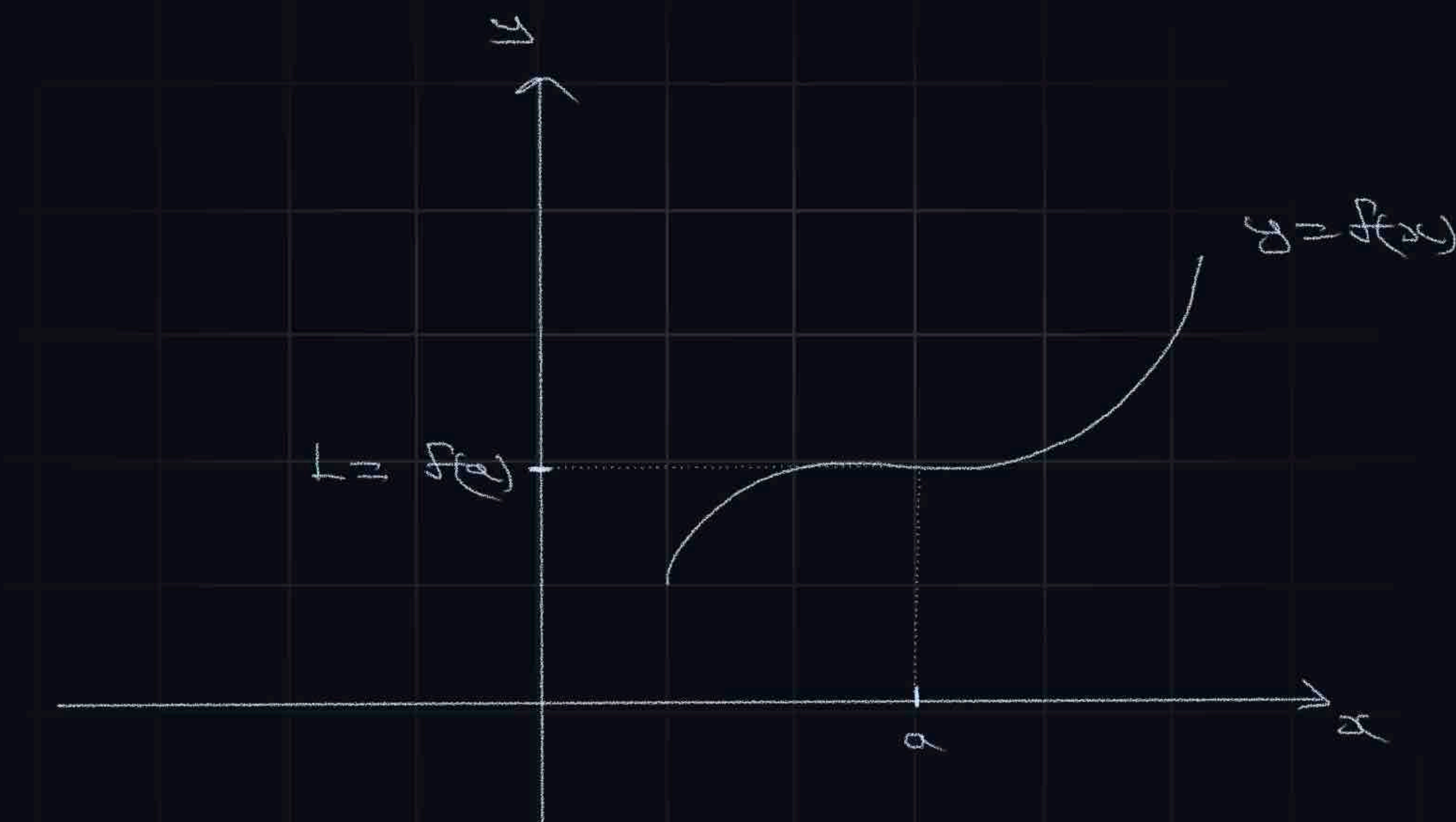
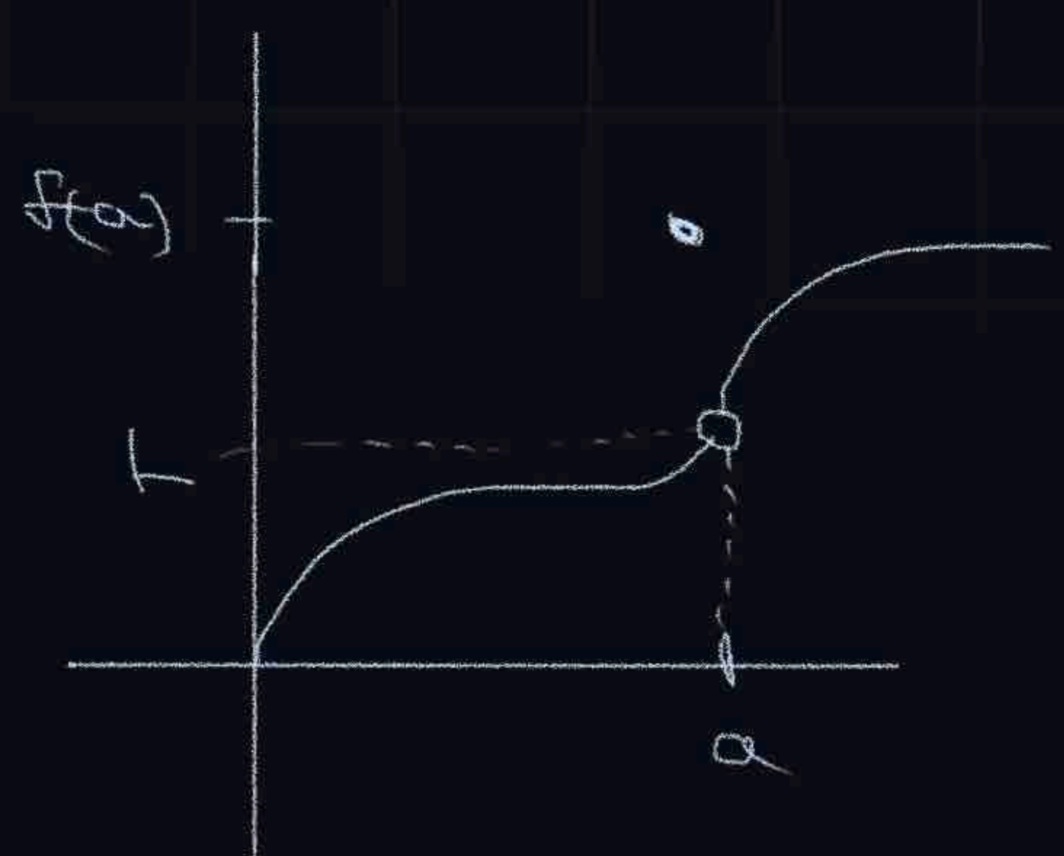
