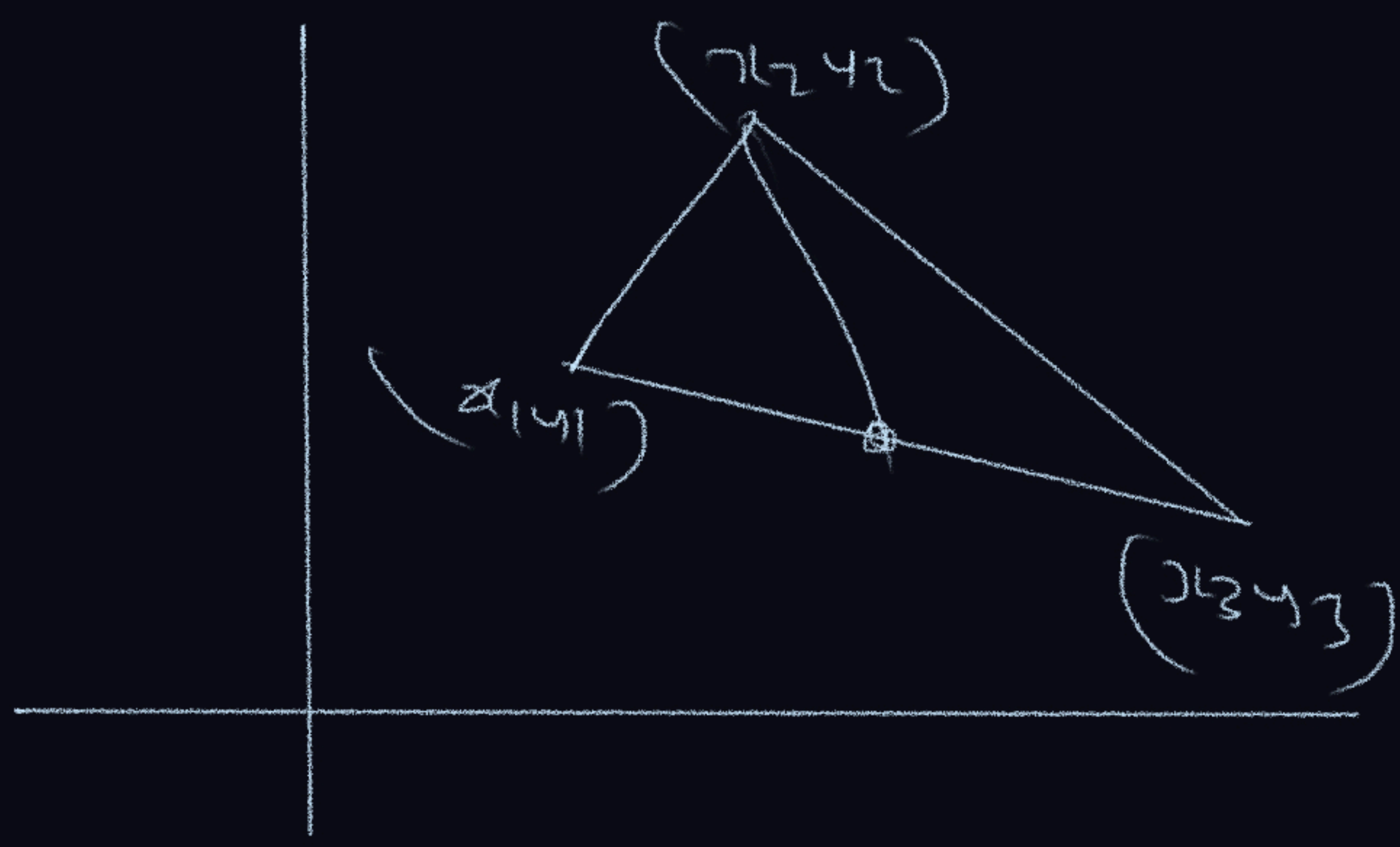
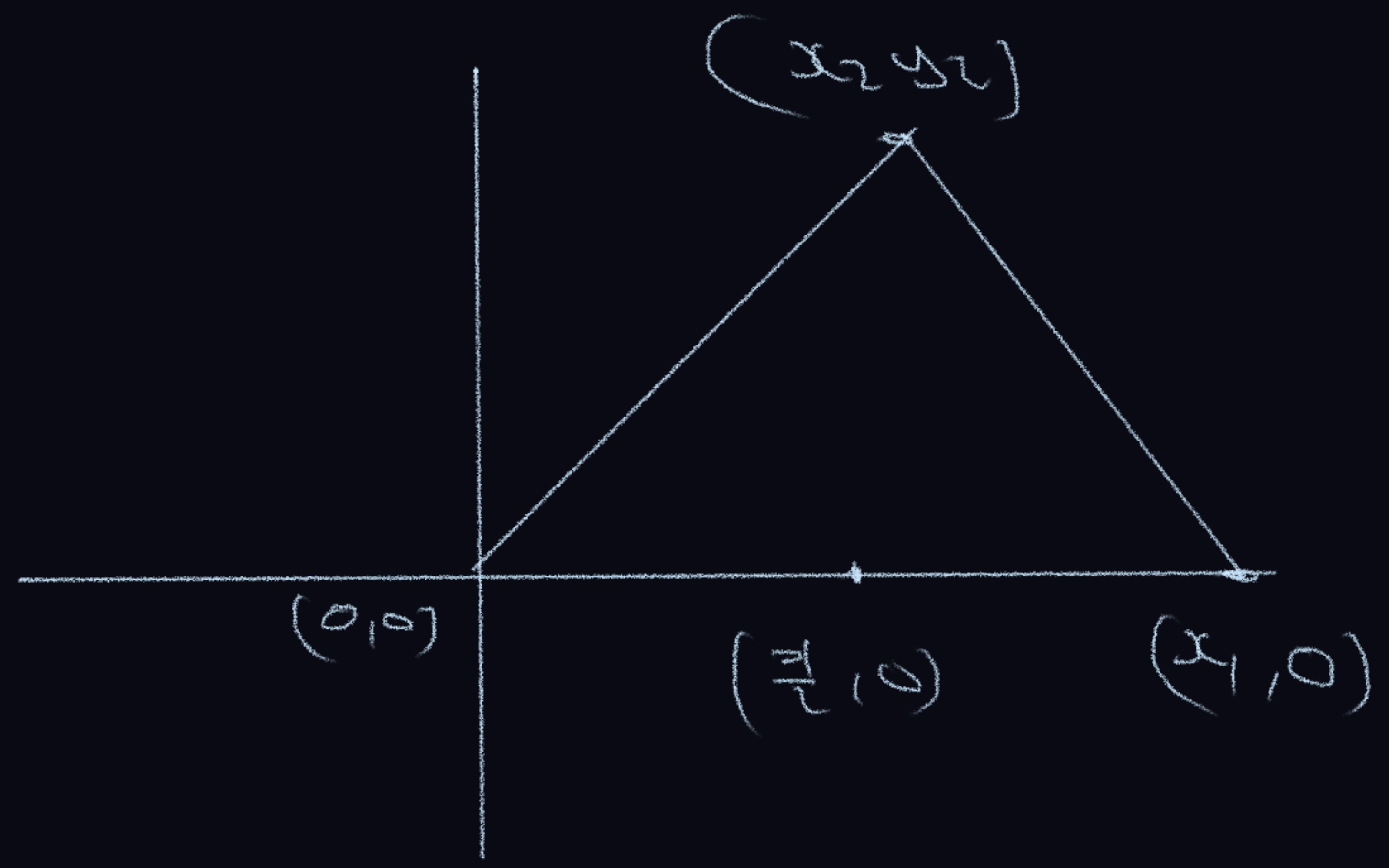
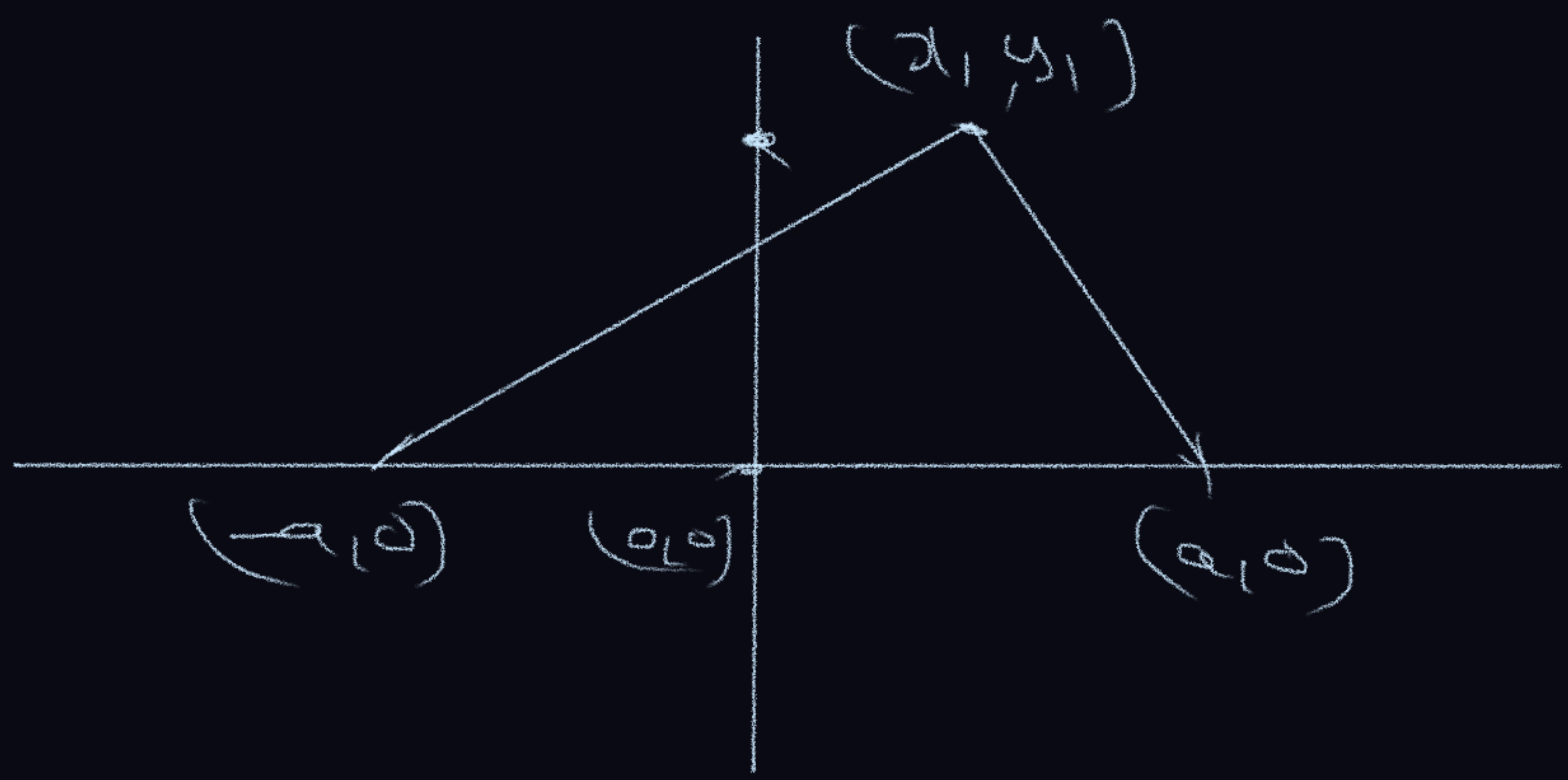
Coordinate geometry method

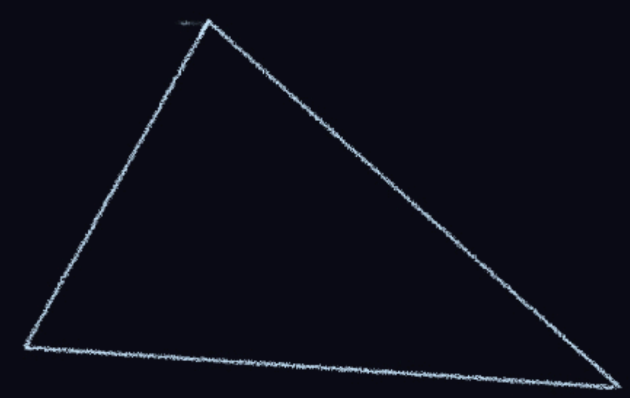
In a ΔABC prove $AB^2 + AC^2 = 2(AD^2 + BD^2)$ 'D' is middle point of BC.

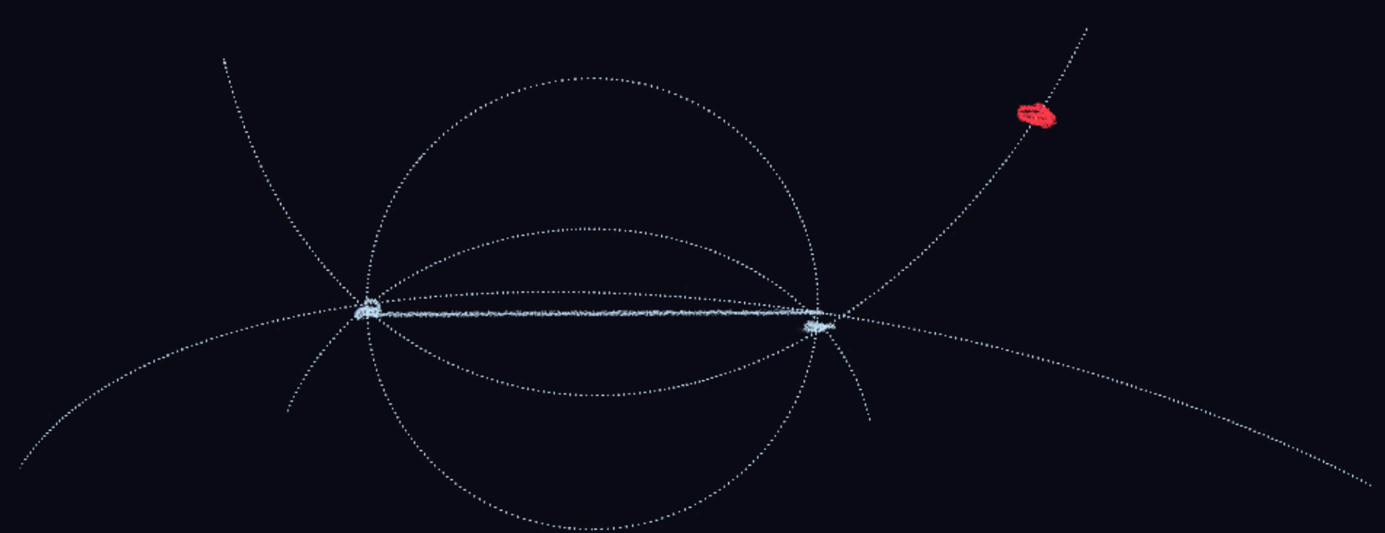


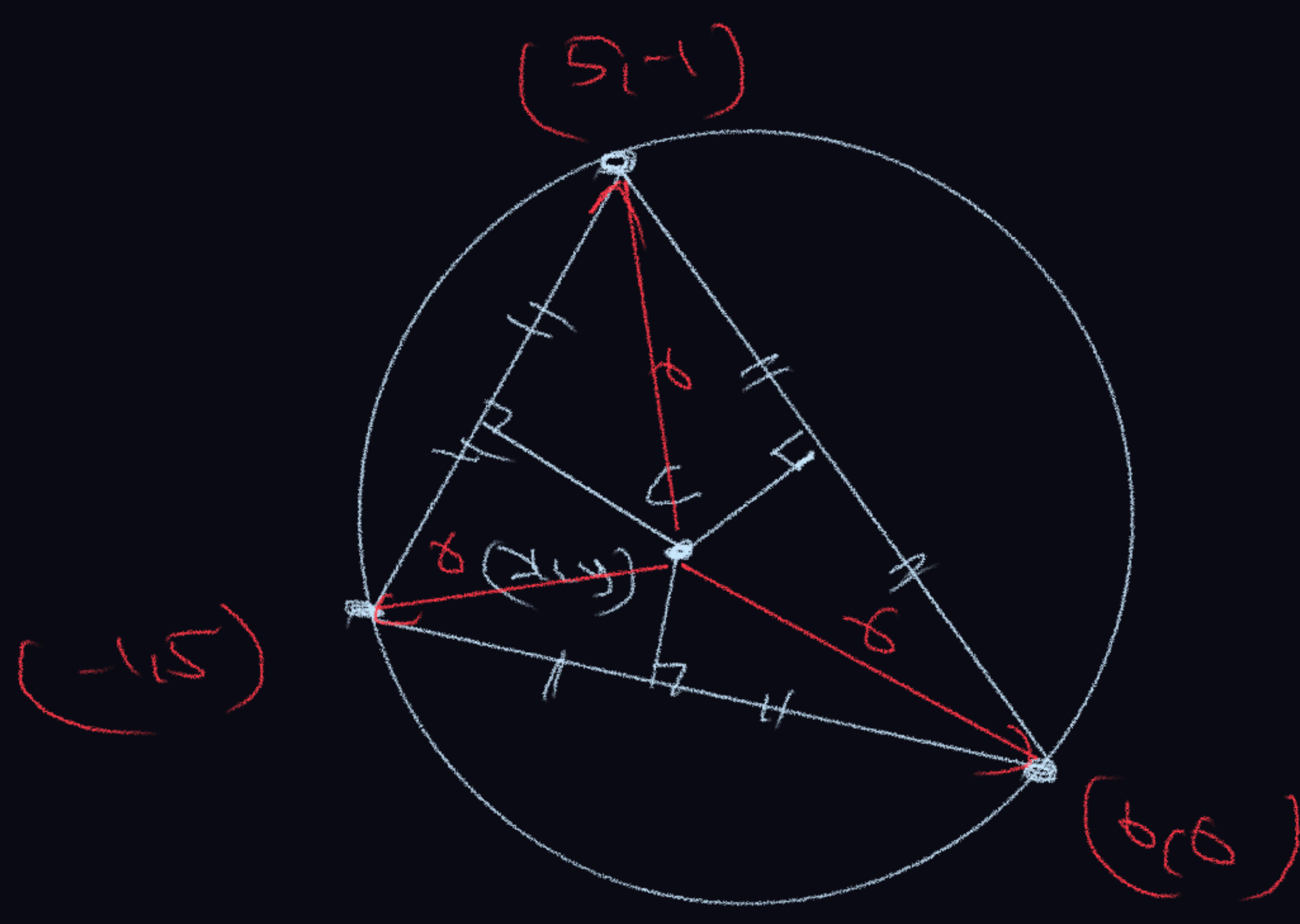




A (5, -1)   B (-1, 5)   C (6, 6)   find its circumcentre?







$$(y-6)^2+(x-6)^2 = (y-5)^2+(x+1)^2 = (y+1)^2+(x-5)^2$$

(x,y)

(x,y)

$x=$

$y=$

Two points

$(0,0)$ $(3,\sqrt{3})$ and P form equilateral triangle. find P.

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\text{Quadril Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \\ x_1 & y_1 \end{vmatrix}$$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right|$$

cyclic order

A B C

Prove that area of $(t, t-2)$ $(t+2, t+2)$ $(t+3, t)$ is independent of t.

are collinear?

