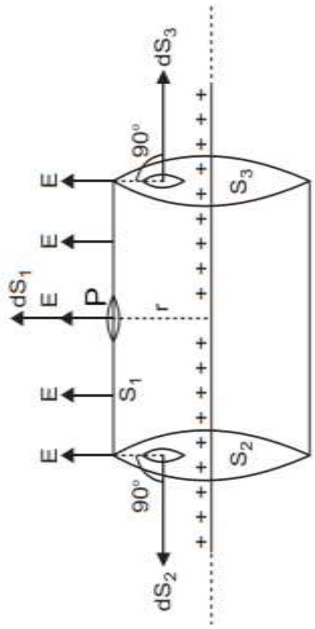
But $\oint_S dS$= surface area of the sphere of radius r= $4\pi r^2$

So, $\phi = \frac{1}{4\pi\,\varepsilon_o}\frac{q}{r^2}(4\pi r^2) = \frac{q}{\varepsilon_o}$

This is the expression of Gauss's law or theorem in electrostatics.

1.6.3 Applications:

(i) Electric field intensity due to an infinitely long straight uniformly charged wire:



Consider an infinite and thin straight wire having uniform linear charge density (i.e., charge per unit length) λ. This wire is symmetrical about the axis of the wire. To calculate the electric field intensity $\vec{E}$ at a point P, distant r from the line. Draw an imaginary cylinder (Gaussian surface) of radius r and length l around the charged wire.
The charge enclosed by the Gaussian surface $q = \lambda l$

According to Gauss's theorem, $\oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_o} = \frac{\lambda l}{\varepsilon_o}$ (1)

The cylinder Gaussian surface is divided into three parts S_1, S_2 and S_3, i.e., curved surface, left and right face respectively.

So, Equation (1) can be written as

$$\oint_S \vec{E}.d\vec{S} = \int_{S_1}\vec{E}.d\vec{S} + \int_{S_2}\vec{E}.d\vec{S} + \int_{S_3}\vec{E}.d\vec{S} = \frac{\lambda l}{\varepsilon_o} \quad (2)$$

For surfaces S_2 and S_3, angle between $\vec{E}$ and $d\vec{S}$ is 90^o. So, $\vec{E}.d\vec{S} = EdScos90^o = 0$ for these surfaces. So, electric flux will cross through the curved surface only.

So, Equation (2) becomes

$$\oint_S \vec{E}.d\vec{S} = \int_{S_1}\vec{E}.d\vec{S} = \frac{\lambda l}{\varepsilon_o} \quad \Rightarrow \quad \int_{S_1} EdScos0^o = \frac{\lambda l}{\varepsilon_o} \quad \Rightarrow \quad \int_{S_1} EdS = \frac{\lambda l}{\varepsilon_o}$$

Since, E is constant over the Gaussian surface.

So, $E\int_{S_1} dS = \frac{\lambda l}{\varepsilon_o}$ ➔ $E(2\pi r l) = \frac{\lambda l}{\varepsilon_o}$

($\int_{S_1} dS$ = area of the curved surface of the cylinder $= 2\pi r l$)

$E = \frac{\lambda}{2\pi\varepsilon_o r}$ clearly, $E \propto \frac{1}{r}$

In vector form, $\vec{E} = \frac{\lambda}{2\pi\varepsilon_o r}\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to the curved surface of the wire.

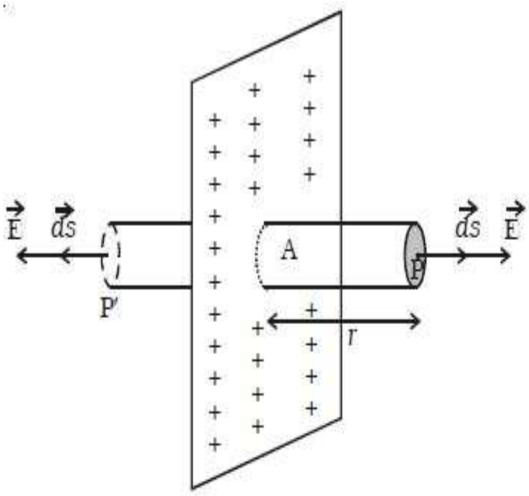
If $\lambda > 0$, the direction of electric field at every point is radially outwards.