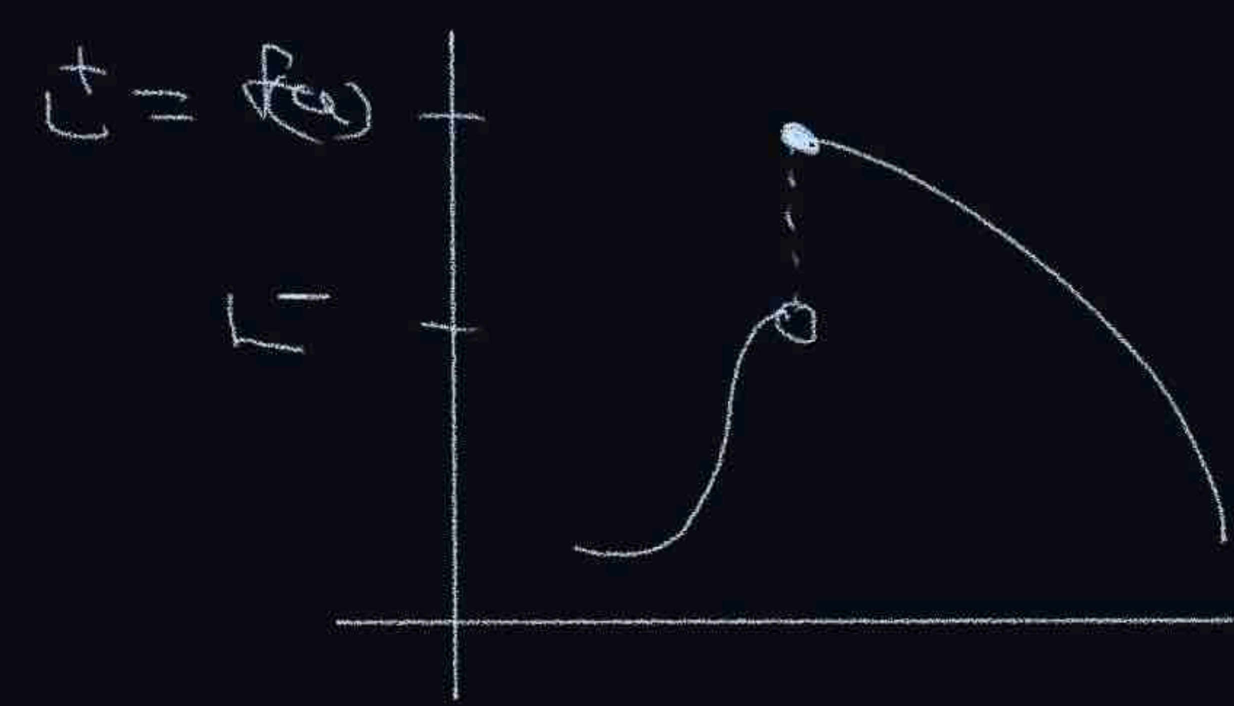