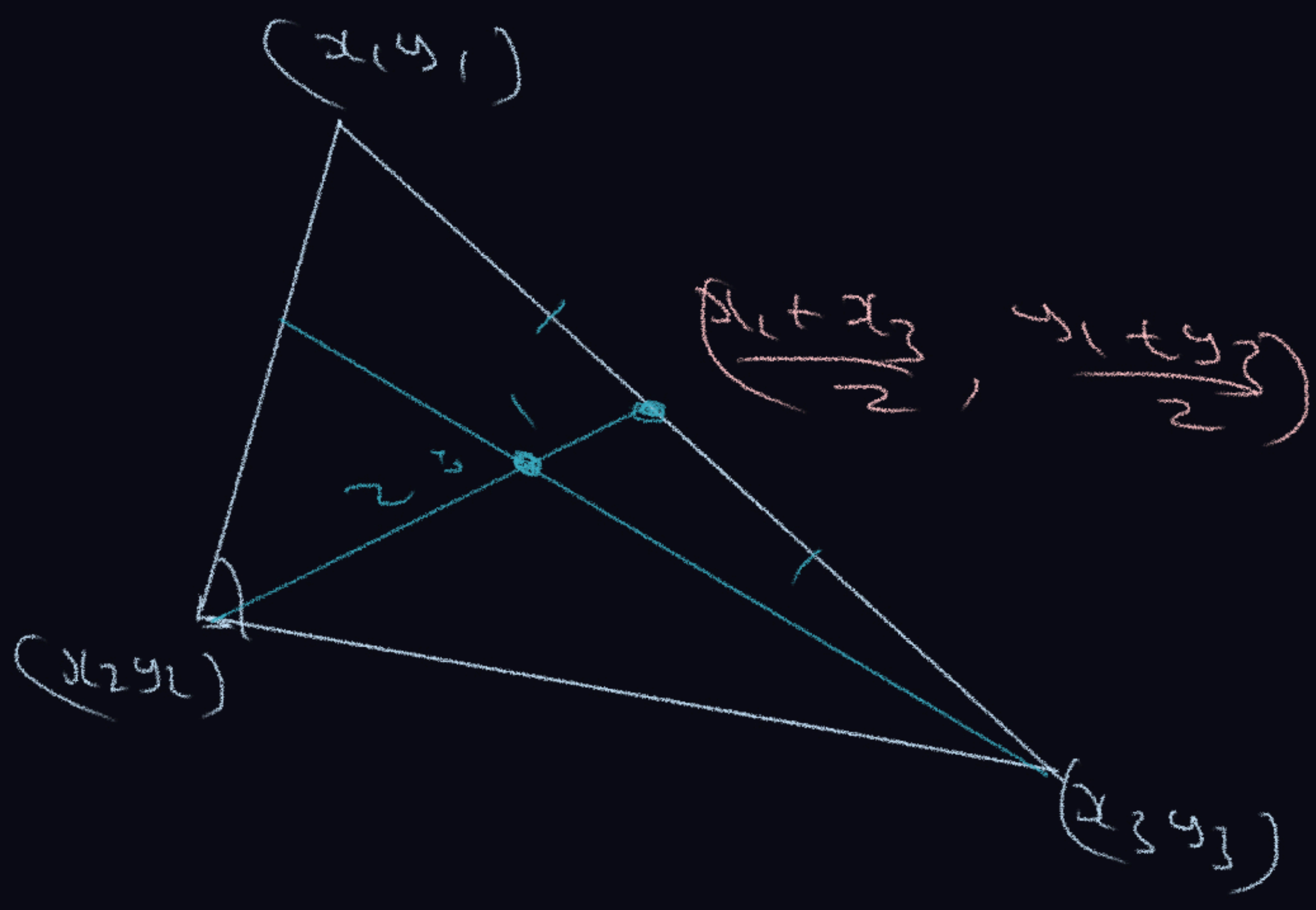
If the vertices of a triangle have rational coordinates, then prove that triangle cannot be equilateral.

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} = \frac{1}{2}\left[(x_1y_1 + x_2y_3 + x_3y_1) - (\qquad)\right]$$

Area = Rational number

$$A = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}(\text{rational})^2 = \text{irrational} = \text{irrat}$$

$$a = \text{side lenght} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \text{rational}$$

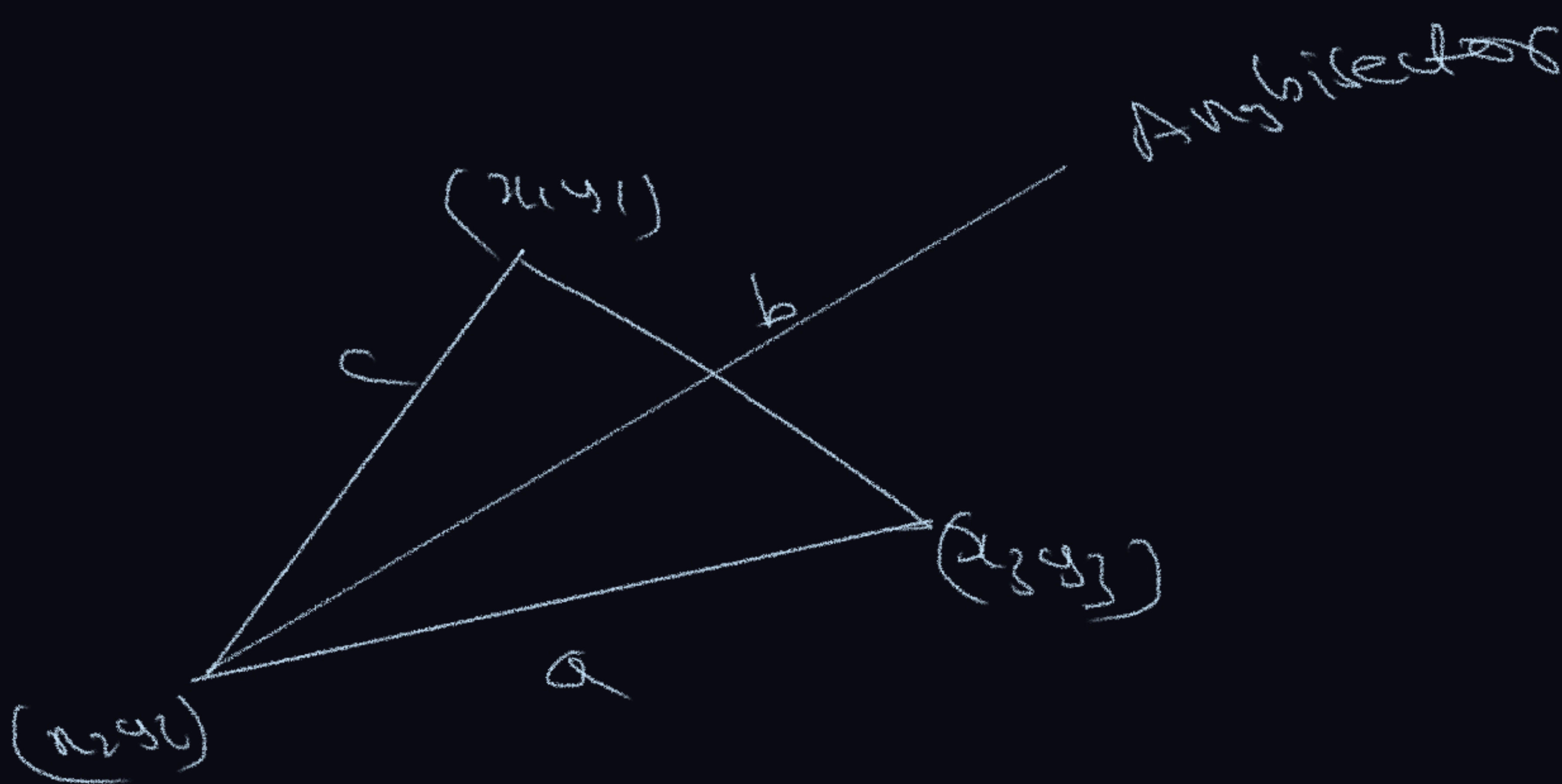


Centroid ?

↓

(medians)

$$X = \frac{x_1+x_2+x_3}{3} \qquad Y = \frac{y_1+y_2+y_3}{3}$$

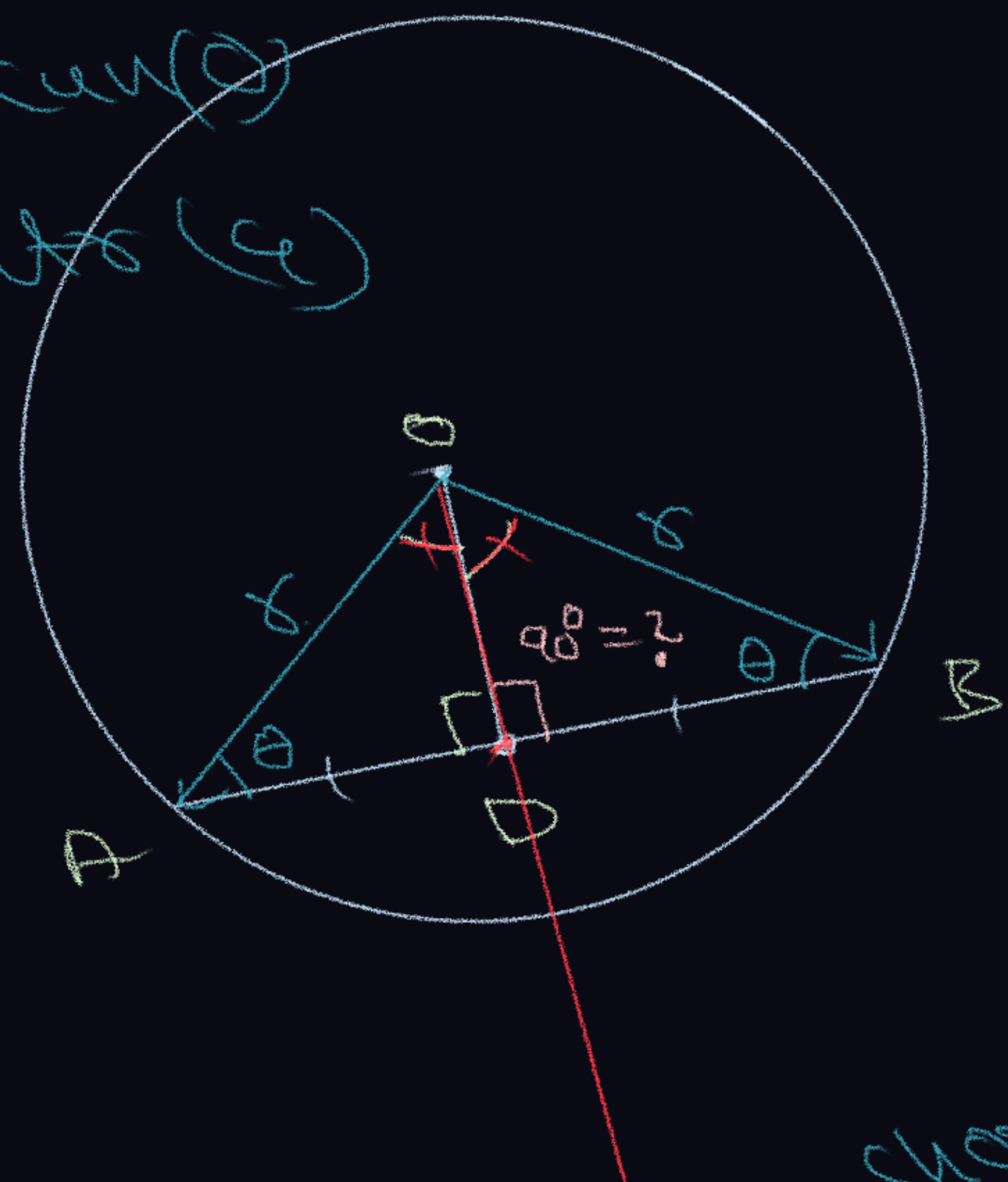


$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

H = Altitude ⇒ ortho

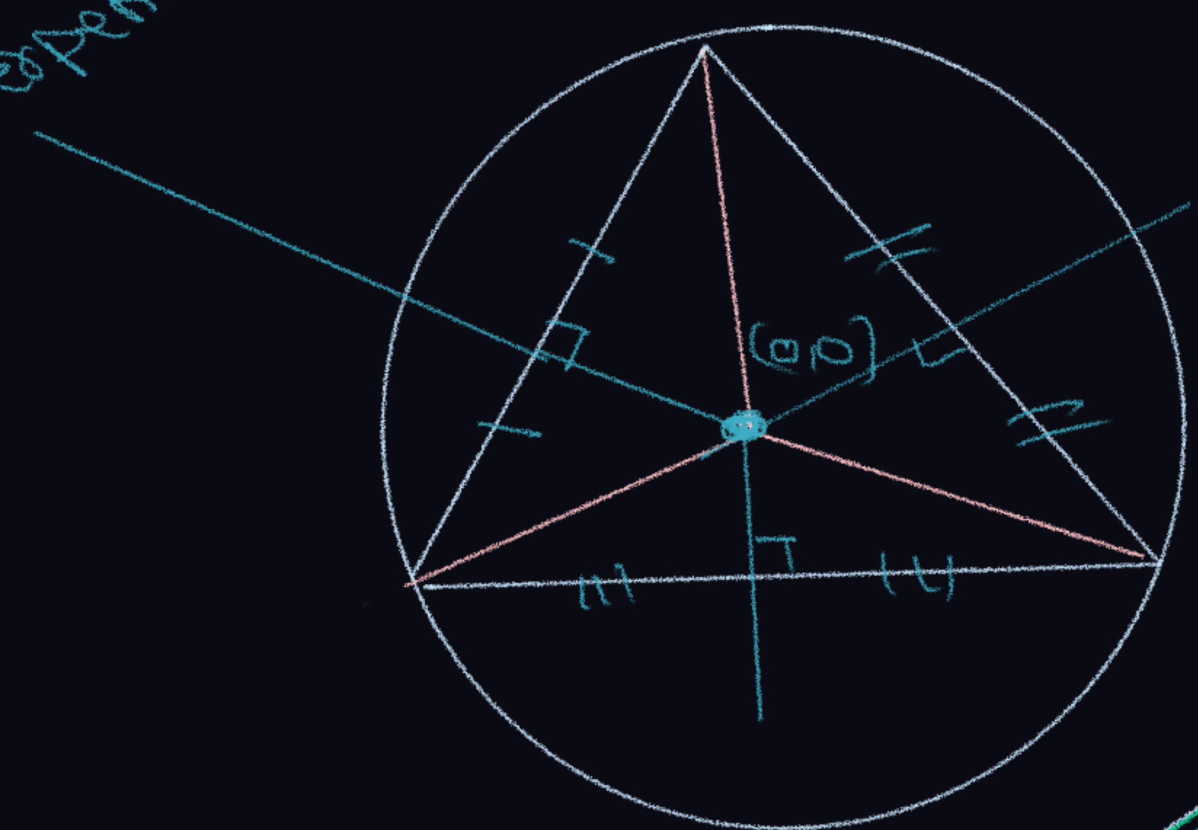
circum(O)

centre (G)



– perpendicular on chord from centre bisects the chord

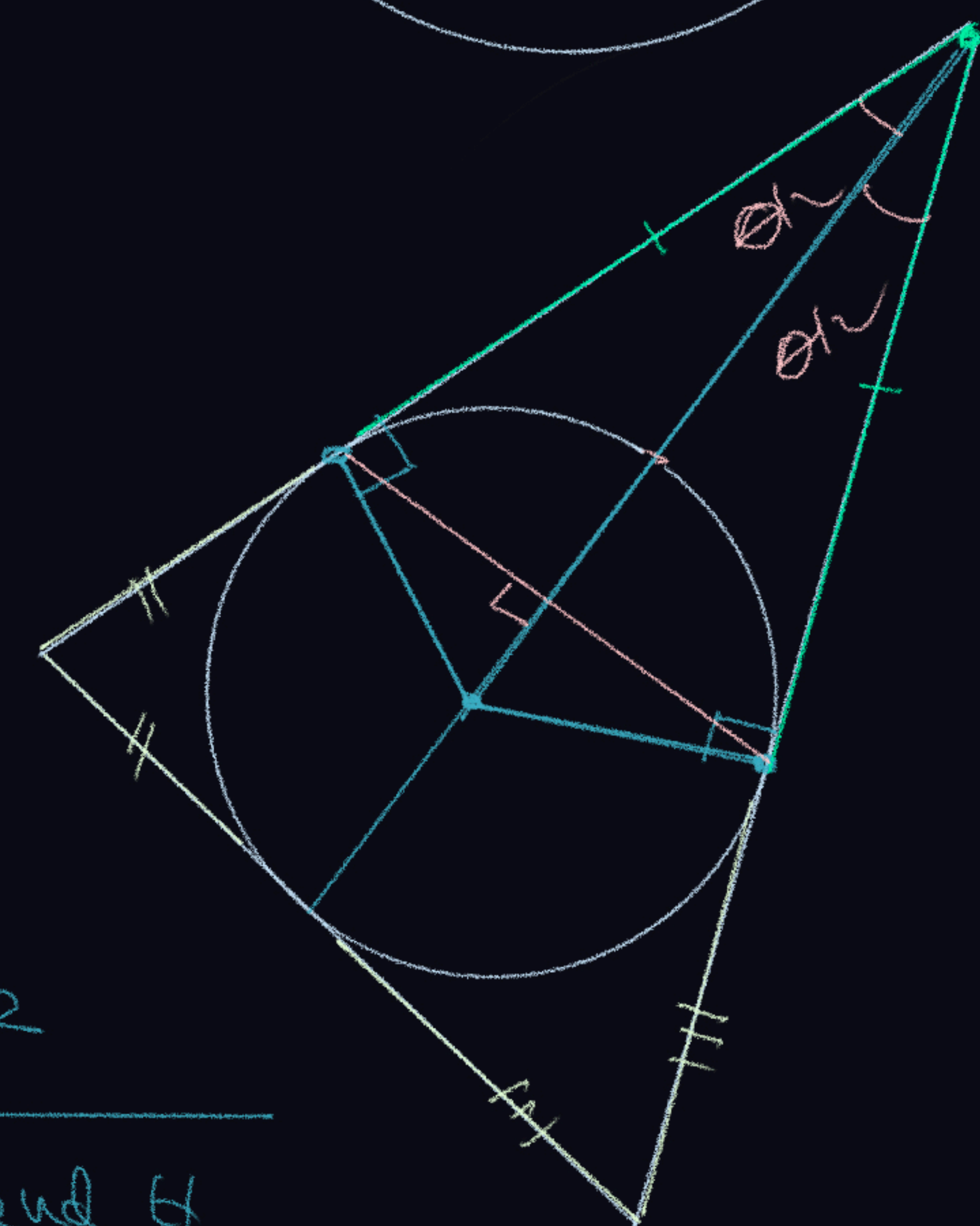
perpend bisect



Perp bisect

↓

circumcentre



Interior angle bisect

↓

incentre

$$\boxed{G = \frac{2(O) + H}{3}}$$

$3G = H$

$OG : GH = 1:2$

G divides O and H in 2:1 ratio

$H = (x_1 + x_2 + x_3, y_1 + y_2 + y_3)$

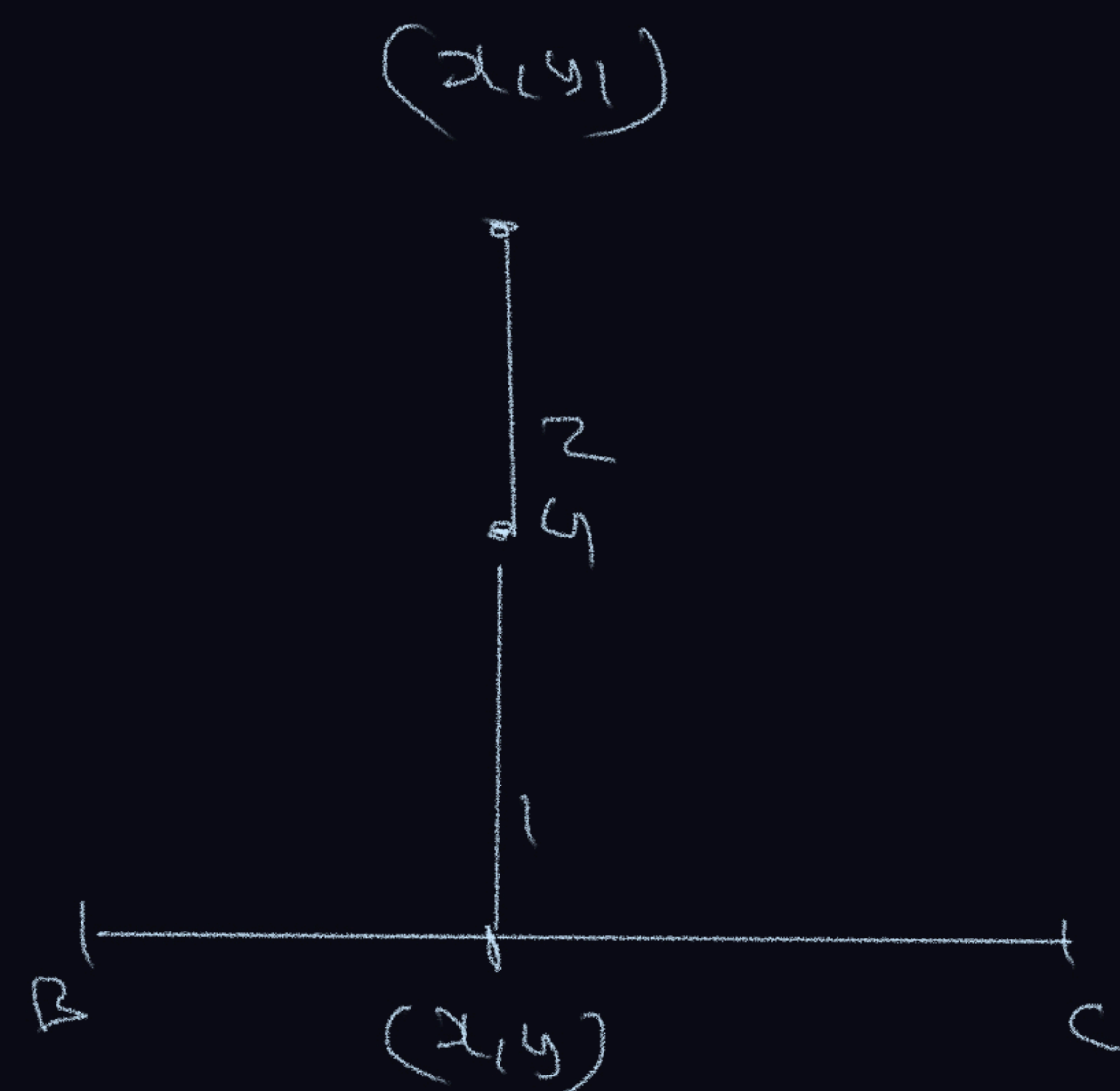
ΔABC

Find coordinates of middle point of BC.

