METHODS AND MEANINGS

MATH NOTES

The Elimination Method for Solving Systems of Equations

One method of solving systems of equations is the **Elimination Method**. This method involves adding or subtracting both sides of two equations to eliminate a variable. Equations can be combined this way because balance is maintained when equal amounts are added to both sides of an equation. For example, if $a = b$ and $c = d$, then if you add a and c you will get the same result as adding b and d. Thus, $a + c = b + d$.

Consider the system of linear equations shown at right. Notice that when both sides of the equations are added together, the sum of the x-terms is zero and so the x-terms are eliminated. (Be sure to write both equations so that x is above x, y is above y, and the constants are similarly matched.)

$$\begin{aligned} 3x + 2y &= 14 \\ -3x + 5y &= 14 \\ \hline \frac{7y}{7} &= \frac{28}{7} \\ y &= 4 \end{aligned}$$

Now that you have one equation with one variable ($7y = 28$), you can solve for y by dividing both sides by 7. To find x, you can substitute the answer for y into one of the original equations, as shown at right. You can then test the solution for x and y by substituting both values into the other equation to verify that $-3x + 5y = 14$.

$$\begin{aligned} 3x + 2(4) &= 14 \\ 3x + 8 &= 14 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

$$-3(2) + 5(4) = 14 \ \checkmark$$

Since $x = 2$ and $y = 4$ is a solution to both equations, it can be stated that the two lines cross at the point (2, 4).

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More Solving Systems by Elimination

In Chapter 6, you learned how to solve systems of equations by eliminating a variable. Suppose you want to solve the system of equations shown at right.

$$\begin{aligned} 3x + 2y &= 11 \\ 4x + 3y &= 14 \end{aligned}$$

The goal is to *eliminate* either x or y when you add the equations together. In this case, you need to do something to *both* equations before you add them. To eliminate y, you can multiply the first equation by 3 and multiply the second equation by –2.

multiply by 3

multiply by – 2

$$\begin{aligned} 9x + 6y &= 33 \\ -8x - 6y &= -28 \end{aligned}$$

$$\begin{aligned} 9x + 6y &= 33 \\ -8x - 6y &= -28 \\ \hline x &= 5 \end{aligned}$$

Then eliminate the y-terms by adding the two new equations, as shown above.

Since you know that $x = 5$, you can substitute to find that $y = -2$. Therefore, the solution to the system of equations is (5, –2).

You could also solve the system by multiplying the first equation by 4 and the second equation by –3. This would cause x to be eliminated when the equations are added together.