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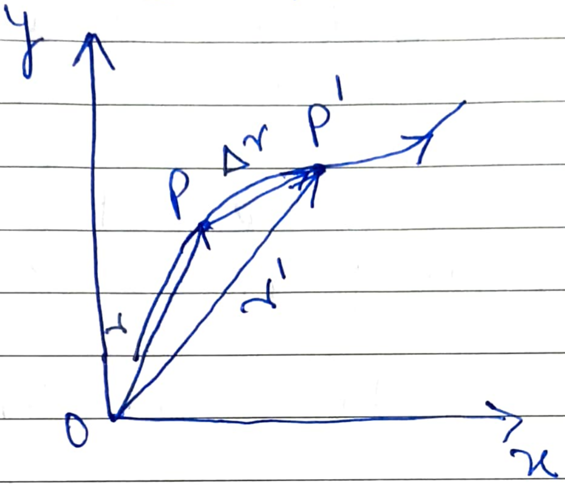
# MOTION IN A PLANE

- Quantities are either scalar or vector
- Vector quantities have magnitude and direction. Scalar quantities have magnitude only.
- Vector quantities obey triangle law of Addition & Parallelogram law of Addition.
- Position & Displacement Vector



If an object moves from P to P', then PP' is the displacement vector. Point P is represented by position vector r and point P' is represented by r'. Length of vector r is the magnitude of the vector

⇒ OP is the position vector of the object at Point P & OP' is the position vector of object at P'.

⇒ $r + \Delta r = r'$ ⇒ $\Delta r = r' - r$

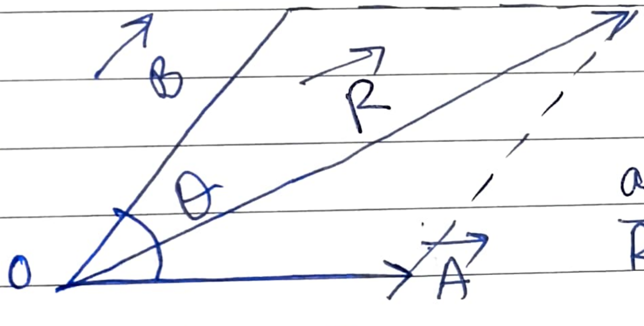
⇒ $\Delta r = (x' - x)\hat{i} + (y' - y)\hat{j} = \Delta x\hat{i} + \Delta y\hat{j}$

- Two vectors are equal if they have same magnitude & same direction
- A vector multiplied by a positive number gives another vector whose magnitude is changed by that factor

$$|\lambda A| = \lambda |A| \text{ if } \lambda > 0$$

- Two vectors and their resultant form the three sides of a triangle
- Vector addition obeys Associative & Commutative law

$\Rightarrow$ $A + B = B + A$ and $(A+B)+C = A+(B+C)$

- Difference of two vectors 'A' & 'B' is the vector sum of A and (−B).
- Parallelogram addition of vector is equivalent to the Triangle Method.
- 

$\vec{A}$ & $\vec{B}$ are two vectors inclined at an angle 'θ'.
$\vec{R}$ = Resultant Vector

$$\vec{R} = \vec{A} + \vec{B}$$
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

- Unit Vector along x-axis, Y-axis & Z-axis are denoted by $\hat{i}$, $\hat{j}$ & $\hat{k}$. Unit vectors are perpendicular to each other.

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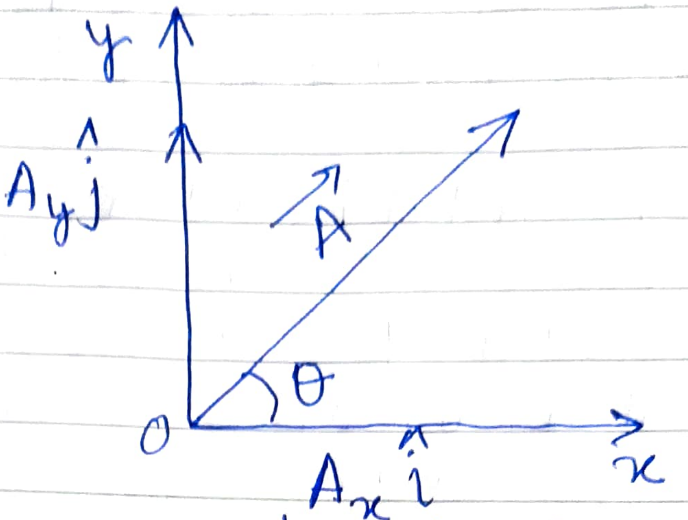
- In general vector $A = |A|\hat{n}$
where $\hat{n}$ = Unit vector in the direction of A

- 

Consider a vector A in x-y plane.