If $\lambda < 0$, the direction of electric field at every point is radially inwards.

(ii) <u>Electric field intensity due to uniformly charged infinite plane sheet:</u>

Consider a thin infinite plane sheet having uniform surface charge density (i.e., charge per unit area) 'σ'. We have to calculate electric field intensity $\vec{E}$ at any point P distant r from the sheet. Draw a imaginary cylinder of cross-sectional area dS and length r on each side of sheet. Electric field $\vec{E}$ is perpendicular to the sheet. At the two cylindrical edges P and P', $\vec{E}$ and outward normal $\hat{n}$ are parallel to each other.

So, electric flux over these edges= $2\vec{E}.\hat{n}dS = 2EdS$



But on the curved surface of the cylinder, $\vec{E}$ and outward normal $\hat{n}$ are perpendicular to each other. So, no contribution to electric flux is made by the curved surface of the cylinder.

So, total electric flux over the entire surface of the cylinder $\phi_E = 2EdS$

Total charge enclosed by the cylinder, $q = \sigma dS$

Acc. To Gauss's theorem, $\phi_E = 2EdS = \frac{q}{\varepsilon_o} = \frac{\sigma dS}{\varepsilon_o}$

So, $E = \frac{\sigma}{2\varepsilon_o}$

In vector form, $\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{n}$ where $\hat{n}$ is a unit vector perpendicular to plane of the sheet pointing away from it.

Note: (i) the electric field is directed away from the sheet if it is positively charged and it is directed towards the sheet if it is negatively charged.

(ii) E is independent of r, the distance of the point from the plane charged sheet.

(iii) It depends upon surface charge density and acts perpendicular to the sheet. i.e., $E \propto \sigma$.

1.7 <u>Conductors and insulators:</u>

1.7.1 Conductors: an electrical conductor is defined as materials that allow electricity to flow through them easily.

Examples of Conductors

- Material such as silver is the best conductor of electricity. But it is costly and so, we don't use silver in industries and transmission of electricity.
- Copper, Gold, and Aluminium are good conductors of electricity. We use them in electric circuits and systems in the form of wires.
- Mercury is an excellent liquid conductor.

1.7.2 Insulators: Insulators are the materials or substances which resist or don't allow the current to flow through them.

Examples of Insulators

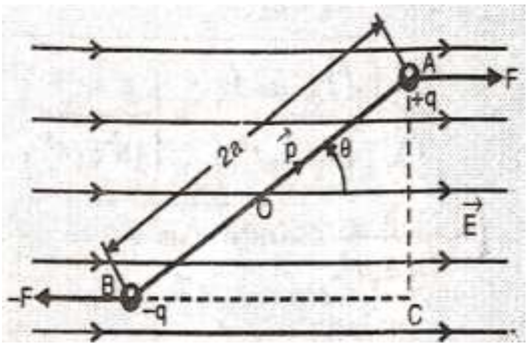
- Glass is the best insulator as it has the highest resistivity.
- Plastic is a good insulator and it finds its use in making a number of things.
- Rubber is a common material used in making tyres, fire-resistant clothes and slippers. This is because it is a very good insulator.

1.7.3 Differences Between Conductor and Insulators:

Conductor	Insulator
Materials that permit electricity or heat to pass through it	Materials that do not permit heat and electricity to pass through it
A few examples of a conductor are silver, aluminium and iron	A few examples of an insulator are paper, wood and rubber
The electrons move freely within the conductor	The electrons do not move freely within the insulator

1.8. <u>Force and torque experienced by a dipole(in uniform electric field):</u>

Consider an electric dipole consisting of two equal and opposite point charge -q at A and +q at B separated by a small distance AB=2a, having dipole moment $|\vec{p}| = q \times 2a$



Let this dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.

Force on charge +q at A= $q\vec{E}$, along the direction of $\vec{E}$.

Force on charge -q at B= $q\vec{E}$, in a direction apposite to $\vec{E}$.

Since $\vec{E}$ is uniform, ∴ net force on the dipole is $(qE - qE) = 0$,

As the forces are equal, unlike and parallel acting at different points. So, they form a couple which rotates the dipole in the clockwise direction.