ortho (a, b)

circum (c, d)

$A\ (x_1, y_1)$



$$\frac{2x + x_1}{3} = \frac{2(c) + a}{3}$$

$$\frac{2y + y_1}{3} = \frac{2d + b}{3}$$

assume if needed

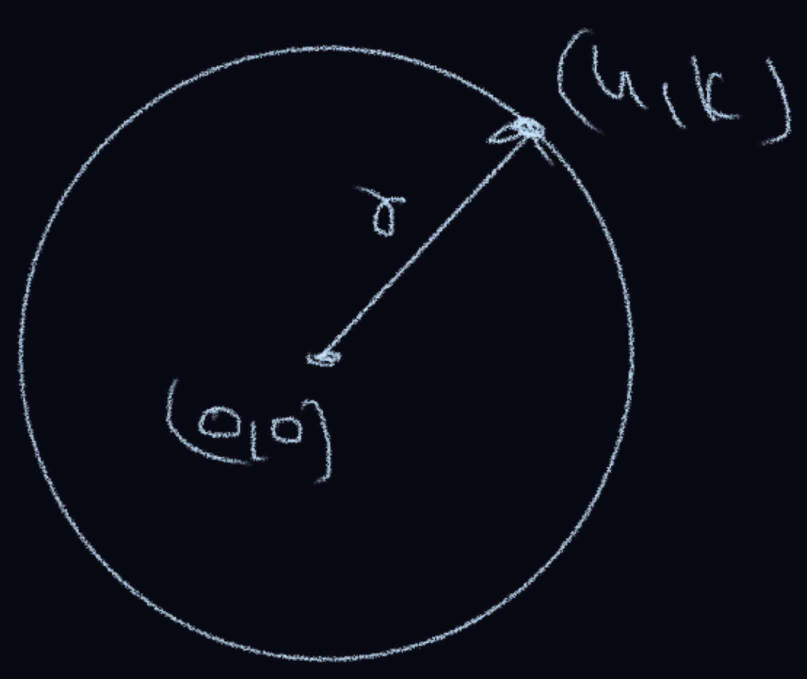
Locus (h,k)

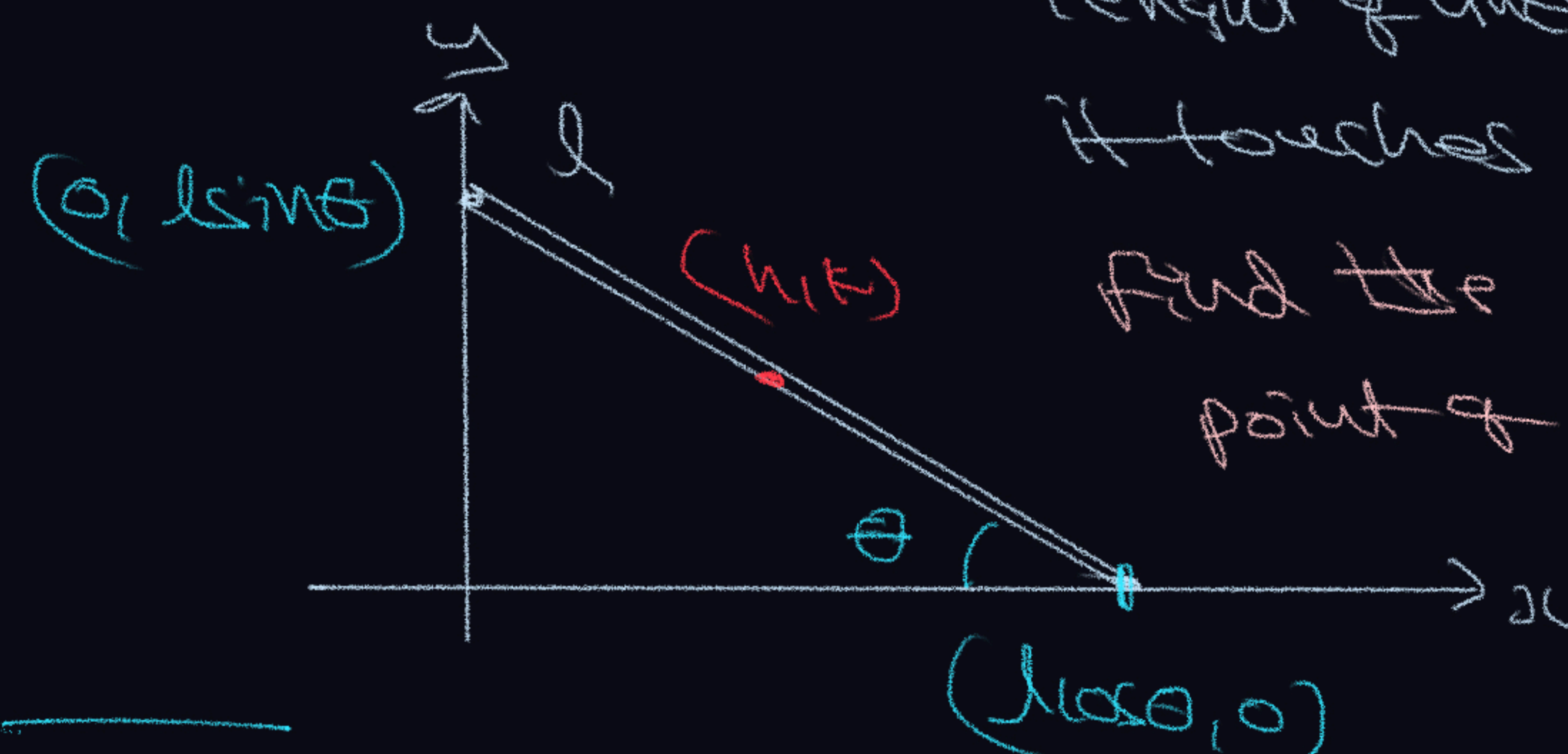
conditions ⟶ Equations with (h,k) (t)

$h = f(t)$ ⟶ t = parameter

$k = g(t)$

$(h, k) = 0$ locus





length of line = l

it touches axis

find the locus of mid point of rod?

2var

$h = \frac{x}{2}$

$k = \frac{y}{2}$

$x^2 + y^2 = l^2$

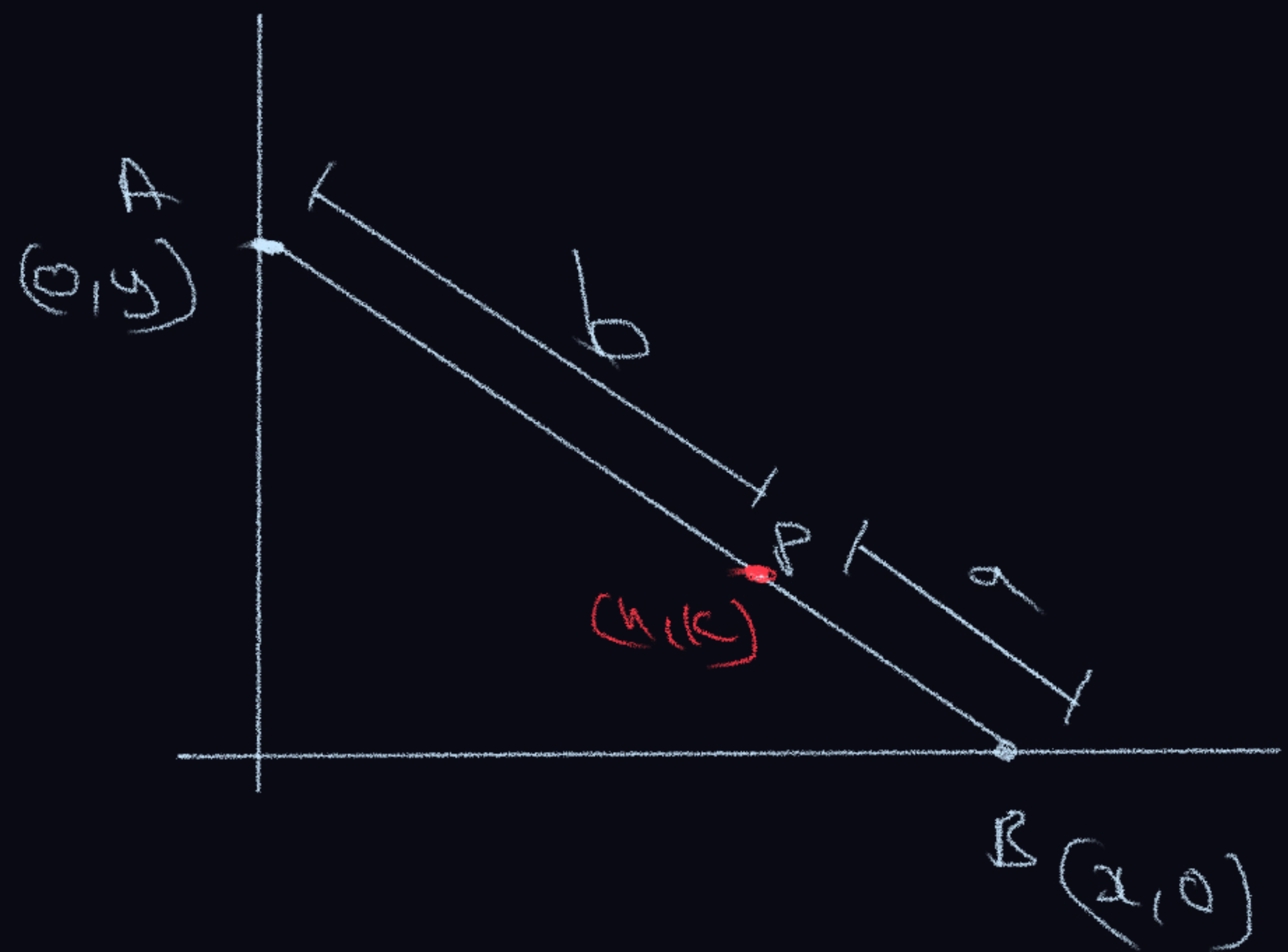
$$\boxed{(2h) + (2k)^2 = l^2}$$

θvar

$h = f(\theta)$

$k = f(\theta)$

$\sin^2\theta + \cos^2\theta = 1$



AB is a variable line

P is a variable point on AB

$PA = b \quad PB = a \quad AB = a+b$

Find locus of P?

$$x^2 + y^2 = (a+b)^2$$

$$h = \frac{a(0) + b(x)}{a+b} \qquad k = \frac{ay + b(0)}{a+b}$$

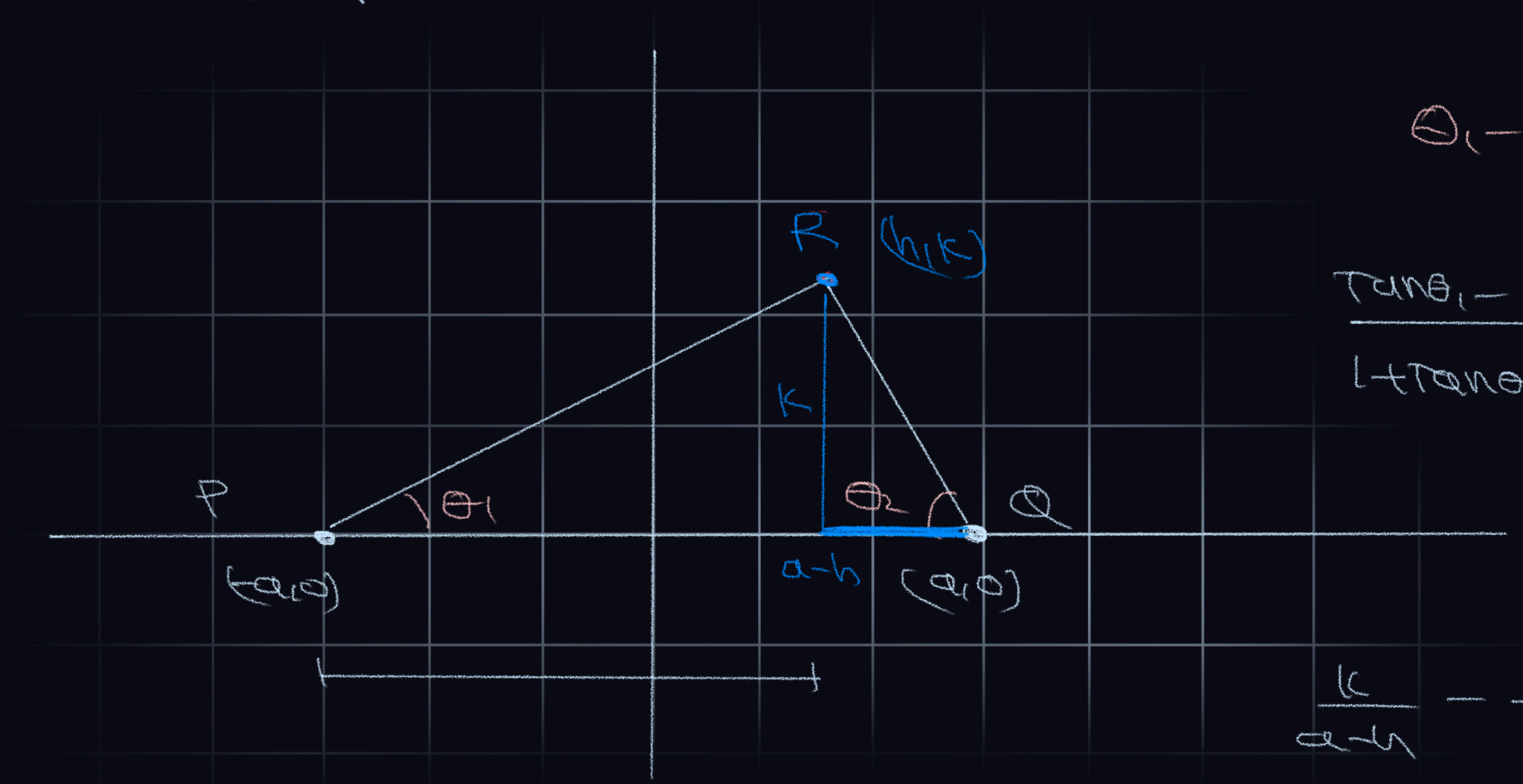
$$= \frac{b}{a+b}x \qquad = \frac{a}{a+b}y$$

$$\left(\frac{h}{b}(a+b)\right)^2 + \left(\frac{k}{a}(a+b)\right)^2 = (a+b)^2$$

$$\frac{h^2}{b^2} + \frac{k^2}{a^2} = 1$$

Two points P and Q are given. R is a variable point on one side of the line PQ such that $\angle RPQ - \angle RQP$ is a positive constant 2α. Find the locus of point R.

Good geom method



$\theta_1 - \theta_2 = 2\alpha$

$$\frac{\tan\theta_1 - \tan\theta_2}{1+\tan\theta_1 \tan\theta_2} = \tan 2\alpha$$

$$\tan\theta_1 = \frac{k}{a+h} \qquad \tan\theta_2 = \frac{k}{a-h}$$

$$\frac{\frac{k}{a-h} - \frac{k}{a+h}}{1+\frac{k^2}{a^2-h^2}} = \tan 2\alpha$$

$2kh = \tan 2\alpha(k^2 - h^2 + a^2)$

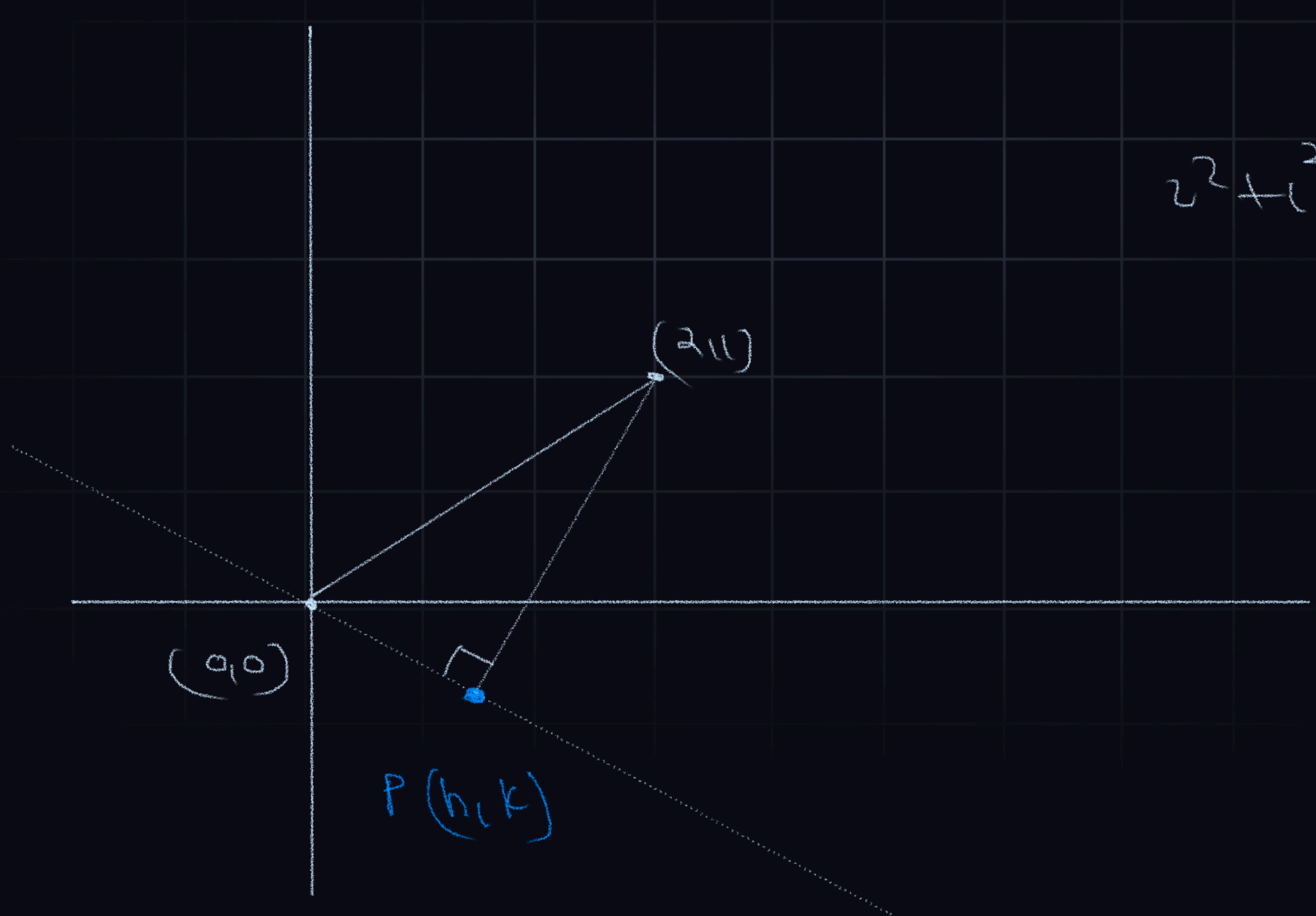
locus of P such that (t^2-t+1, t^2+t+1) is coord of P.