$\Rightarrow$ $$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

where $A_x$ & $A_y$ are the components of vector 'A' along x-axis & y-axis respectively

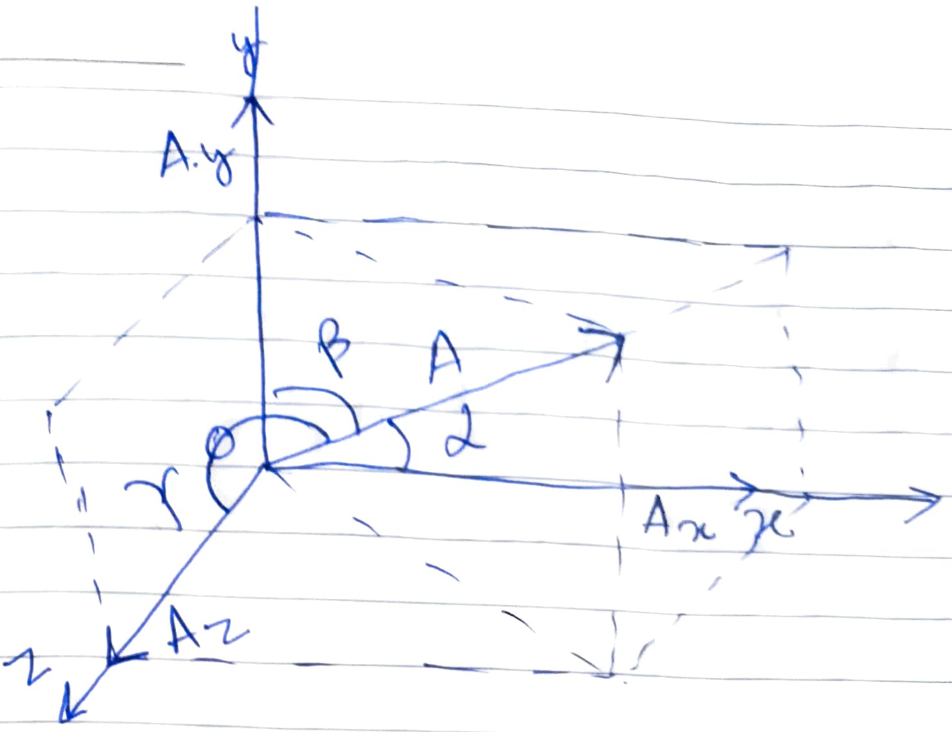
$\Rightarrow$ $$A_x^2 + A_y^2 = A^2$$

$\Rightarrow$ $$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x} \qquad A_x = A\cos\theta,\ A_y = A\sin\theta$$

If a vector lies in three dimensions, then above equation will be

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



Above figure shows a vector A having components along x-axis, Y-axis & Z-axis

$A_x = A\cos\alpha$ $A_y = A\cos\beta$ & $A_z = A\cos\gamma$

$A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Position vector $r = x\hat{i} + y\hat{j} + z\hat{k}$ where x, y & z are the component of r along x, y & z axis

- Relative Velocity in two dimension

Suppose two objects 'A' & 'B' are moving with velocity $V_A$ & $V_B$ respectively with reference to a common frame of reference. Then

$V_{AB} = V_A - V_B$ & $V_{BA} = V_B - V_A$

- If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

  $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then

$$\vec{A}\cdot\vec{B} = |A||B)\, \mathrm{Cos}\,\theta$$

where $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$|B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\vec{A}\times\vec{B} = |A||B)\sin\theta$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A}\cdot\vec{B} = (A_x\cdot B_x)\hat{i} + (A_y\cdot B_y)\hat{j} + (A_z\cdot B_z)\hat{k}$$