Draw, AC perpendicular $\vec{E}$, So, perpendicular distance between the forces = arm of couple AC

$\text{Torque} = \text{moment of force}$

$=\text{Force} \times \text{perpendicular distance AC}$

$=F\times AC=F\times AB\sin\theta$

$=F\times 2a\sin\theta = qE(2a\sin\theta)$

$=(q\times 2a)E\sin\theta$ ➔ $\tau = pE\sin\theta$

In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$

Special cases: (i) When $\vec{p}$ is along $\vec{E}$, $\theta = 0^o$ and $\tau = pE\sin 0^o = 0$

The dipole is on stable equilibrium.

(ii) When dipole is held in a direction opposite to $\vec{E}$, the torque would turn the dipole through 180^o and dipole will be in an unstable equilibrium.

(iii) Torque will be maximum when $\theta = 90^o$, $\tau_{max} = pE\sin 90^o = pE$

1.9 Capacitance:

Capacitance is the ability of conductor to hold the charge and associated electrical energy.

We know that the charge given to a conductor increases its potential, i.e., $Q \propto V$ ➔ $Q = CV$

Where C is a constant of proportionality called capacity or capacitance of conductor.

Symbol of capacitor: The symbol of capacitor is given as

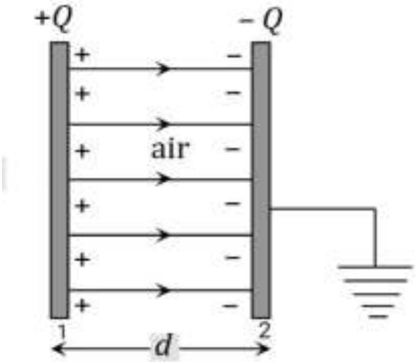
Units: S.I. unit is $\frac{coulomb}{volt} = farad\ (F)$

Commonly used units are mF, µF, nF and pF. (1mF=10^{-3} F, 1µF= 10^{-6} F, 1nF=10^{-9} F, 1pF= 10^{-12}F)

Dimensional formula: $[M^{-1}L^{-2}T^{4}A^{2}]$

1.10 Parallel plate capacitor with air/dielectric medium between the plates:

It consists of two thin conducting plates 1 and 2 each of area A held parallel to each other, distance d apart. One of the plates, 1 is insulated and other plate 2 is earth connected. When a charge Q is given to the insulated plate 1, then a charge -Q is induced on the nearer face of plate 2 and +Q is induced on the farther face of plate 2. As plate 2 is earthed, the charge +Q being free flows to earth. Plate 1 has surface charge density $\sigma = \frac{Q}{A}$ and plate 2 has surface charge density (-σ). In the region on left of plate 1 and on right of plate 2, the electric field is zero. But in the region between the plates separated by air/vacuum, electric field intensity is $E = \frac{\sigma}{\varepsilon_o} = \frac{1}{\varepsilon_0}\frac{Q}{A}$



The potential difference V between the plates is product of the electric field times the distance between the plates, then $V = E \times d = \frac{1}{\epsilon_0}\frac{Q}{A} \times d$

The capacity C_o of parallel plate capacitor is given by $C_o = \frac{Q}{V} = \frac{Q\varepsilon_o A}{Qd} = \frac{\varepsilon_o A}{d}$

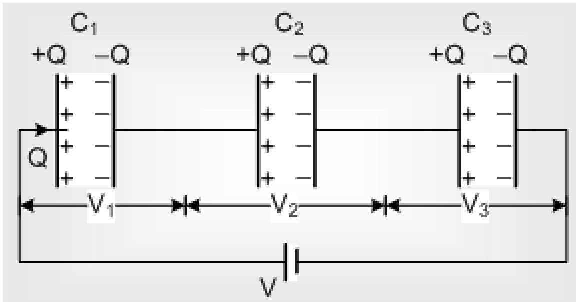
Note: If a dielectric of permittivity ε is placed between the conducting plates,

then capacitance $C = \frac{\varepsilon A}{d}$ where $\varepsilon = \varepsilon_o K$ ➔ $C = \frac{\varepsilon_o K A}{d}$ $C = KC_o$

1.11 Series and parallel combinations of capacitors:

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called series and parallel, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

1.11.1 Capacitors in series: Three capacitors of capacities C_1, C_2 and C_3 are connected in series. V is the potential difference applied across the series combination.