$h = t^2-t+1$ $\qquad$ $k-h = 2t$

$k = t^2+t+1$

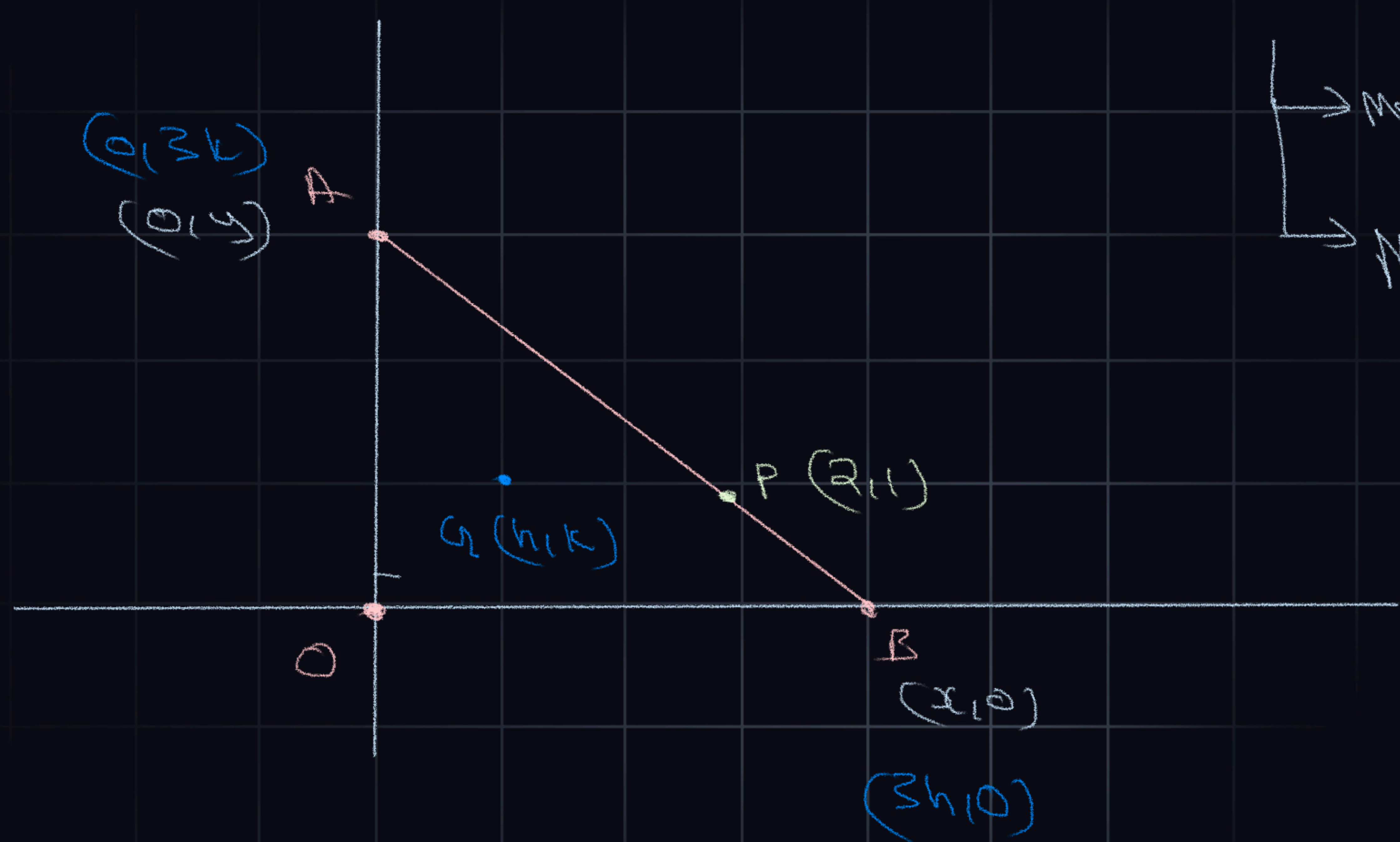
$$\boxed{h = \left(\frac{k-h}{2}\right)^2 - \left(\frac{k-h}{2}\right) + 1}$$

Find the locus of the foot of perpendicular from the point (2,1) on the variable line passing through the point (0,0)



$$2^2+1^2 = h^2+k^2 + (k-1)^2+(h-2)^2$$

A variable line through point $(2,1)$ meets axes at A and B. Find the locus of the centroid of triangle OAB



APB are collinear

→ Method ①: slope = slope

→ Method ②: Area = 0

$$\begin{vmatrix} 0 & 3k \\ 2 & 1 \\ 3h & 0 \\ 0 & 3k \end{vmatrix} = 0$$

$$h = \frac{0+x+0}{3} \qquad x = 3h$$

$$k = \frac{0+0+y}{3} \qquad y = 3k$$

$$9hk - 6k - 3h = 0$$

$$\boxed{3hk = 2k + h}$$

$$\boxed{3 = \frac{2}{h} + \frac{1}{k}}$$

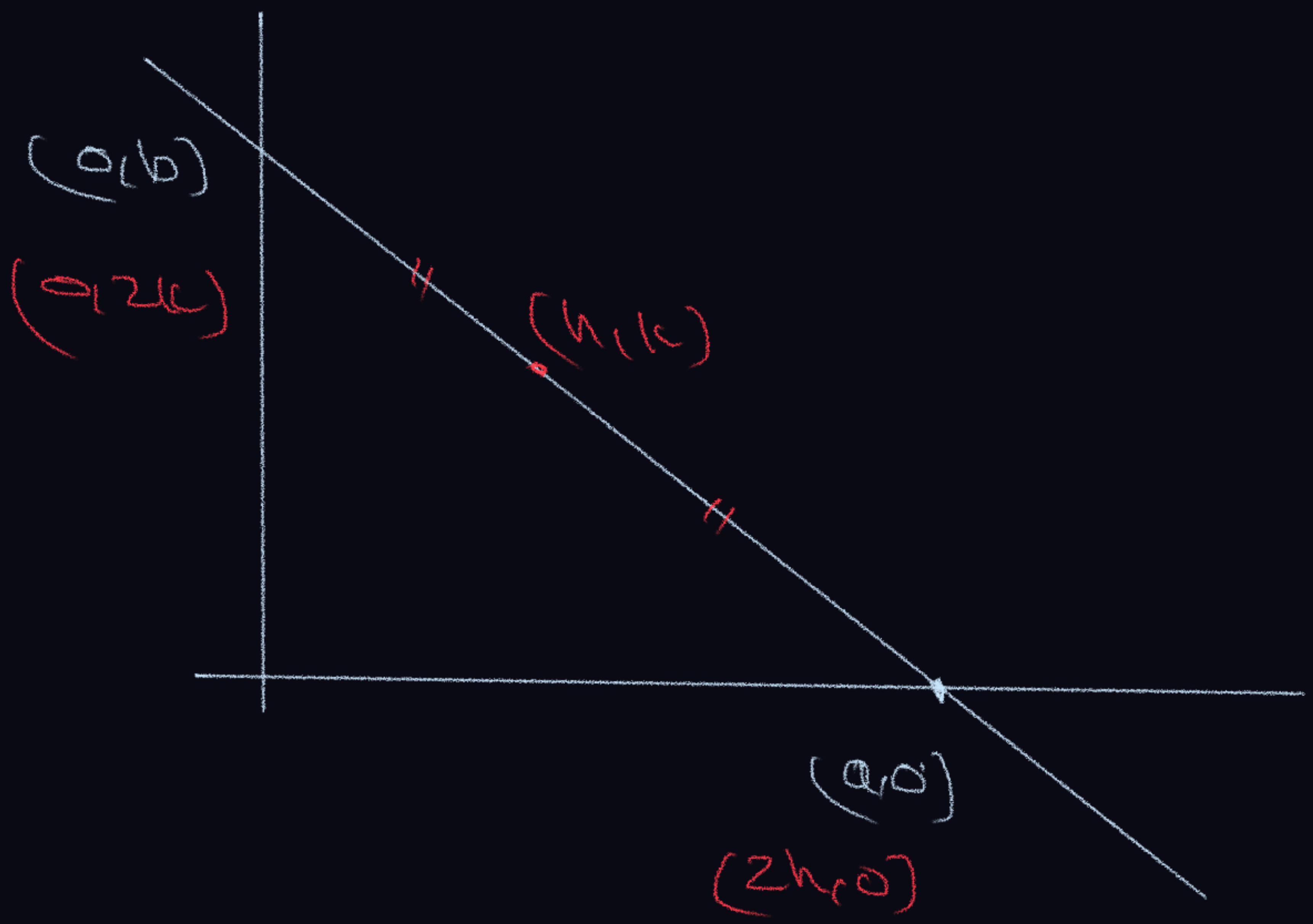
let $A(2,-3)$ and $B(-2,1)$ be the vertices of ΔABC

if centroid of triangle moves on the line

$2x+3y=1$ find locus of vertex C.

α = variable

p = constant



$$\frac{a+0}{2} = h \qquad \frac{0+b}{2} = k$$

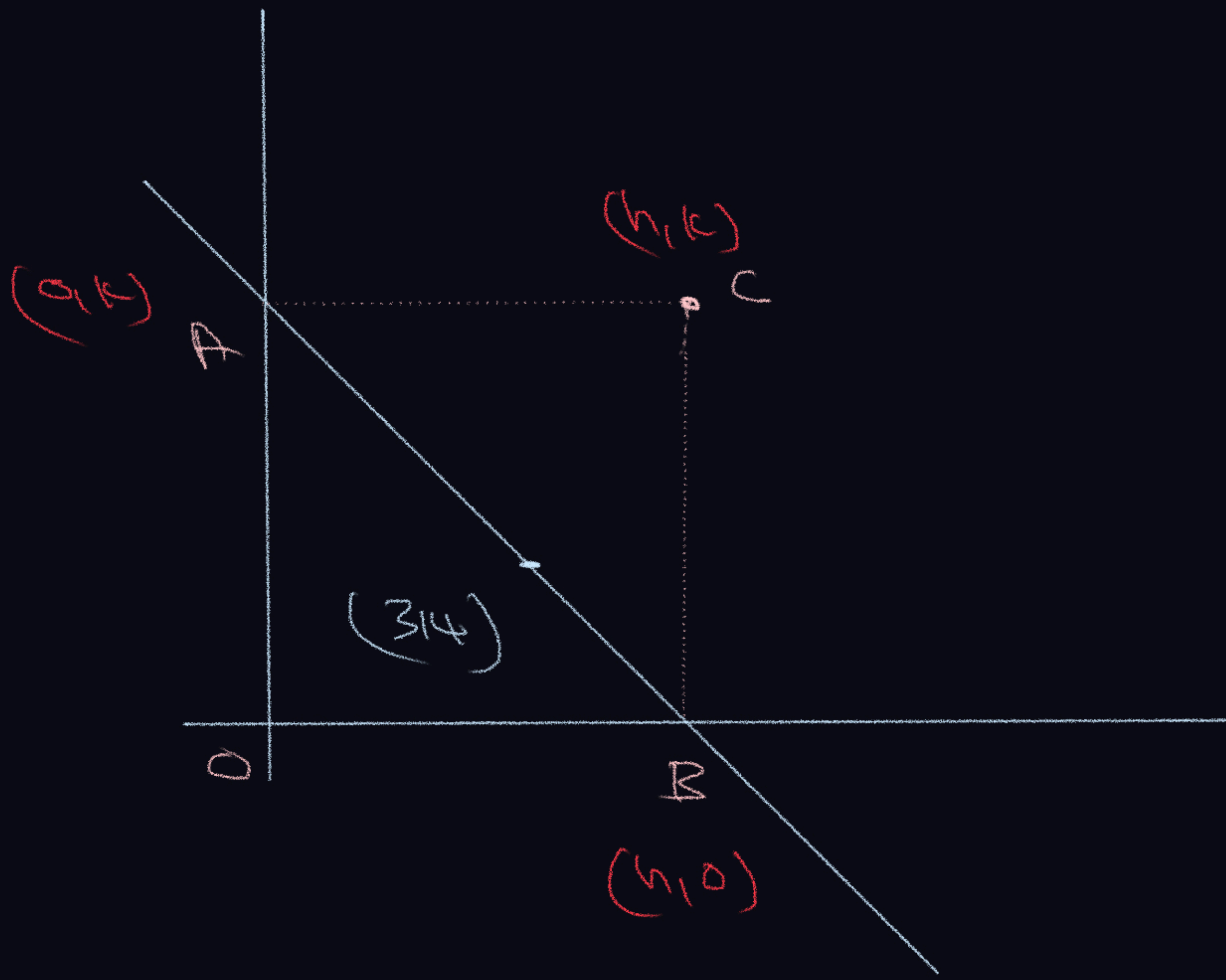
$$a = 2h \qquad b = 2k$$

$$x\cos\alpha + y\sin\alpha = p$$

$$2h\cos\alpha + 0\sin\alpha = p \qquad 0\cos\alpha + 2k\sin\alpha = p$$

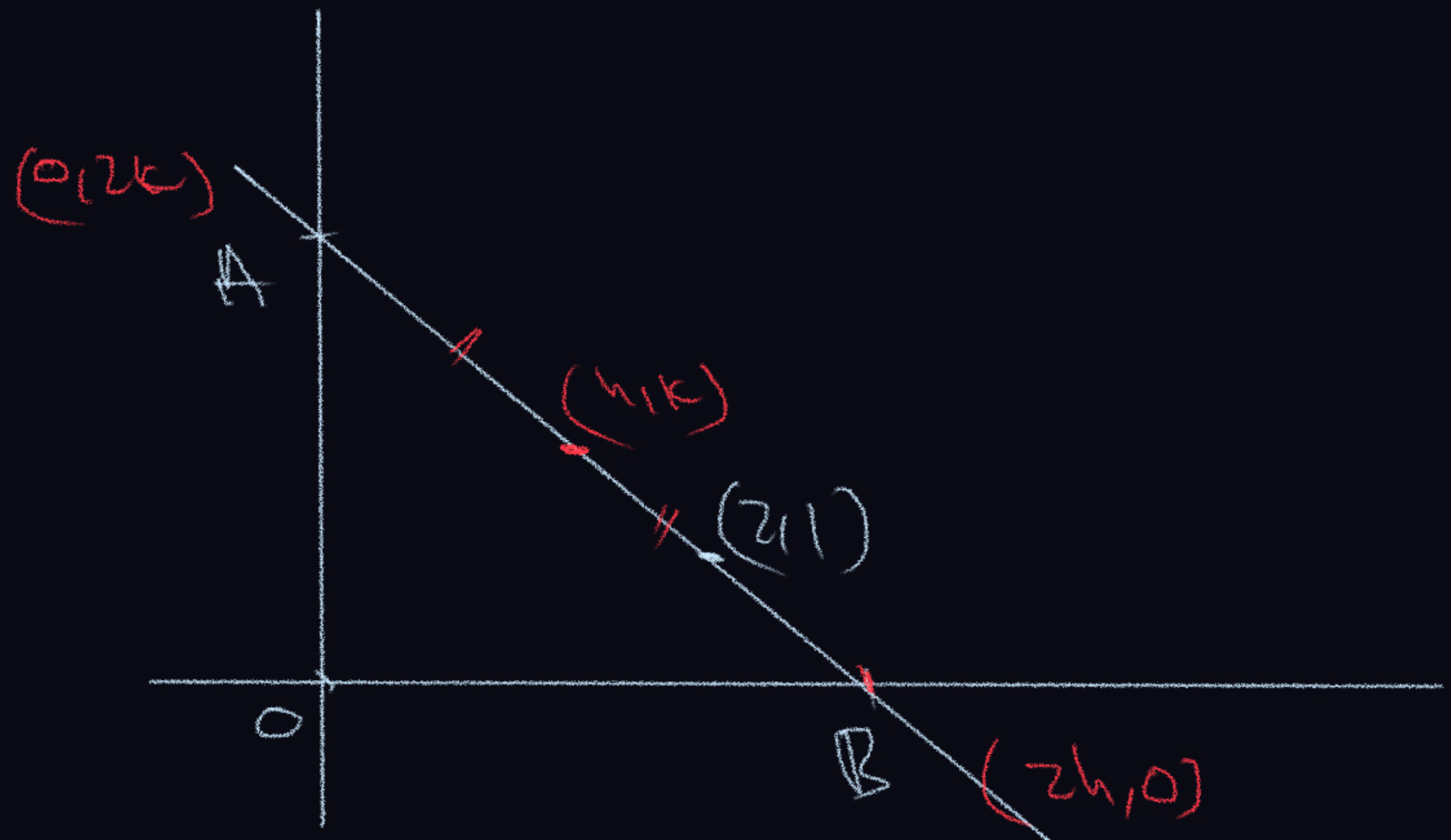
$$\cos\alpha = \frac{p}{2h} \qquad \sin\alpha = \frac{p}{2k}$$

$$\boxed{\left(\frac{p}{2h}\right)^2 + \left(\frac{p}{2k}\right)^2 = 1}$$

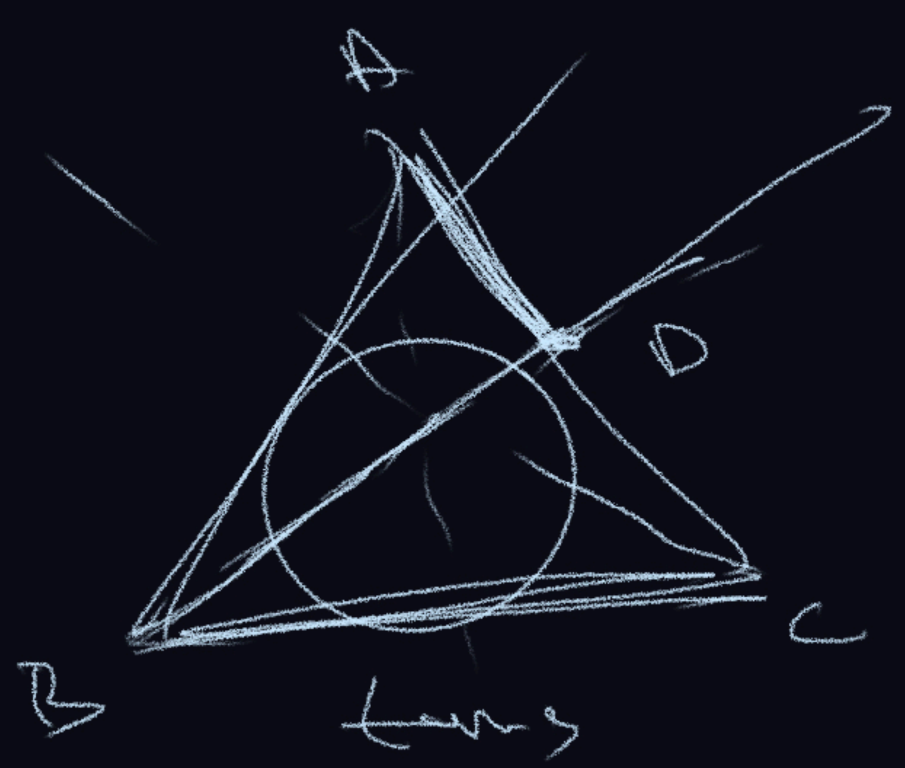


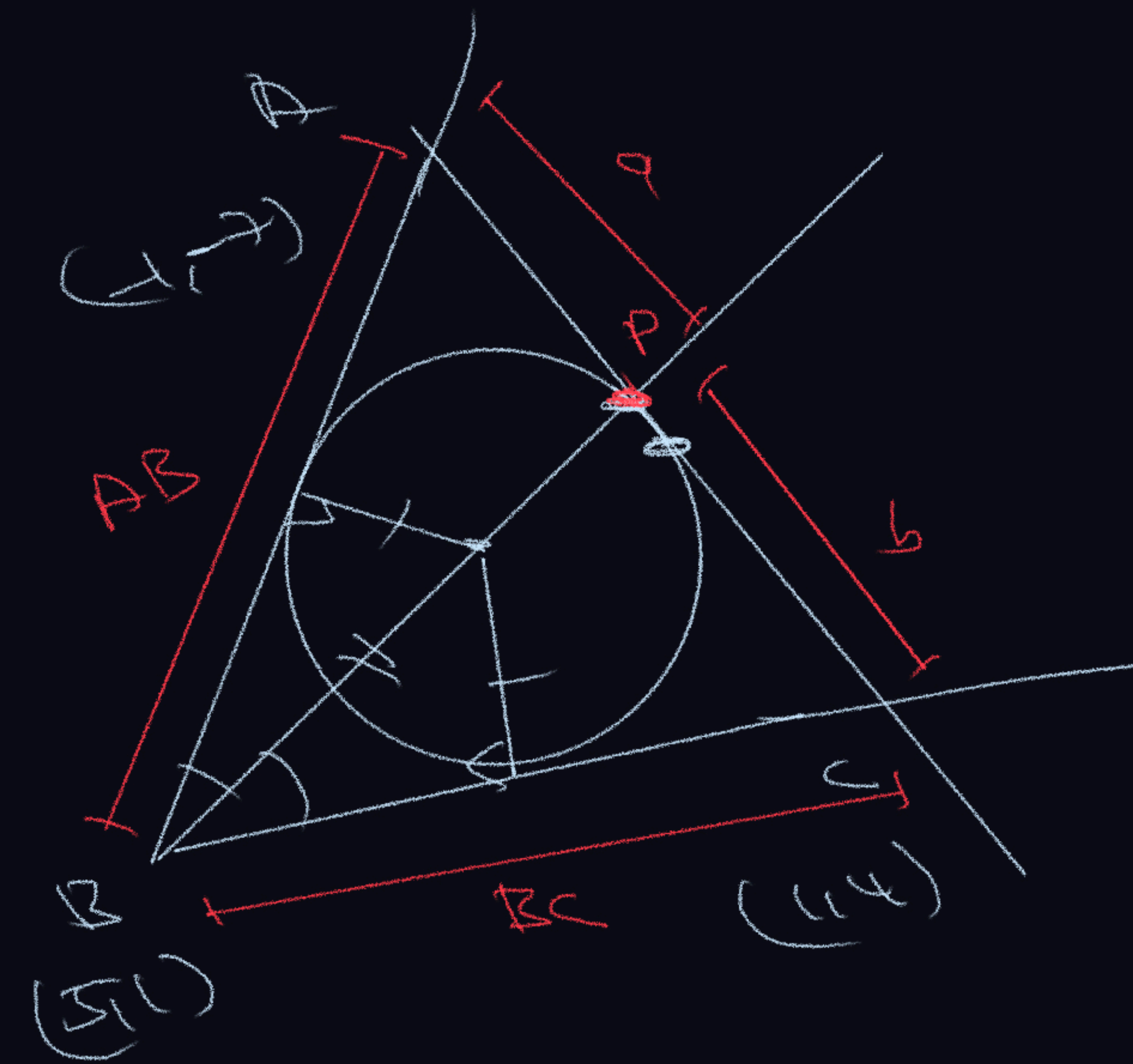
$$\begin{vmatrix} 0 & k \\ 3 & 4 \\ h & 0 \\ 0 & k \end{vmatrix} = 0$$