$$L = \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$$



$$L \neq f(a)$$



Continuity:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

$$f(x) = \frac{\log_e (1+x)^{1+x} - x}{x^2}$$

then find value of $f(0)$ so that $f(x)$ is continuous at $x=0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$f(x) = \begin{cases} \dfrac{a + 3\cos x}{x^2} & x < 0 \\ b\tan \dfrac{\pi}{[x+3]} & x \geq 0 \end{cases}$$

$f(x)$ is cont at $x=0$

find a and b.

step function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & x < 0 \\[2ex] \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{b\,x^{3/2}} & x > 0 \\[2ex] c & x = 0 \end{cases}$$

$f(x)$ is cont at $x = 0$

find a, b, c ?

$$f(x) = \begin{cases} \dfrac{e^{px} + \log_e(1+4x) + q}{x^3} & x \neq 0 \\[2ex] r & x = 0 \end{cases}$$

$$f(x) = \begin{cases} \dfrac{e^{px} + \log_e(1+4x) + q}{x^3} & x \neq 0 \\ r & x = 0 \end{cases}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

$$f(x+y) = f(x) \times f(y)$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\lim_{h \to 0} f(1-h) = \lim_{h \to 0} f(1+h) = f(1)$$

$$f(1)\, f(-h) = f(1)\, f(h) = f(1)$$

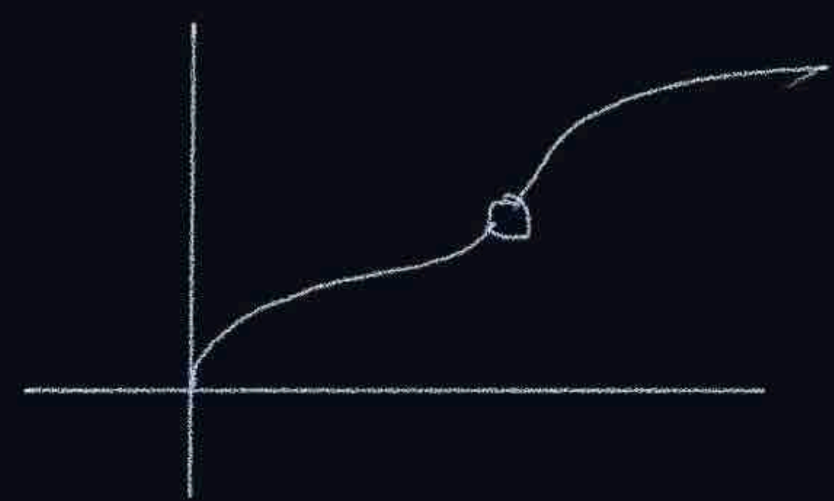
$$\boxed{f(-h) = f(h) = 1}$$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