Each capacitor receives t 6he same amount of charge Q. As, their capacities are different. So, potential difference across the three capacitors are different.

∴ $V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}$ and $V_3 = \frac{Q}{C_3}$

If C_s is the total capacitance of the combination, then $V = \frac{Q}{C_s}$

As $V = V_1 + V_2 + V_3$

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For n capacitors connected in series, total capacitance would be $\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i}$

i.e., the reciprocal of equivalent capacitance of any number of capacitors joined in series is equal to sum of the reciprocals of individual capacitances.

Note: In a series combination, charge can move along only one path. So, that is why charge on each capacitor is same.

1.11.2 Capacitors in parallel: Three capacitors of capacitances $C_1, C_2\ and\ C_3$ are connected in parallel. V is the potential difference applied across the parallel combination.

As potential difference across the three capacitors is the same, So, charges on them will be different say $Q_1, Q_2\ and\ Q_3$ such that

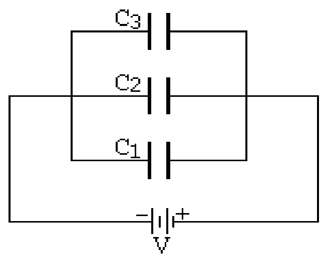
$Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V$

the total charge Q is the sum of the individual charges, then

$Q = Q_1 + Q_2 + Q_3$

If C_p is the equivalent capacitance in parallel, then $Q = C_pV$

So, $C_pV = C_1V + C_3V + C_3V = (C_1 + C_2 + C_3)V \quad \therefore \quad C_p = C_1 + C_2 + C_3$



parallel combination of capacitors

In general, when n capacitors are connected in parallel, then $C_p = \sum_{i=1}^{n} C_i$ i.e., equivalent capacitance of any number of capacitors joined is equal to sum of the individual capacitances.

Note: In a parallel combination, both the plates of every capacitor are connected to the same battery. So, potential difference across each capacitor is same.

1.12 <u>Energy stored in a capacitor:</u>

A capacitor is a system of two conductors carrying charges Q and -Q held some distance apart. Suppose the conductors A and B are initially uncharged. Let positive charge be transferred from conductor B to conductor A in very small instalments of dq each till conductor A gets charge Q. By charge conservation, conductor B would acquire charge -Q. At any intermediate stage, suppose charge on conductor A is +q and charge on conductor is -q. So, potential difference between conductors A and B is $\left(\frac{q}{C}\right)$, where C is the capacity of the capacitor. Small amount of work done in giving an additional charge dq to the capacitor is $dw = \left(\frac{q}{C}\right)(dq)$

Total work done in giving a charge Q to the capacitor

$$W = \int_{q=0}^{q=Q} \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{C}\left[\frac{Q^2}{2} - 0\right] = \frac{1}{C}\left(\frac{Q^2}{2}\right) = \frac{Q^2}{2C}$$

This work is stored in the form of potential energy (U) of the capacitor, $U = W = \frac{Q^2}{2C}$

Put $Q = CV$ So, U$= \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$ ➔ $U = \frac{1}{2}CV^2$

Put $C = \frac{Q}{V}$ So, $U = \frac{1}{2}QV$

So, $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

1.12.1 Energy density of a parallel plate capacitor:

Energy density (u) is defined as the total energy stored per unit volume of the capacitor.

i.e., $u = \frac{total\ energy\ (U)}{Volume} = \frac{\frac{1}{2}CV^2}{Ad}$

Using $C = \frac{\epsilon_o A}{d}$ and $V = Ed$

We get, u=$\frac{1}{2}\left(\frac{\epsilon_o A}{d}\right)\left(\frac{E^2 d^2}{Ad}\right) = \frac{1}{2}\varepsilon_o E^2$ ➔ $u = \frac{1}{2}\varepsilon_o E^2$

1.12.2 Total energy stored in a combination of capacitors:

In series combination of capacitors, every capacitor carries the same charge Q i.e., Q is constant.

∴ Total energy $U = \frac{Q^2}{2C_s} = \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots.\right]$

$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + \ldots$$

$$U = U_1 + U_1 + U_1 + \ldots..$$

In parallel combination of capacitors, potential difference across each capacitor is same i.e., V is constant.