$$\boxed{hk - 3k - 4h = 0}$$

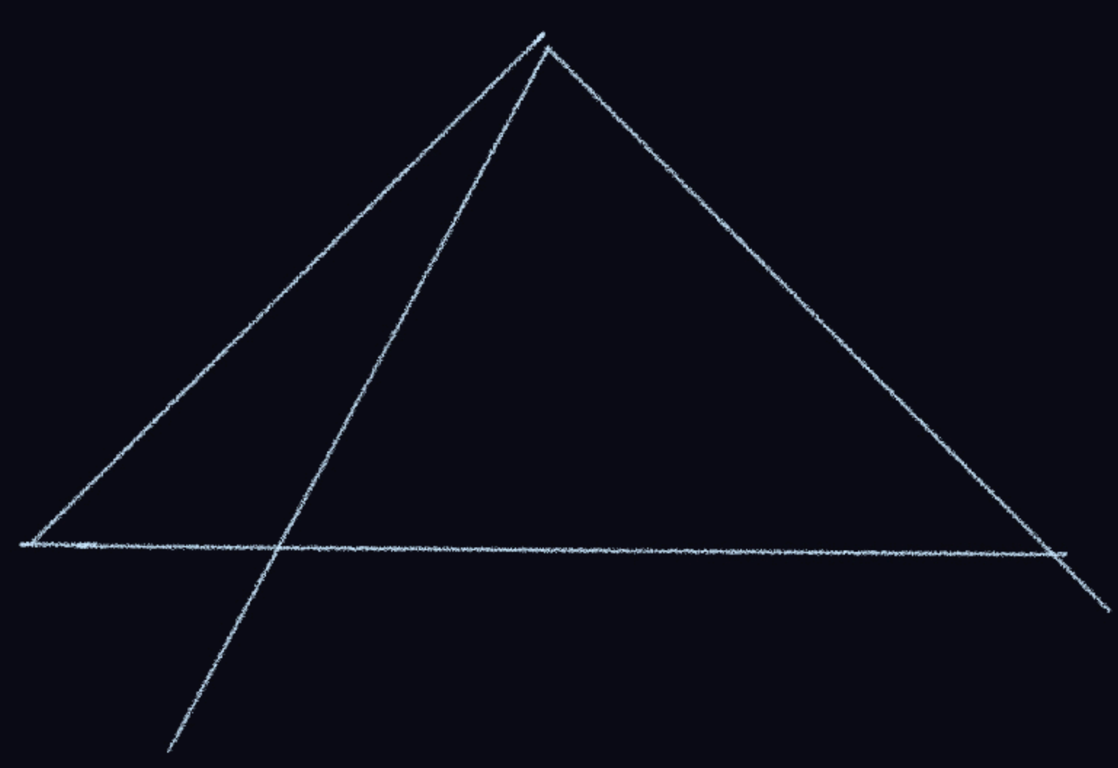


$$\begin{vmatrix} 0 & 2k \\ 2 & 1 \\ 2h & 0 \\ 0 & 2k \end{vmatrix} = 0$$

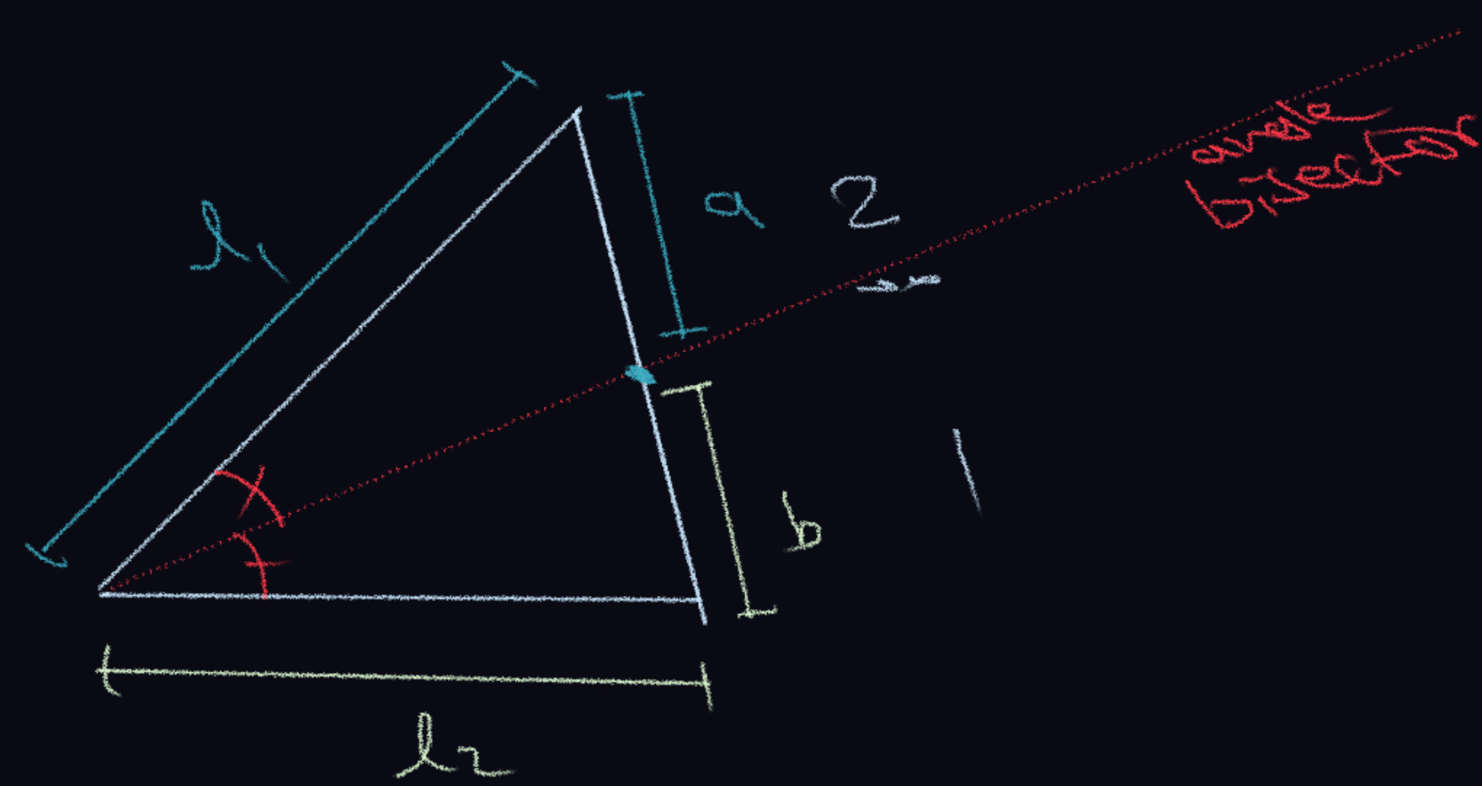




$$\frac{a}{AB} = \frac{b}{BC} \qquad \frac{a}{b} = \frac{AB}{BC}$$



Angular bisector theorem

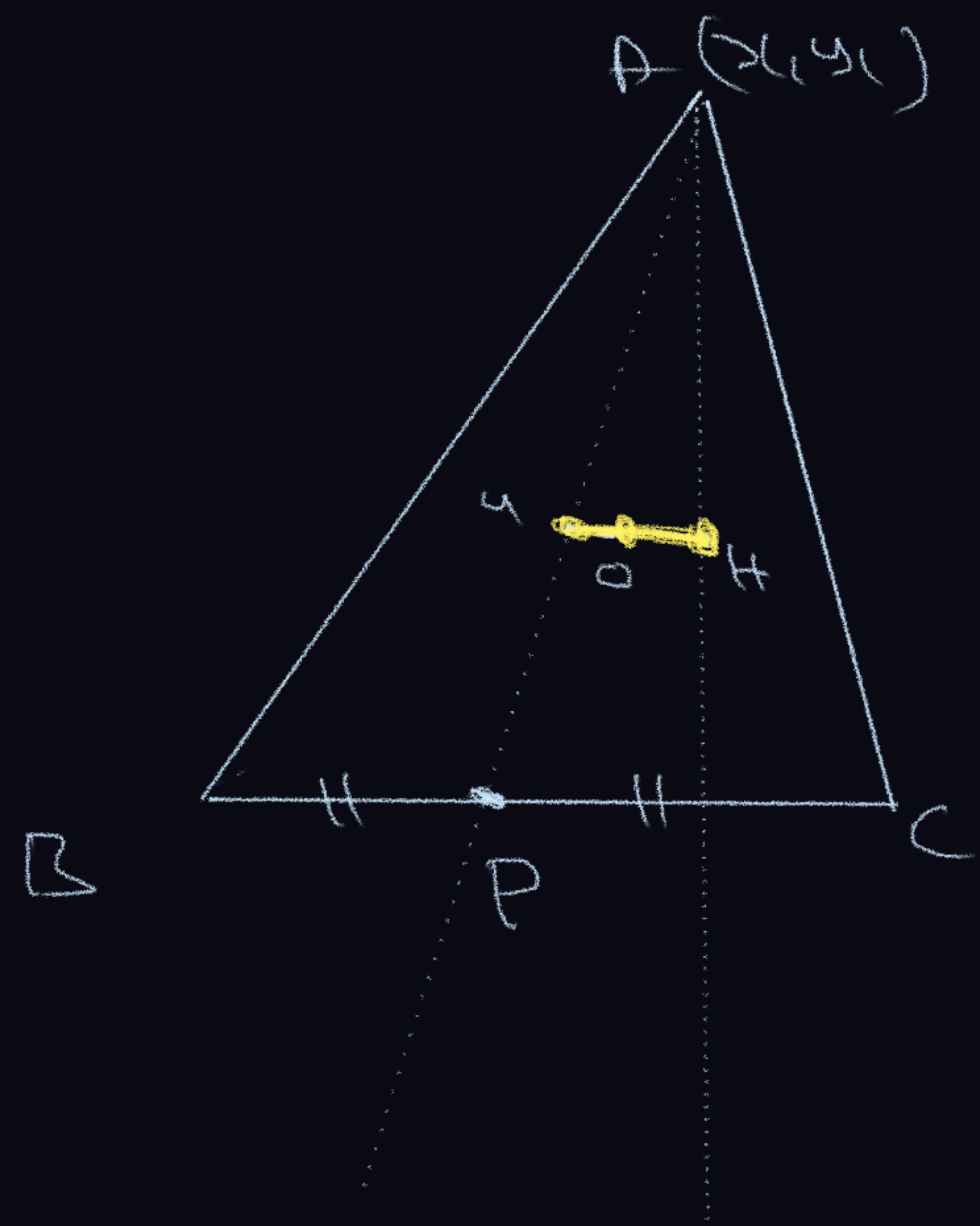


$$\frac{a}{l_1} = \frac{b}{l_2}$$

$$\frac{l_1}{l_2} = 2$$

orthocentre, circum, centroid = collinear

find coord of P?



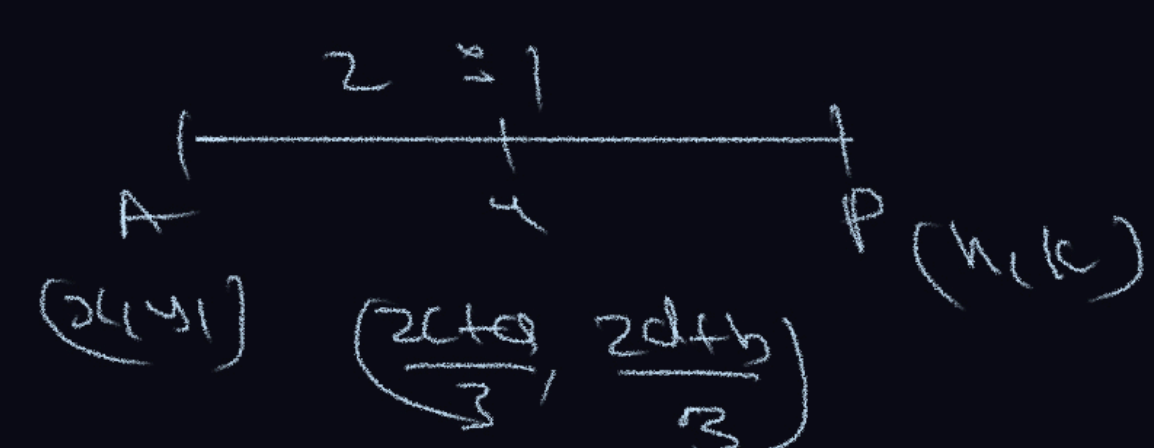
median

$H(a, b)$

$O(c, d)$

$$G = \frac{2O + H}{3} = \left(\frac{2c+a}{3}, \frac{2d+b}{3}\right)$$

$AG : GP = 2 : 1$



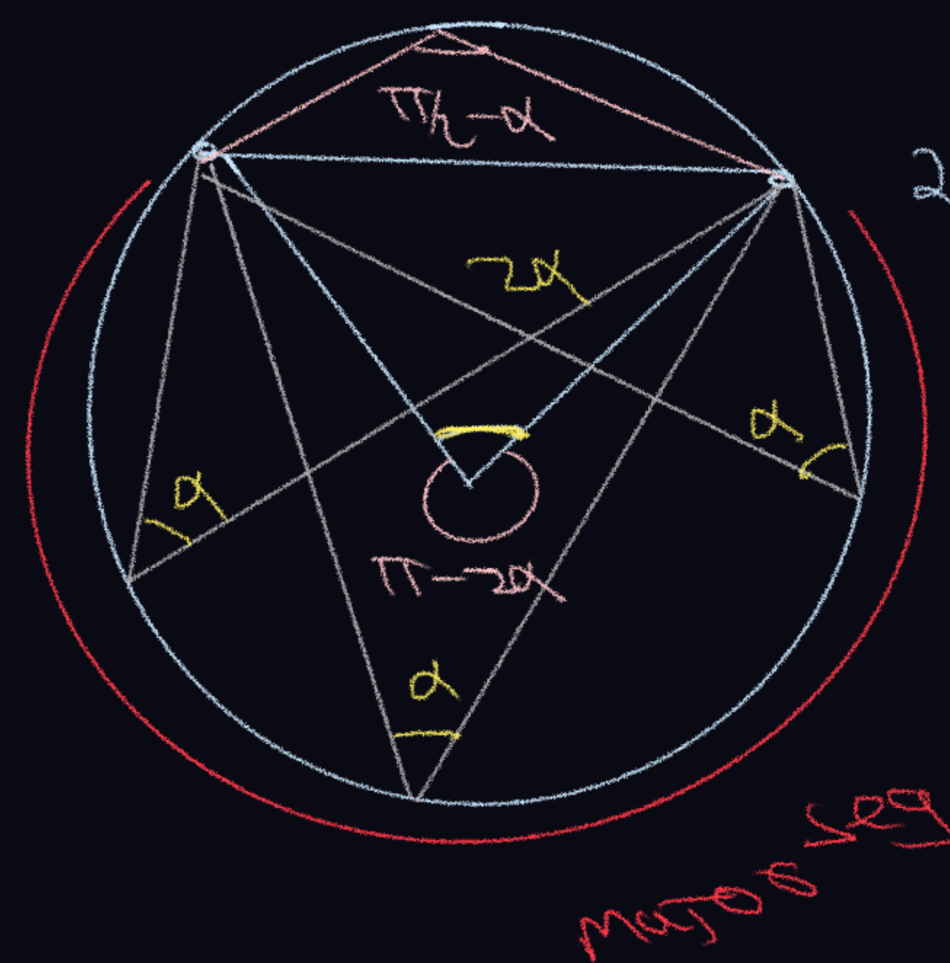
$$(h, k) = (2c + a - x_1, \ 2d + b - y_1)$$

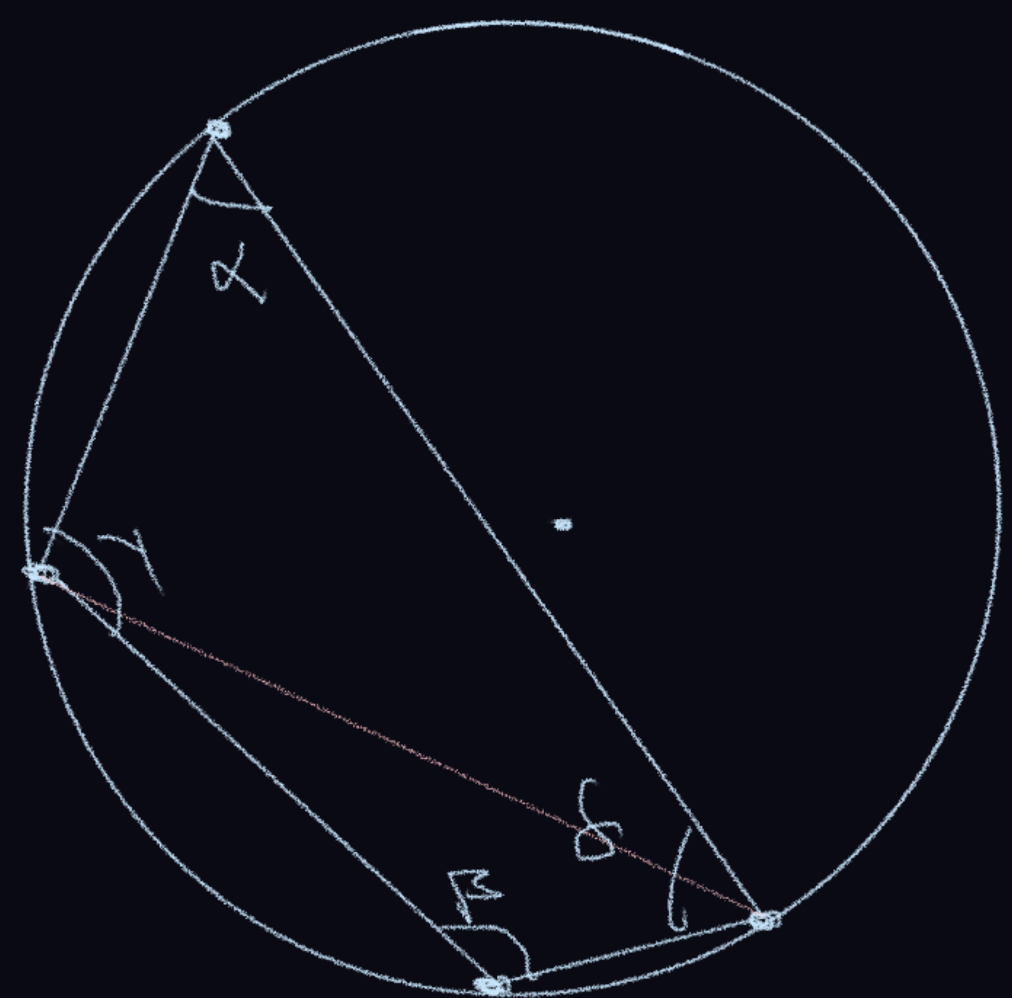
$$\frac{2c+a}{3} = \frac{2h + x_1}{3} \qquad \frac{2d+b}{3} = \frac{2k + y_1}{3}$$