$$f(a)\, f(-h) = f(a)\, f(h) = f(a)$$

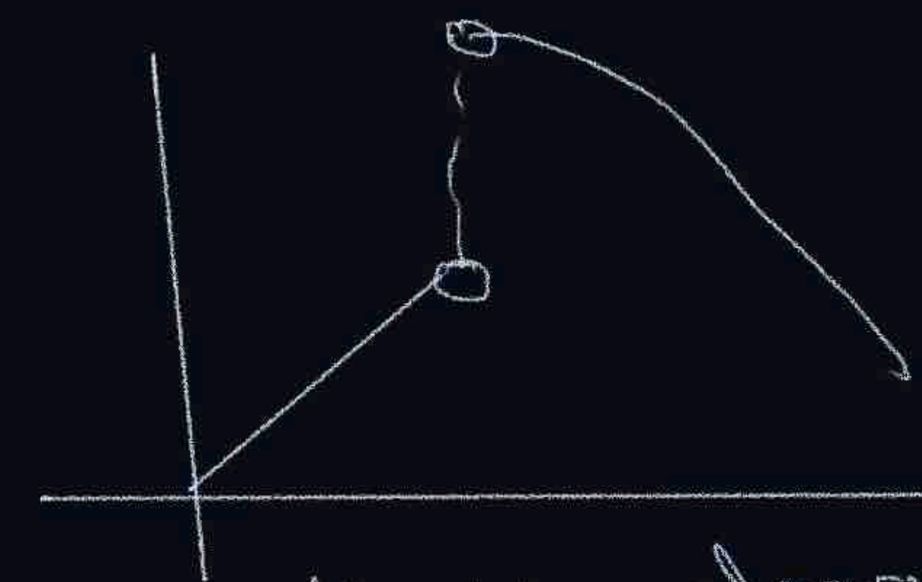
Discontinuity

Removable



Limit exist

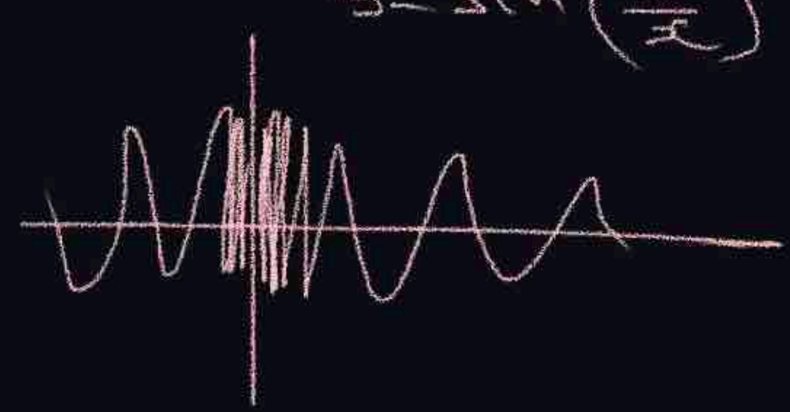
Nonremovable



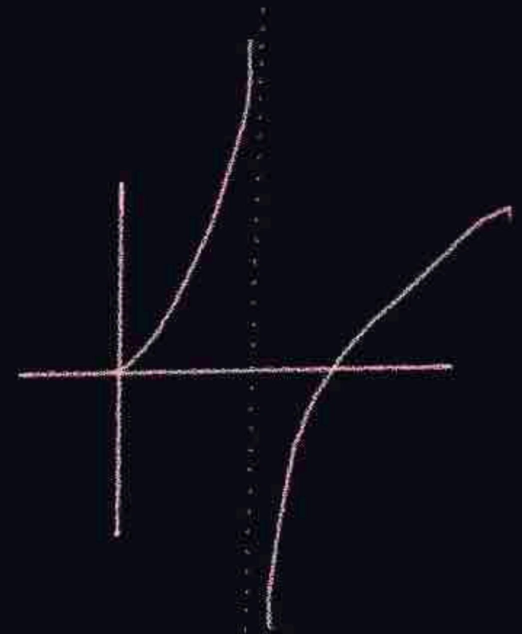
Limit doesn't exist

Oscillatory

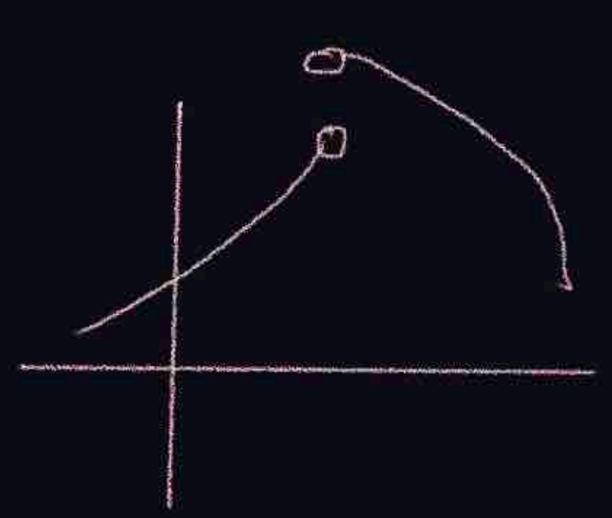
$y = \sin\left(\frac{\pi}{x}\right)$



Infinite



Finite



Oscillatory

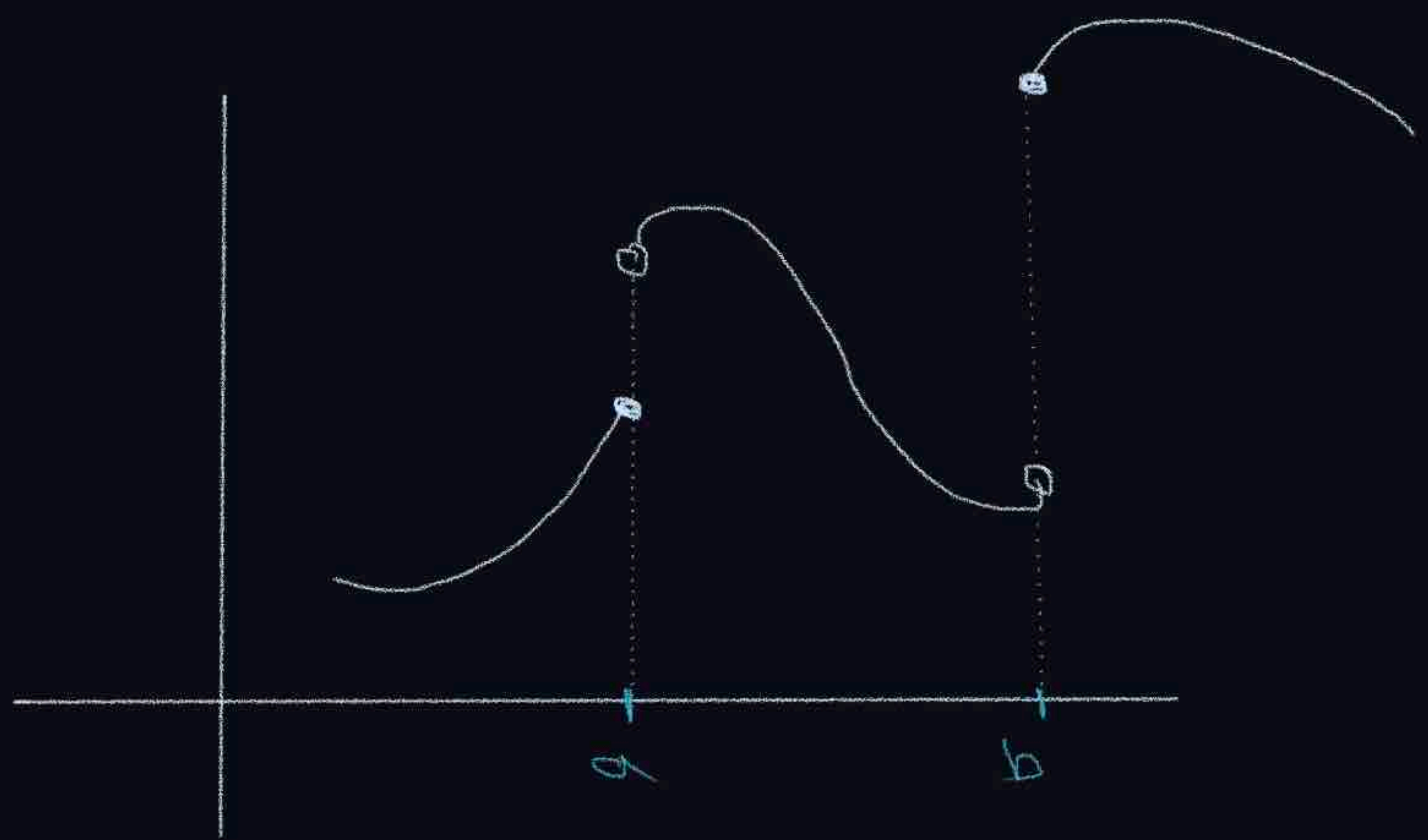
Infinity

Jump

Isolated point

removable

Directional continuity



at $x=a$ $\quad \lim_{x\to a^-} f(x) = f(a)$ $\quad$ Left continuous

at $x=b$ $\quad \lim_{x\to b^+} f(x) = f(b)$ $\quad$ Right continuous

Continuity in an interval

closed interval

$x \in [a,b]$