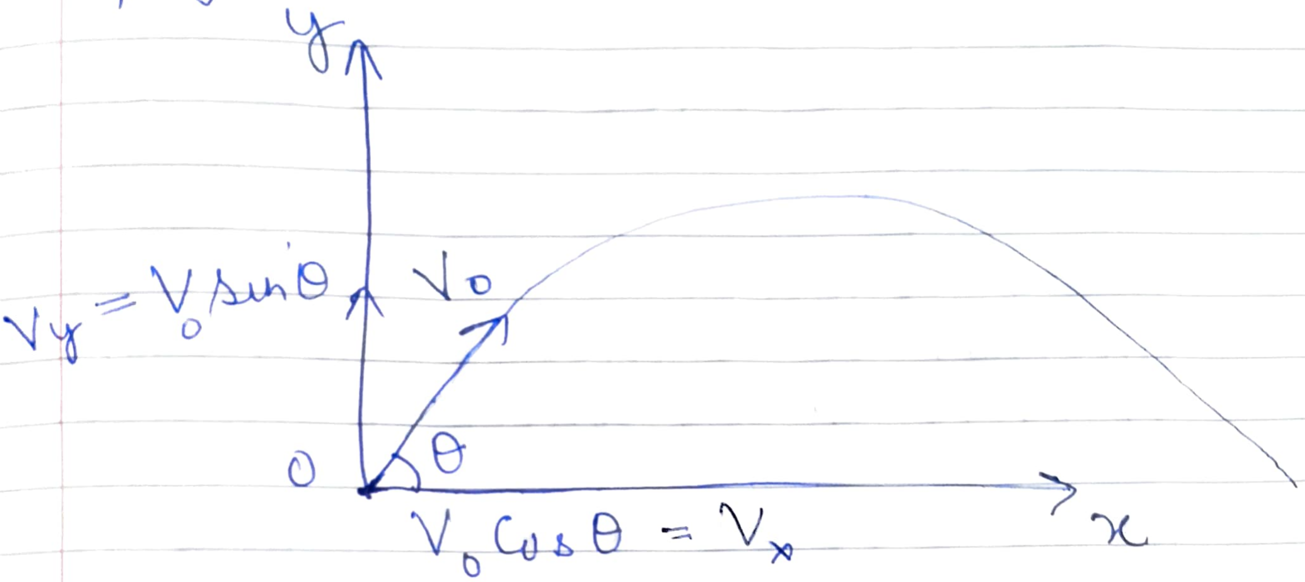
$\hat{i}\cdot\hat{i} = 1$ $\hat{j}\cdot\hat{j} = 1$ $\hat{k}\cdot\hat{k} = 1$

$\hat{i}\times\hat{i} = 0$ $\hat{j}\times\hat{j} = 0$ $\hat{k}\times\hat{k} = 0$

- Cross product of two vectors gives a third vector which is perpendicular to both the vectors.

# Projectile Motion

Any object projected at an angle with the horizontal will follow a projectile motion



If $V_0$ = Initial velocity
$\theta^0$ = Angle of Inclination with the horizontal

Then $V_y = V_0 \sin\theta$ (Vertical Componen

$V_x = V_0 \cos\theta$ (Horizontal Component)

Range $= V_0 \cos\theta \cdot t$ (where t= time o flight)
$= V_0^2 \sin 2\theta / g$

Maximum height $= \dfrac{(V_0 \sin\theta)^2}{2g}$

Time taken to reach Maximum height $= \dfrac{V_0 \sin\theta}{g}$

$y = (V_0 \sin\theta)t - \frac{1}{2} g t^2$

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## Circular Motion

Object moving in a circular path at Constant speed is called Uniform circular Motion.

Force (Centripetal or Centrifugal) $= \frac{mv^2}{r}$

where centripetal acceleration $= \frac{v^2}{r} = r$
Direction of Centripetal acceleration is towards the Centre of the circle.

Angular speed 'ω' is the rate of change of Angular distance. It is given by $v = \omega r$

Time period of revolution 'T' is the time taken by the object to complete one revolution

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

Frequency is the number of revolutions per second & is denoted by ν.

$$\nu = \frac{1}{T}$$

⇒ $a_c = \frac{v^2}{r} = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 r}{T^2} = 4\pi^2\nu^2 r$