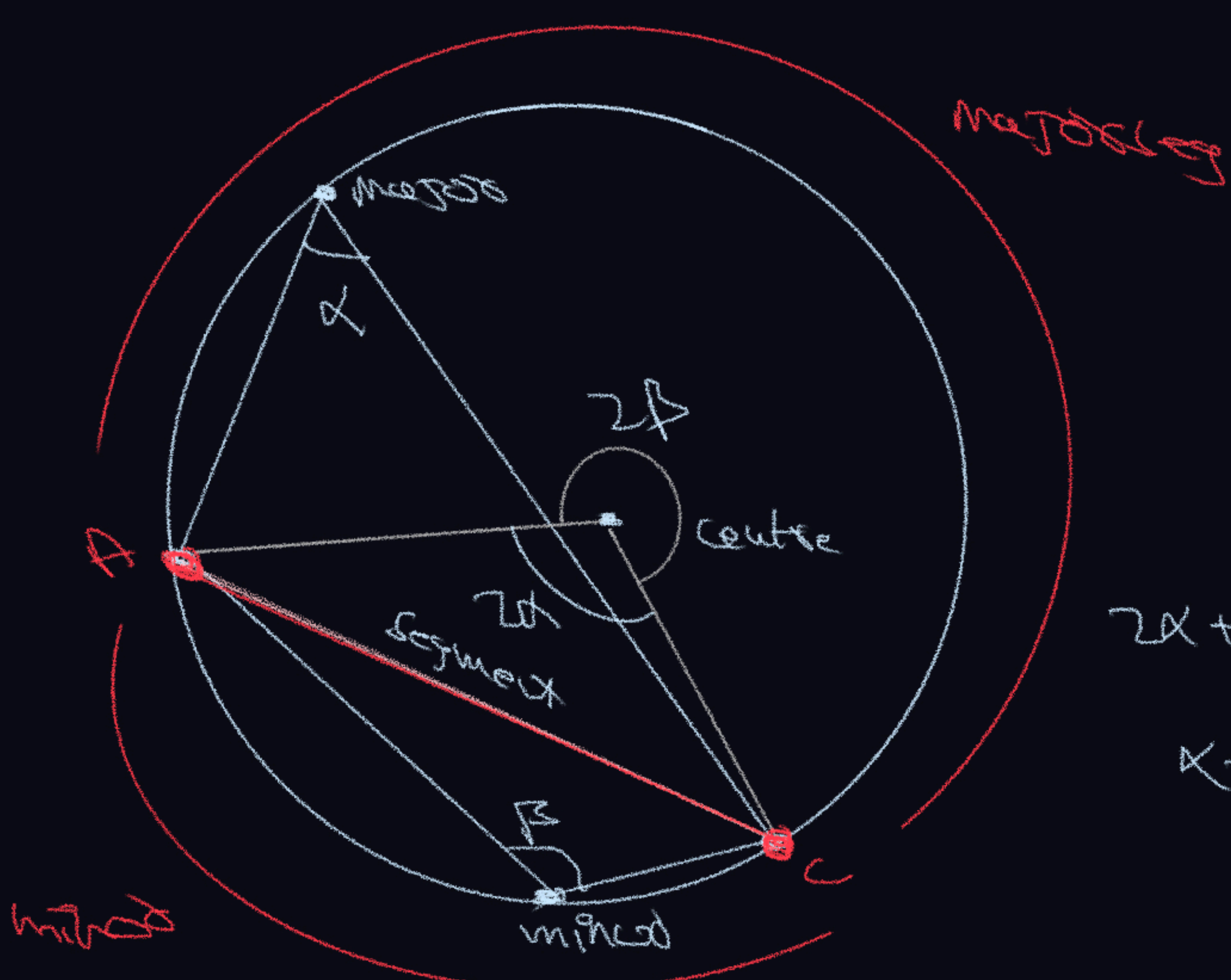
Cyclic Quadrilateral

∞ Cyclic points $\Big\}$ Sum of opposite angles $= \pi$

2(angle at center) = (angle at segment)

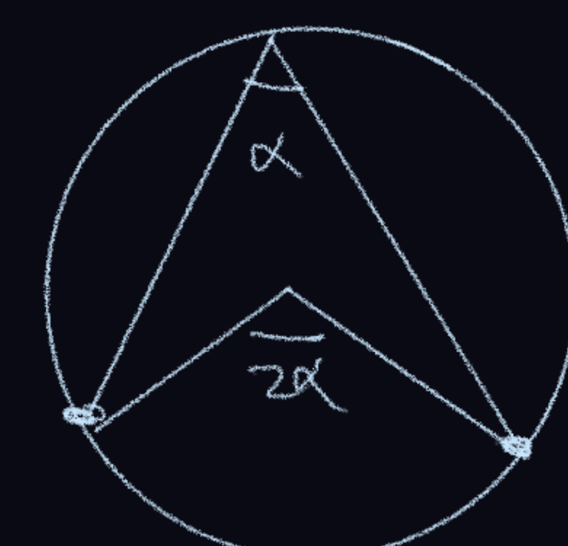


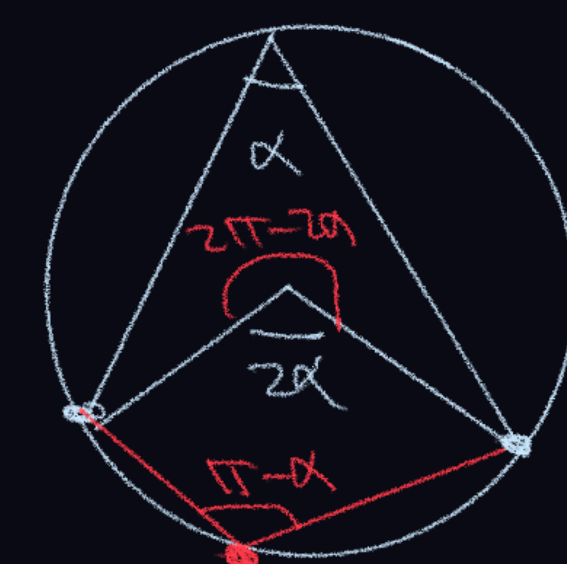




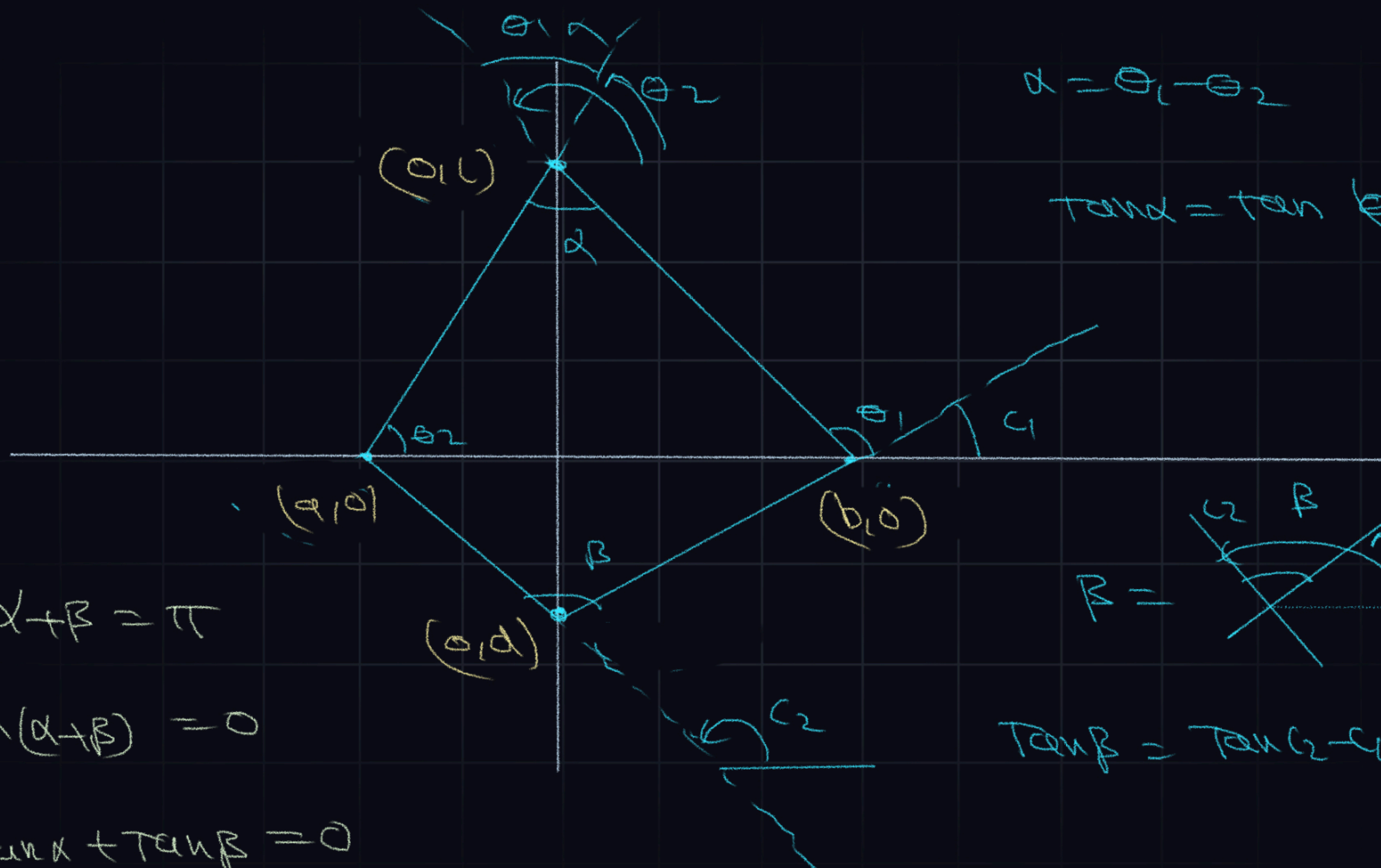
$2\alpha + 2\beta = 2\pi$

$\alpha + \beta = \pi$



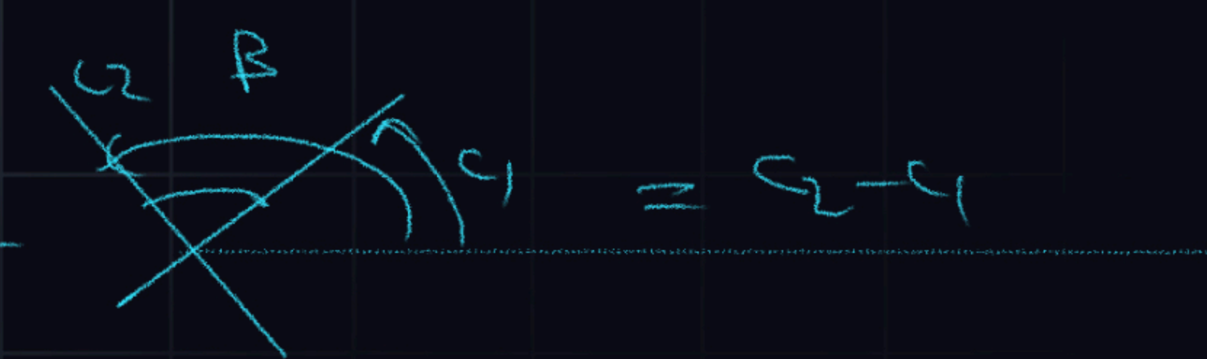


IF points $(a,0)$ $(b,0)$ $(0,c)$ and $(0,d)$ are concyclic points . Prove $(ab=cd)$



$\alpha = \theta_1 - \theta_2$

$\tan\alpha = \tan(\theta_1 - \theta_2) = \dfrac{T\theta_1 - T\theta_2}{1 + T\theta\, T\theta}$

$\beta =$  $= c_2 - c_1$

$\tan\beta = \tan(c_2 - c_1) = \dfrac{Tc_2 - Tc_1}{1 + Tc_2\, Tc_1}$

$\alpha + \beta = \pi$

$\tan(\alpha+\beta) = 0$

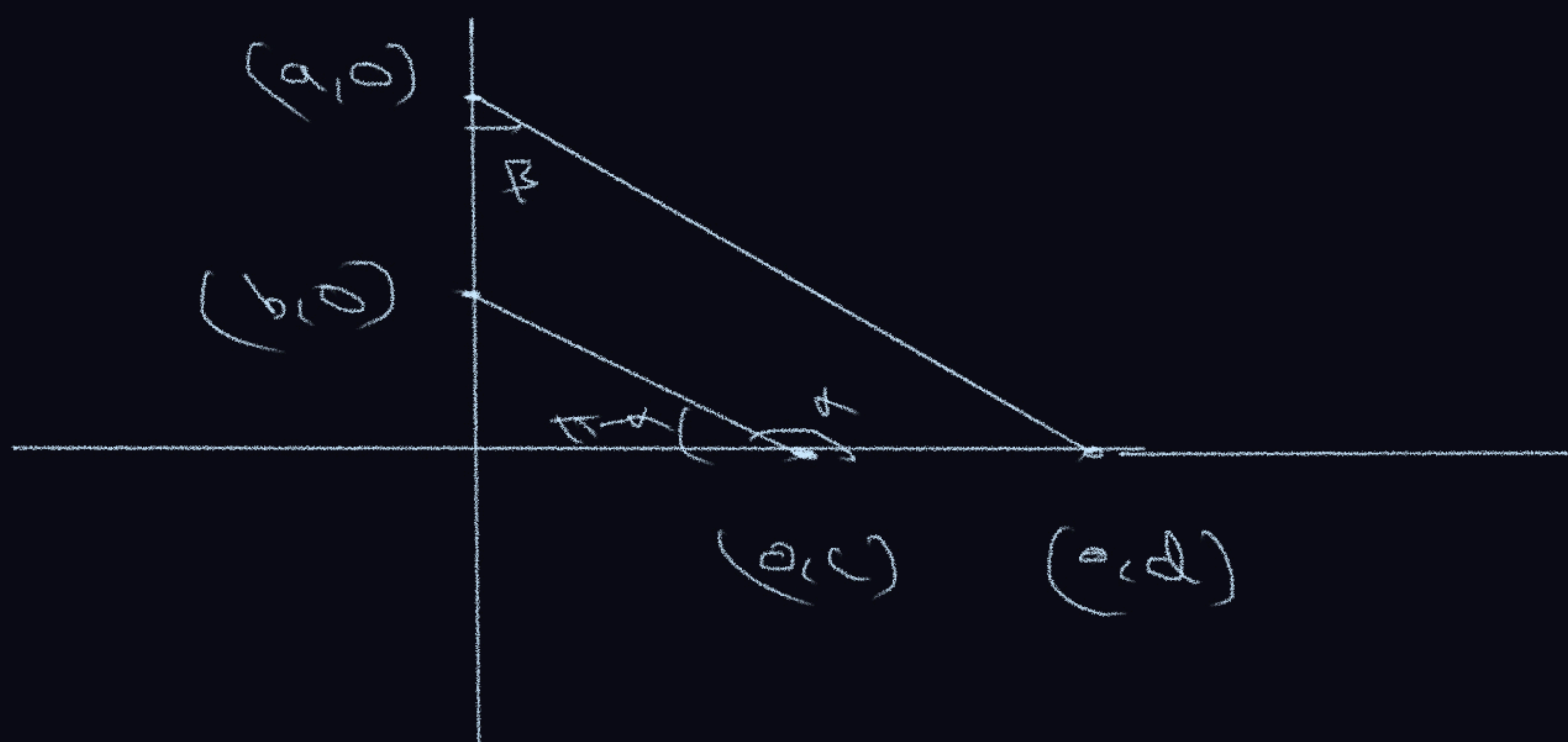
$\therefore \tan\alpha + \tan\beta = 0$

$$\frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 \tan\theta_2} + \frac{\tan c_2 - \tan c_1}{1 + \tan c_2 \tan c_1} = 0$$

$$\frac{\left(-\frac{c}{b}\right) - \left(-\frac{c}{a}\right)}{1 + \frac{c^2}{ab}} + \frac{\left(\frac{-d}{a}\right) - \left(\frac{-d}{b}\right)}{1 + \frac{d^2}{ab}} = 0$$

$$\frac{\frac{c}{a} - \frac{c}{b}}{1 + \frac{c^2}{ab}} + \frac{\frac{d}{b} - \frac{d}{a}}{1 + \frac{d^2}{ab}} = \frac{c(b-a)}{ab + c^2} + \frac{d(a-b)}{ab + d^2} = 0$$