∴ Total energy, $U = \frac{1}{2}C_p V^2 = \frac{1}{2}(C_1 + C_2 + C_3 + \ldots.)V^2$

$$= \frac{1}{2}C_1 V^2 + \frac{1}{2}C_2 V^2 + \frac{1}{2}C_3 V^2 + \cdots$$

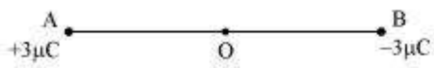
$$U = U_1 + U_1 + U_1 + \ldots.$$

i.e., total energy stored in series and parallel combination of capacitors is equal to sum of the energies stored in the individual capacitors.

Solved Problems

1. **Which physical quantity has its S.I. unit (i) C-m (ii) N/C (iii) Nm^2C^{-1}?**
   Solution: (i) Electric dipole moment (ii) Electric field intensity (iii) Electric flux.
2. **A charged rod P attracts rod R whereas P repels another charged rod Q. What type of force is developed between Q and R?**
   Solution: Q and R will develop attractive forces because R is attracted by P whereas Q is repelled by P.
3. **What is the S.I. unit of permittivity in vacuum?**
   Solution: $\varepsilon_o = 8.854 \times 10^{-12}\ C^2N^{-1}m^{-2}$
4. **If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?**
   Solution: The situation is represented in the given figure. O is the mid-point of line AB.

   Distance between the two charges, AB = 20 cm
   ∴AO = OB = 10 cm
   Net electric field at point O, = E,
   Electric field at point O caused by +3μC charge,



   $E_1 = \frac{3\times10^{-6}}{4\pi\varepsilon_0(AO)^2} = \frac{3\times10^{-6}}{4\pi\varepsilon_0(10\times10^{-2})^2}$ N/C along OB
   Magnitude of electric field at point O caused by −3μC charge,
   $E_2 = \frac{3\times10^{-6}}{4\pi\varepsilon_0(OB)^2} = \frac{3\times10^{-6}}{4\pi\varepsilon_0(10\times10^{-2})^2}$ N/C along OB
   So, $E = E_1 + E_2 = 2\left[9\times10^9\frac{3\times10^{-6}}{(10\times10^{-2})^2}\right] = 5.4\times10^6$ N/C along OB
   (since the values of E_1 and E_2 are same, the value is multiplied with 2)

Therefore, the electric field at mid-point O is 5.4×10^6 N C^{-1} along OB.
A test charge of amount 1.5×10^{-9} C is placed at mid-point O. q = 1.5×10^{-9} C, Force experienced by the test charge = F= qE= $1.5 \times 10^{-9} \times 5.4 \times 10^6 = 8.1 \times 10^{-3}$ N
The force is directed along line OA. This is because the negative test charge is repelled by the charge placed at point B but attracted towards point A.
Therefore, the force experienced by the test charge is 8.1×10^{-3} N along OA.

5. An infinite line charge produces a field of 9 × 10⁴ N/C at a distance of 2 cm. Calculate the linear charge density.

Solution: Electric field produced by thee infinite line charges at a distance d having linear charge density λ is given by the relation,

$E = \frac{\lambda}{2\pi\varepsilon_o d}$ ➔ $\lambda = (2\pi\varepsilon_o d)E$

Where, d = 2 cm = 0.02 m, E=9×10^4N/C, ϵ_0 = Permittivity of free space, $\frac{1}{4\pi\varepsilon_o}$= 9×10^9 N m^2 C^{-2}

$\lambda = \frac{0.02\times9\times10^4}{2\times9\times10^9}$ = = 0.1 µC/m

Therefore, the linear charge density is 0.1 µC/m.

6. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Solution: Capacitor of the capacitance, C = 12 pF = 12×10^{-12} F
Potential difference, V = 50 V
Electrostatic energy stored in the capacitor is given by the relation,

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = 1.5 \times 10^{-8}\,J$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

7. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel. What is the total capacitance of the combination? Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Solution: Capacitances of the given capacitors are $C_1 = 2$ pF, $C_2 = 3$ pF, $C_3 = 4$ pF
For the parallel combination of the capacitors, equivalent capacitor C′ is give n by the algebraic sum,
$C' = 2 + 3 + 4 = 9$ pF
Therefore, total capacitance of the combination is 9 pF.
Supply voltage, V = 100 V
The voltage through all the three capacitors is same, V = 100 V
Charge on a capacitor of capacitance C and potential difference V is given by the relation,
$q = CV$ (1)
For C = 2 pF, charge = $VC = 100 \times 2 = 200$ pC $= 2 \times 10^{-10}$C
For C = 3 pF, charge = $VC = 100 \times 3 = 300$ pC $= 3 \times 10^{-10}$C
For C = 4 pF, charge = $VC = 100 \times 4 = 400$ pC $= 4 \times 10^{-10}$C

8. Three capacitors each of capacitance 9 pF are connected in series. What is the total capacitance of the combination? What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Solution: Capacitance of each of the three capacitors, C = 9 pF

9. Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3} \quad \Rightarrow \quad C' = 3\,pF$$

Therefore, total capacitance of the combination is 3 pF.
Supply voltage, V = 100 V
Potential difference (V’) across each capacitor is equal to one-third of the supply voltage,

$$V' = \frac{V}{3} = \frac{120}{3} = 40 \text{ V}$$

Therefore, the potential difference across each capacitor is 40 V.

10. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m². (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Solution: Diameter of the sphere, d = 2.4 m

Radius of the sphere, r = 1.2 m

Surface charge density, $\sigma = 80.0\ \mu C/m^2 = 80 \times 10^{-6}\ C/m^2$

Total charge on the surface of the sphere,

Q = Charge density × Surface area $= \sigma(4\pi r^2) = 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2$

$Q = 1.447 \times 10^{-3}$ C. Therefore, the charge on the sphere is 1.447×10^{-3} C.

Total electric flux (ϕ_{total}) leaving out the surface of a sphere containing net charge Q is given by the relation, $\phi_{total} = \frac{q}{\varepsilon_0}$

Where, ϵ_0 = Permittivity of free space $= 8.854 \times 10^{-12}\ N^{-1}C^2\ m^{-2}$

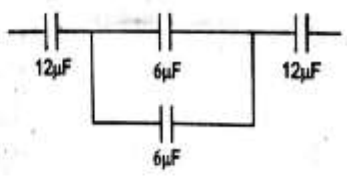
$Q = 1.447 \times 10^{-3}$ C

$\phi_{total} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}} = 1.63 \times 10^{8}\ N\ C^{-1}\ m^2$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^{8}\ N\ C^{-1}\ m^2$.

Assignment

1. Find the number of electrons that constitute one coulomb of charge?
2. What is the value of permittivity of free space?
3. What is electric dipole and electric dipole moment?
4. Define electric field intensity? What is its S.I. unit? Is it a vector or scalar quantity?
5. Derive the expression for the torque acting on an electric dipole placed in a uniform electric field ?
6. Define electric flux? Write its S.I. unit? Is it a vector quantity?
7. What are dielectrics? Distinguish between polar and non-polar dielectrics?
8. Use Gauss's theorem to drive an expression for the electric field at a point due to an infinite plane sheet of charge of uniform charge density σ?
9. Using Gauss's theorem, derive an expression for the electric field due to a thin infinitely long straight line of charge?
10. Derive an expression for the capacitance of a parallel plate capacitor?
11. Find the equivalent capacitance of three capacitors of capacitances C_1, C_2 and C_3 connected in (i) series (ii) parallel.
12. For a parallel plate capacitor, prove that the total energy stored in the capacitor is $\frac{1}{2}CV^2$?
13. Two capacitors of capacitances C_1= 3μF and C_2=6μF arranged in series are connected in parallel with a third capacitor C_3=4 μF. The arrangement is connected to a 6 V battery. Calculate the total energy stored in capacitors.
14. Find the resultant capacitance of the capacitors connected as shown in fig.



15. The plates are in vacuum of a parallel plate capacitor are 5 mm apart and 2 m² in area. A potential difference of 1000 volt is applied across the capacitor. Calculate (i) the capacitance (ii) the charge on each plate and (iii) electric intensity in the space between the two plates.
16. Calculate the capacitance of the capacitor C. The equivalent capacitance of the combination between P and Q is 30μF.