$(a,0)$ $(b,0)$ $(0,c)$ and $(0,d)$



$-\tan\alpha = \tan(\pi-\alpha) = \frac{b}{c}$

$\tan\alpha = -\frac{b}{c}$

$\tan\beta = \frac{d}{a}$

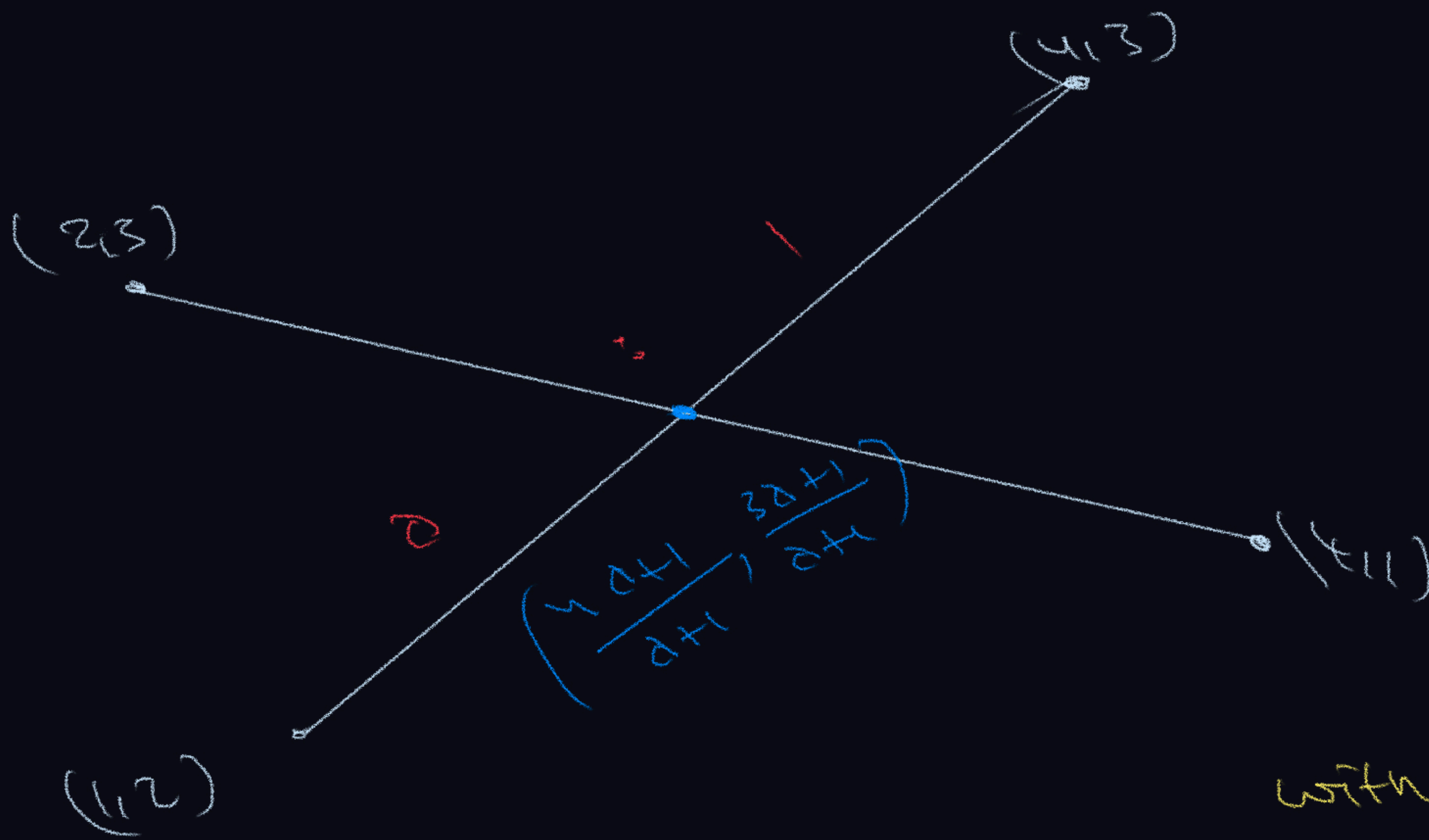
$\alpha + \beta = \pi$

$\tan\alpha + \tan\beta = 0$

$\frac{d}{a} - \frac{b}{c} = 0$

$\boxed{ab = cd}$

Straight Lines

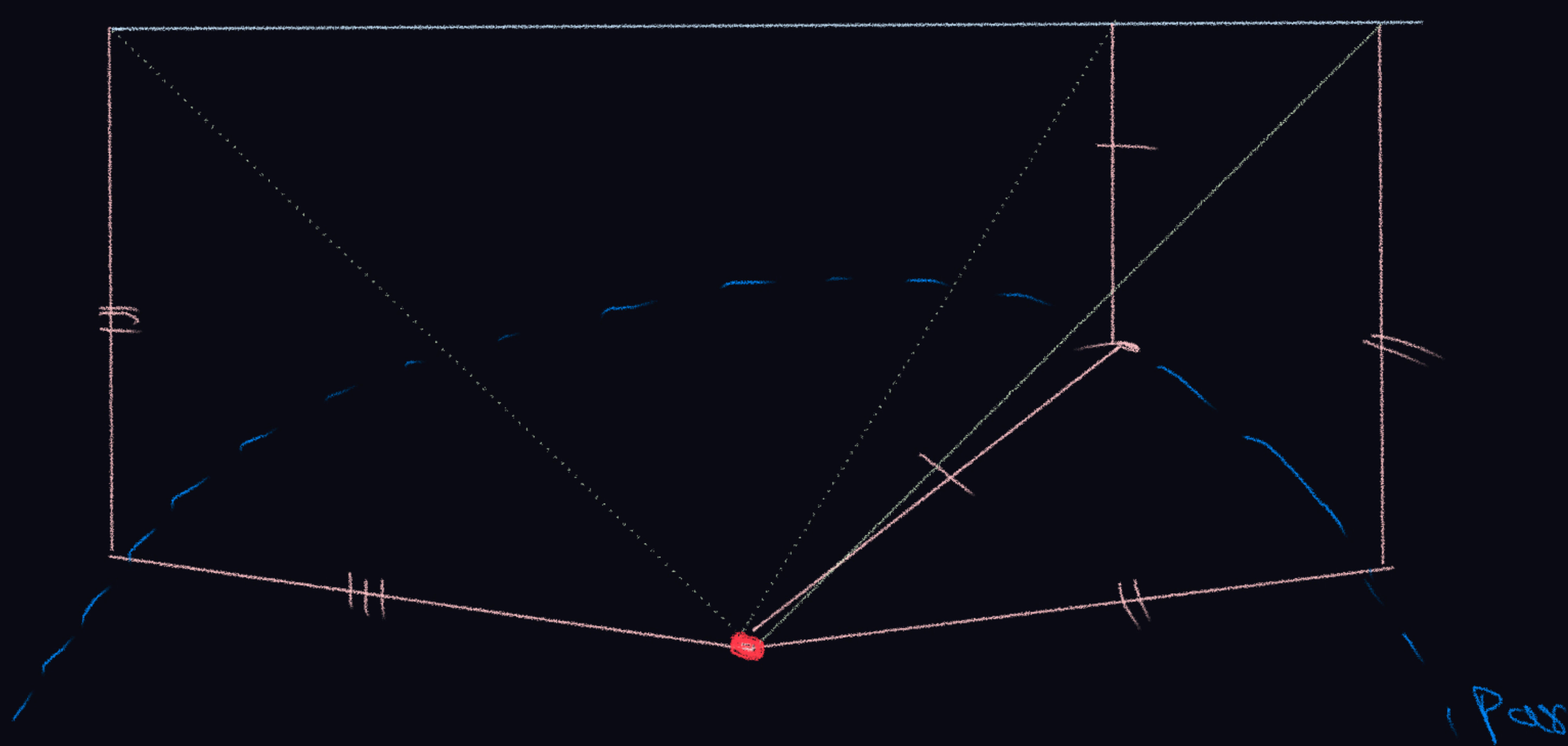


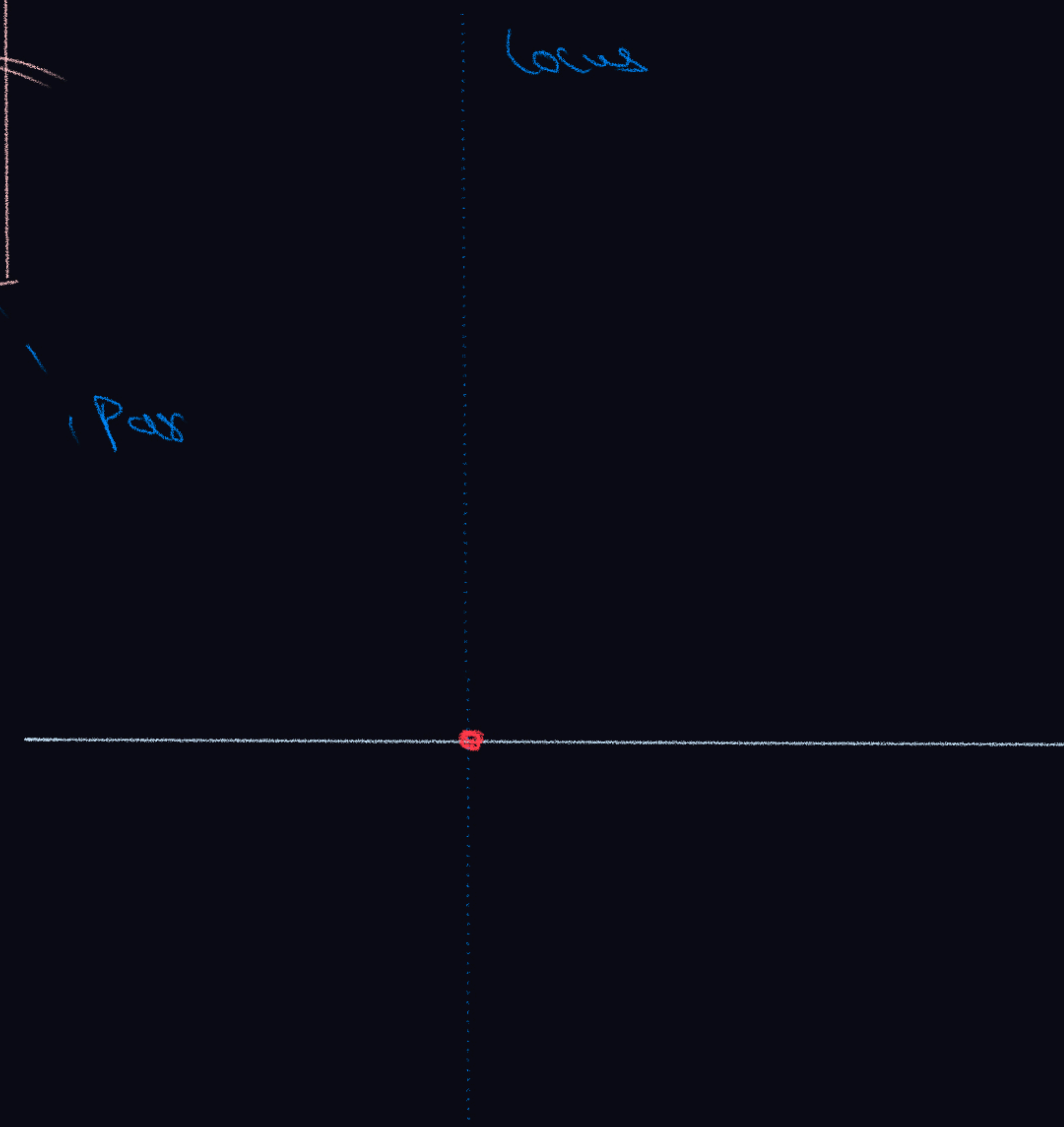
$$\frac{y-1}{x-4} = \frac{3-1}{2-4} = \frac{2}{-2} = -1$$

$$x+y-5=0$$

without losing generality

Check always



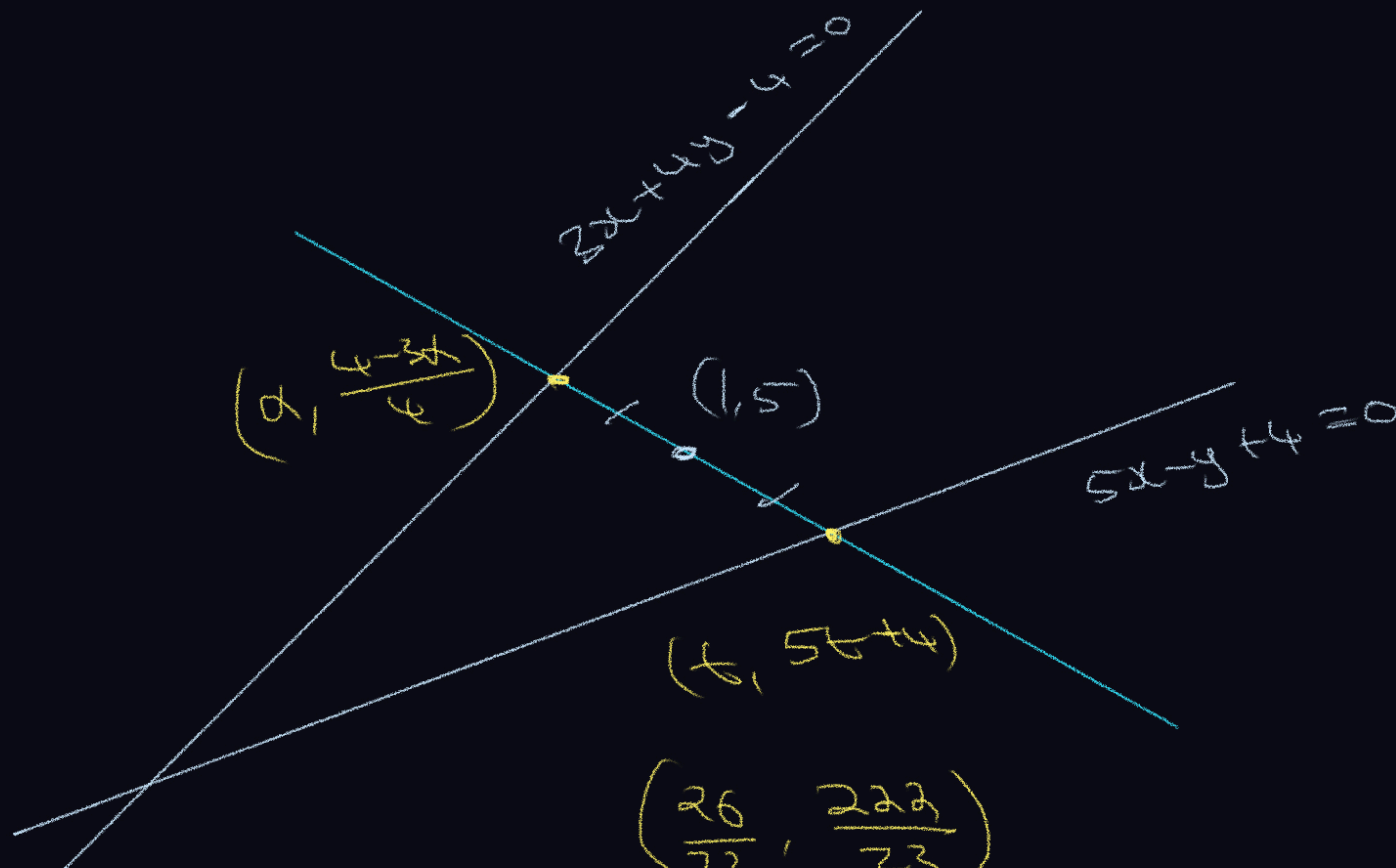


$(2\sqrt{3}, -1)$

$y = \sqrt{3}x - 7$

$m\sqrt{3} = -1$

$y = -\frac{1}{\sqrt{3}}x + c$



$\left(\frac{26}{23}, \frac{222}{23}\right)$

$\alpha+t=2$

$\frac{4-3\alpha}{4}+5t+4=10$

$\cancel{1}-\frac{3}{4}\alpha+5t+\cancel{4}=\cancel{10}\ 5$

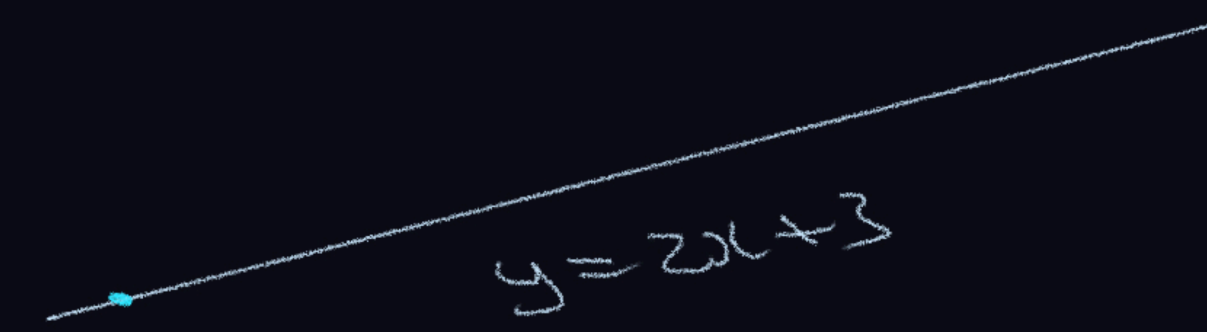
$20t-3\alpha=20$

$20t-3(2-t)=20$

$20t+3t-6=20$

$t=\frac{26}{23}$

Parametric point on a line



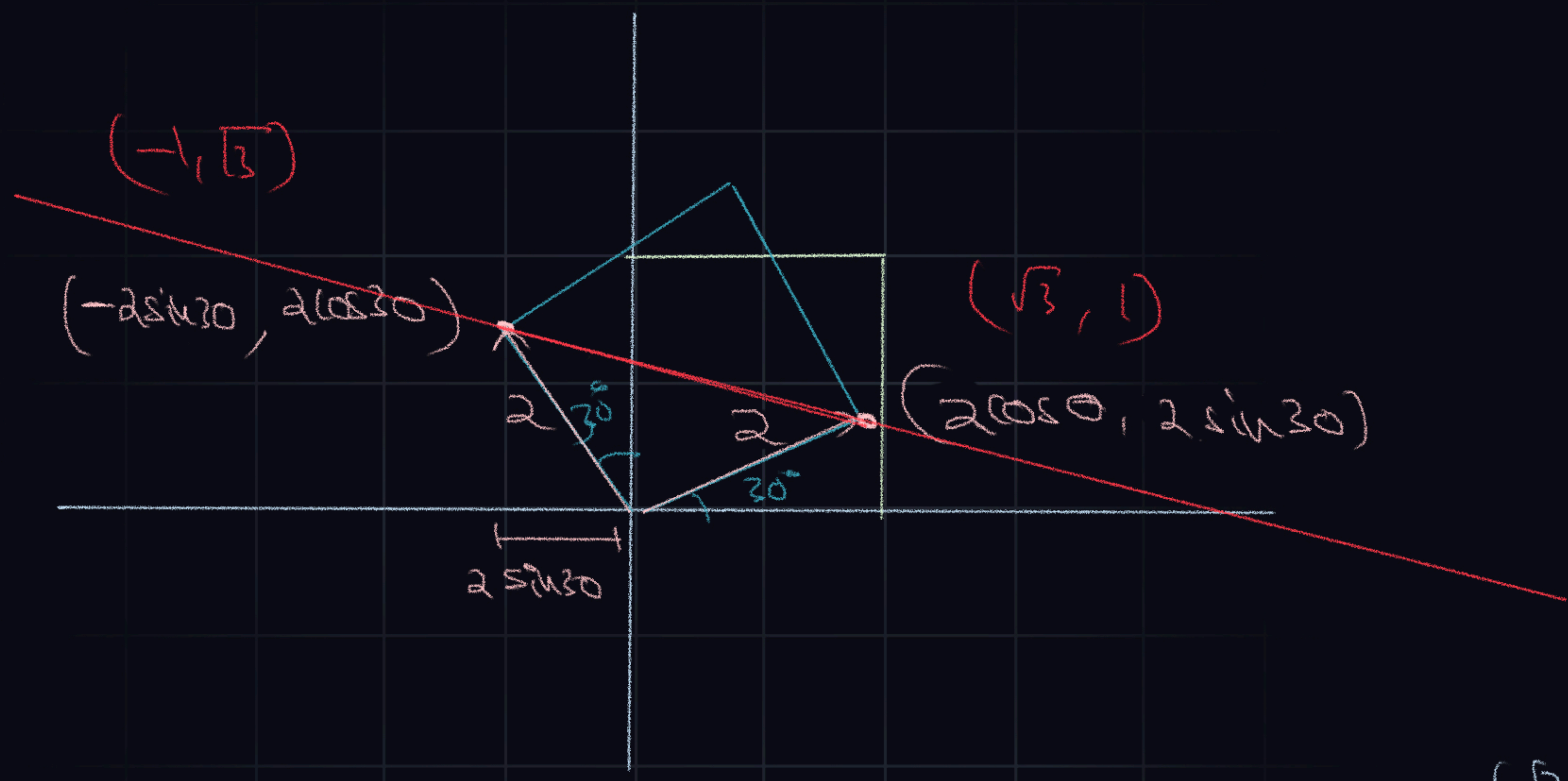
$(\alpha, 2\alpha+3)$

$2x-3y+2=0$

$\left(t, \frac{2t+2}{3}\right)$

$\frac{y-5}{x-1}=\frac{\frac{222}{23}-5}{\frac{26}{23}-1}=\frac{107}{3}$

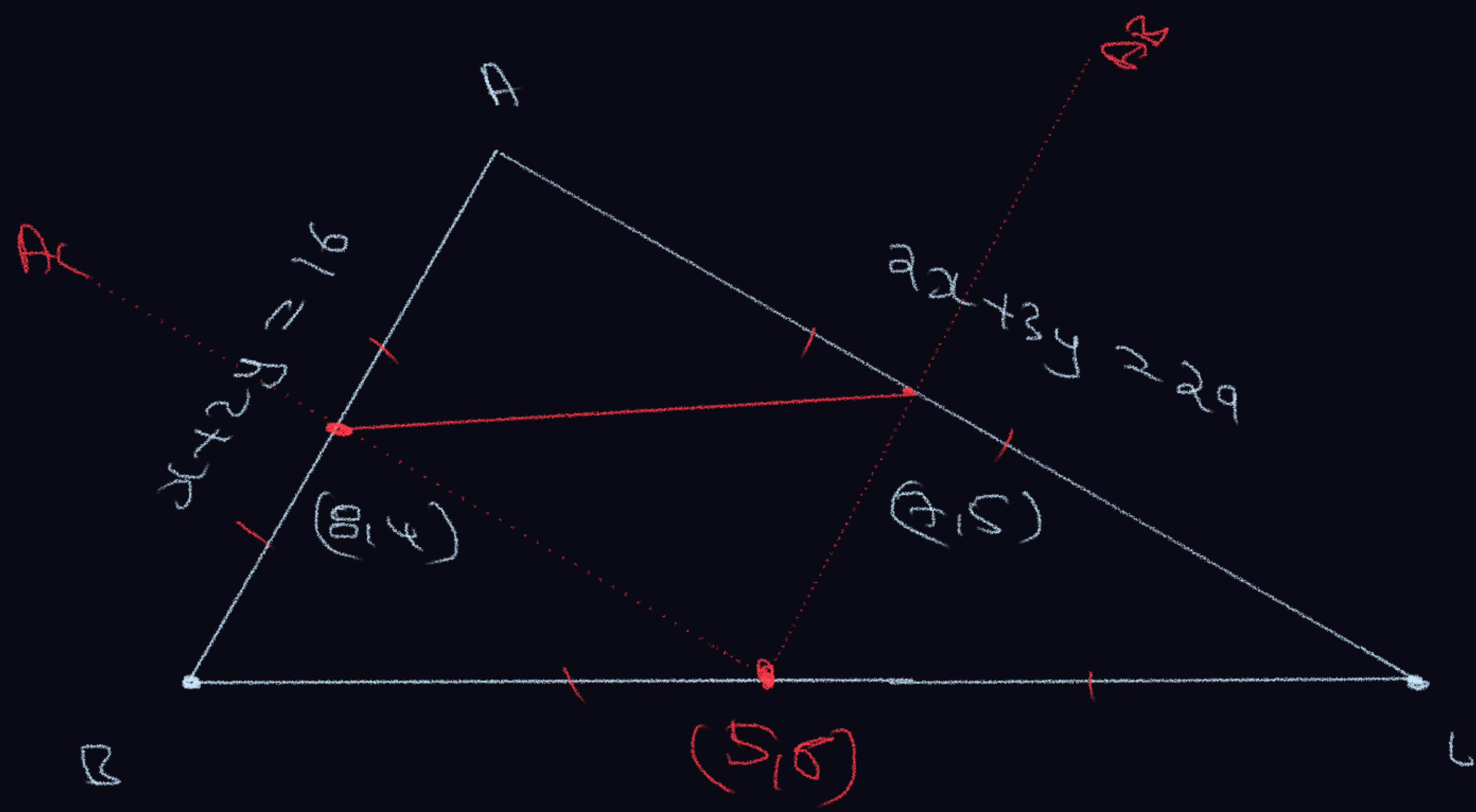
$3y-107x+92=0$



$$\frac{y-1}{x-\sqrt{3}} = \frac{1-\sqrt{3}}{\sqrt{3}+1}$$

$$(\sqrt{3}+1) - (1-\sqrt{3})x = (\sqrt{3})^2 + 1$$

$$= 4$$



AB $\equiv$ $x+2y=17$

AC $\equiv$ $2x+3y=28$

$$2x+3y=29$$

$$2 \times \quad x+2y=17$$

$$-y=-5$$

$$y=5$$

$$x=7$$

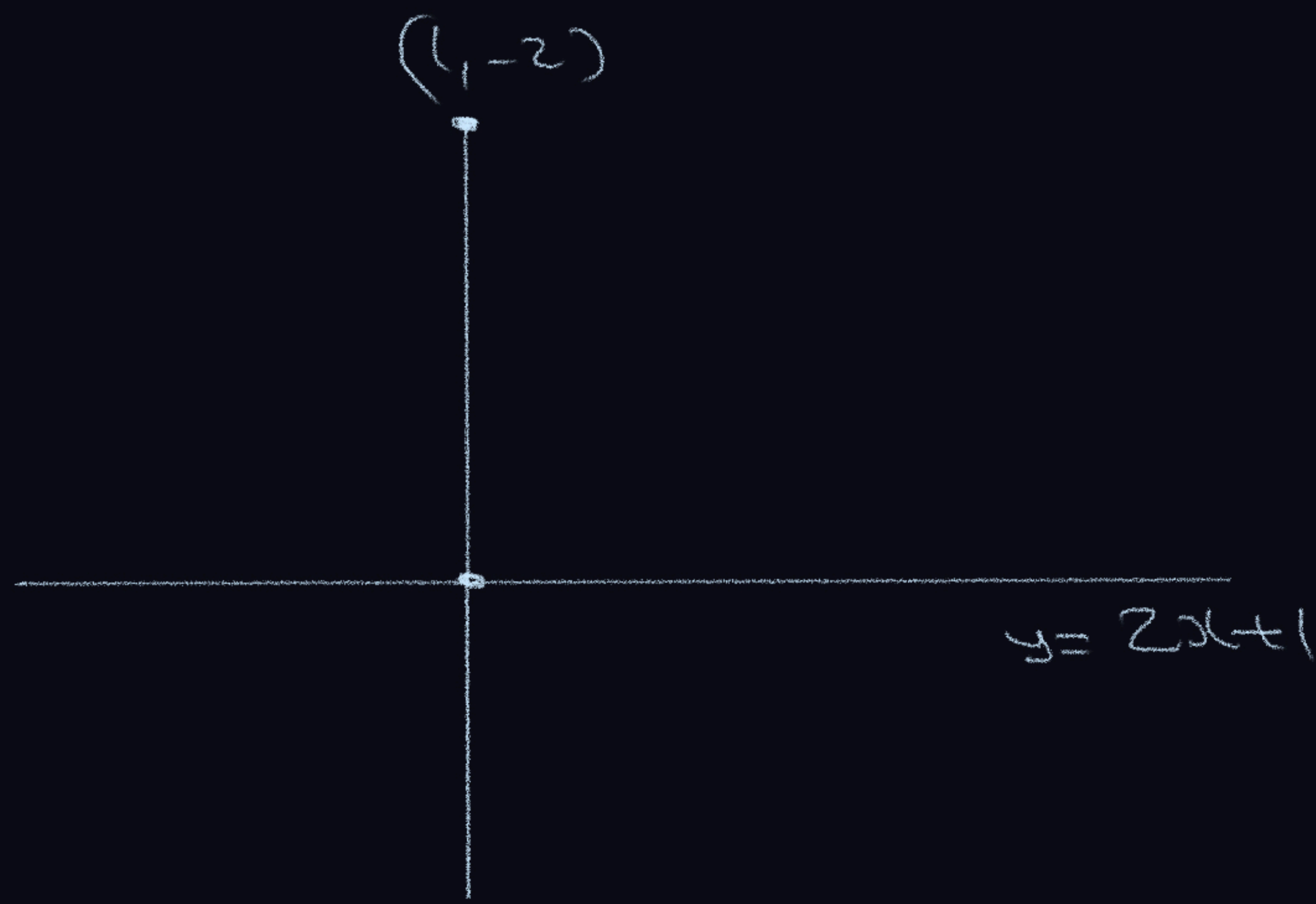
Equation of BC

$$m=-1$$

$$y=-x+c$$

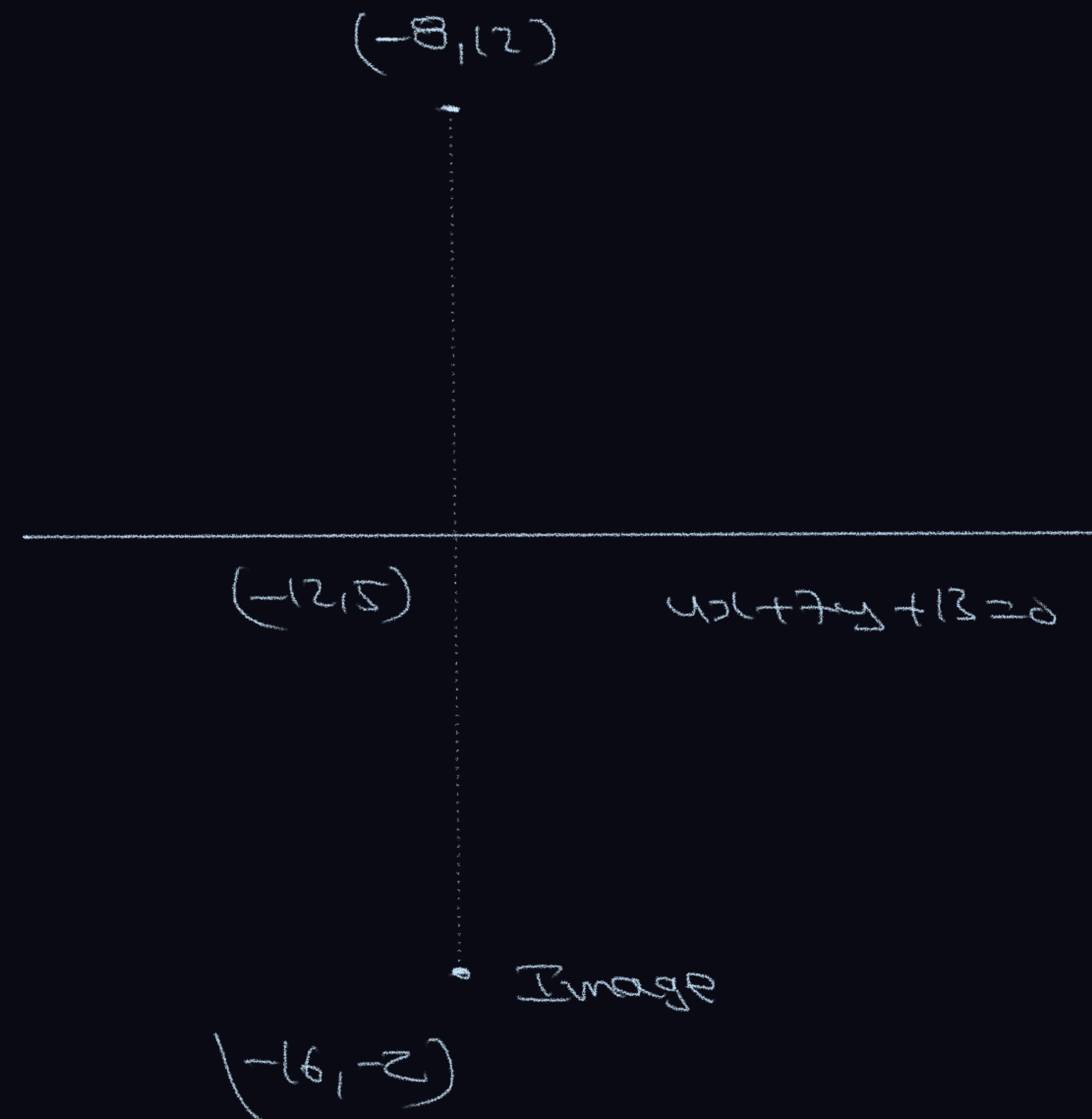
$$6=-5+c$$

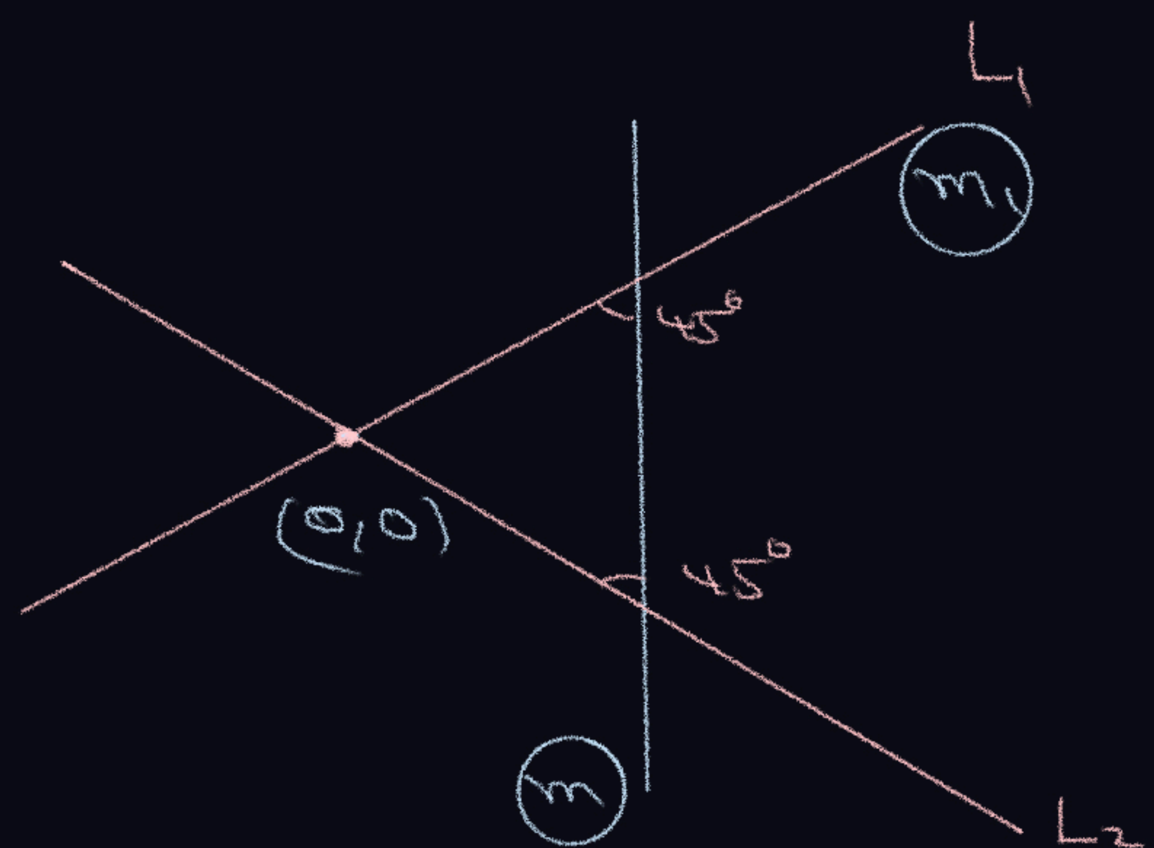
$$\boxed{x+y=1}$$



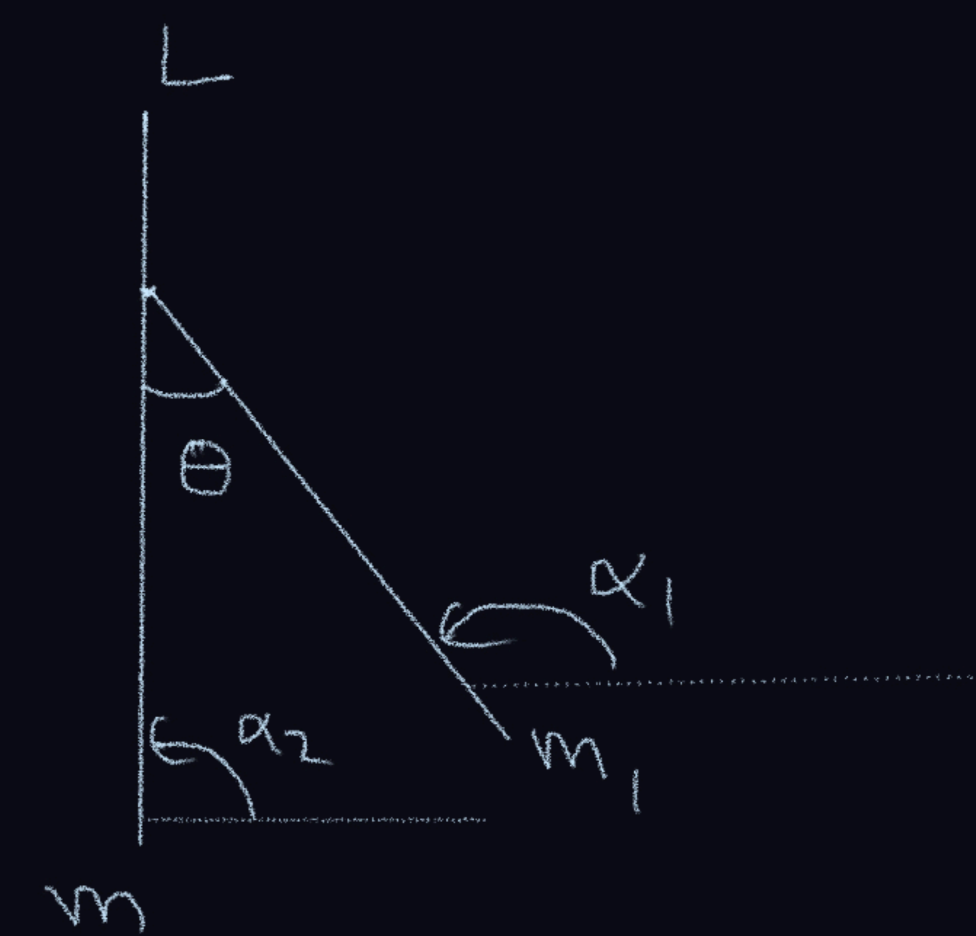
Eq of line

$y = -\frac{1}{2}x + K$





$3x-4y-12=0$



$$\tan 45 = \left|\frac{m_1-m}{1+m_1m}\right|$$

$$\tan\theta = \tan(\alpha_1-\alpha_2)$$

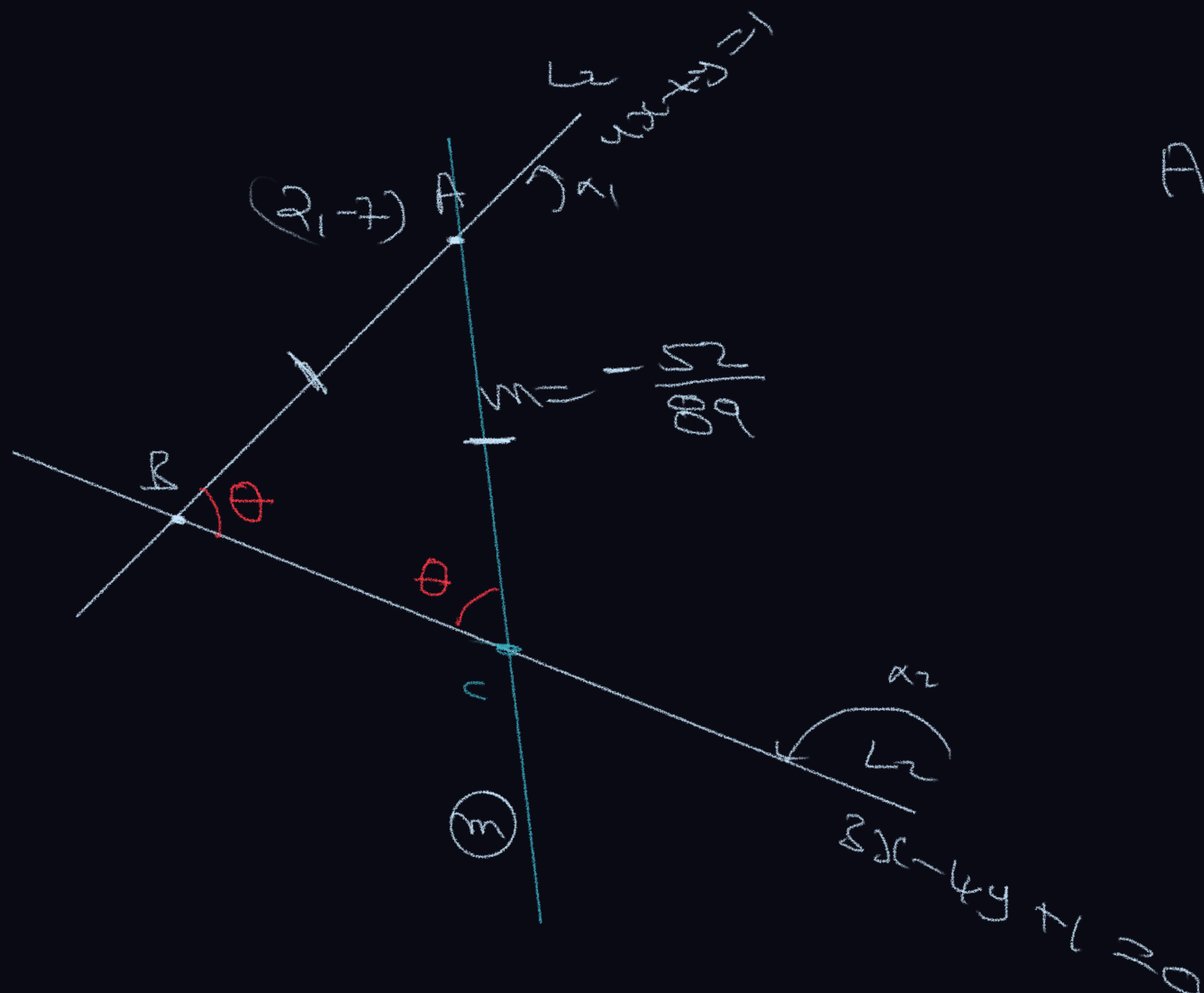
$$\tan\theta = \left|\frac{\tan\alpha_1-\tan\alpha_2}{1+\tan\alpha_1\tan\alpha_2}\right|$$

$$1 = \left|\frac{m_1-3/4}{1+\frac{3}{4}m_1}\right| = \left|\frac{4m_1-3}{4+3m_1}\right|$$

$$\boxed{\tan\theta = \left|\frac{m_1-m}{1+m_1m}\right|}$$

angle betw lines (m, m_1)

$$\frac{4m_1-3}{4+3m_1} = 1 \qquad \frac{4m_1-3}{4+3m} = -1$$



$$AB = AC$$

$$\tan(\alpha_2-\alpha_1) = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

$$\operatorname{Tan}\theta = \left|\frac{\frac{3}{4}-(-4)}{1+\frac{3}{4}(-4)}\right| = \left|\frac{\frac{3}{4}-m}{1+\frac{3}{4}m}\right|$$

$$\left|\frac{19}{-8}\right| = \left|\frac{\frac{3}{4}-m}{1+\frac{3m}{4}}\right|$$

$$\frac{\frac{3}{4}-m}{1+\frac{3}{4}m} = -\frac{19}{8} \text{ or } \frac{19}{8}$$

$$\frac{\frac{3}{4}-m}{1+\frac{3m}{4}} = \frac{19}{8}$$

$$\frac{3-4m}{4+3m} = \frac{19}{8}$$

$$24-32m = 76+57m$$

$$\boxed{\frac{-52}{89} = m}$$