$f(a) = f(a^+) \Rightarrow$ cont at $x = a$

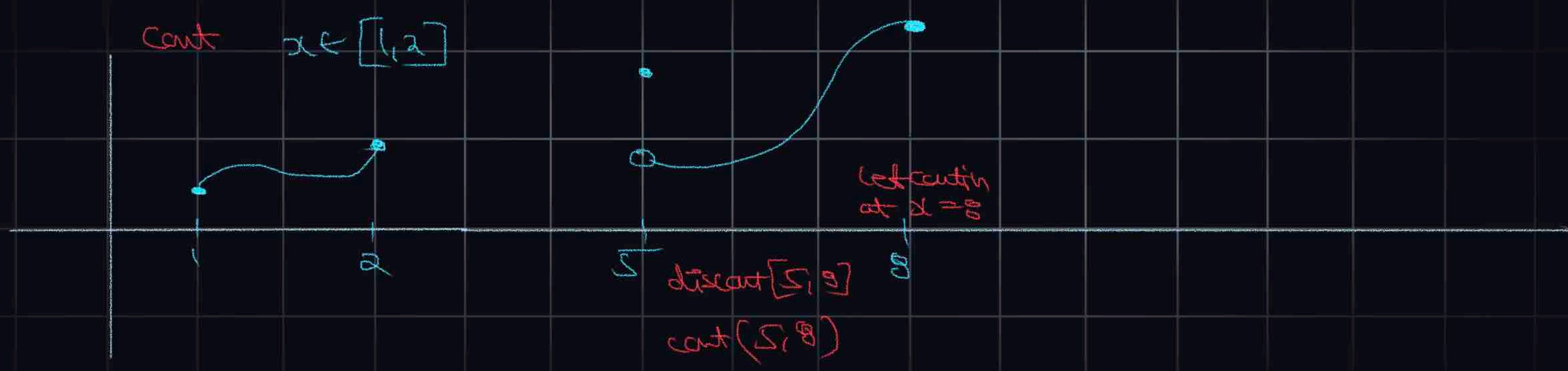
$f(b) = f(b) \Rightarrow$ cont at $x = b$

cont at all $x \in (a,b)$

$\Rightarrow$ cont at $x \in [a,b]$

open interval

cont at all $x \in (a,b)$



Find points of discontinuity:

$f(x) = \dfrac{1}{2\sin x - 1}$ discont at $x = n\pi + (-1)^n \dfrac{\pi}{6}$

$f(x) = \dfrac{1}{x^2 - 3|x| + 2}$ disc at $x = \pm 1, \pm 2$

$f(x) = \dfrac{1}{x^4 + x^2 + 1}$

$$f(x) = \frac{1}{\log|x^2-4|}$$

$$x^2-4 = 0, 1, -1$$

$$x \neq \pm 2, \pm\sqrt{5}, \pm\sqrt{3}$$

$$f(x) = \cos^{-1}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)$$

$$f(x) = \begin{cases} \dfrac{x^2-1}{x^2-2|x-1|-1} & x \neq 1 \\ \dfrac{1}{2} & x = 1 \end{cases}$$

cont at $x=1$

$$f(x) = \begin{cases} \cos^{-1}\{\cot x\} & x < \frac{\pi}{2} \\ \pi[x]-1 & x \geq \frac{\pi}{2} \end{cases}$$

continuity at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} (x+1)^{2-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 1 & x = 0 \end{cases}$$

cont at $x=0$

$$(1^+)^{2-\frac{2}{0^+}} = (1^+)^{2-\infty} = (1^+)^{-\infty} = \frac{1}{(1^\infty)}$$

$$(1+x)^{2-\frac{2}{x}} = \frac{1}{(1+x)^{\frac{2}{x}-2}}$$

$$= \frac{1}{e^{g(x)(f(x)-1)}} = \frac{1}{e^{(\frac{2}{x}-2)(x)}}$$

$$= \frac{1}{e^{2-2x}}$$

$$= \frac{1}{e^{2}}$$

$$f(x) = [\sin x] - [\cos x]$$ cont at $x = \pi$

$$f(x) = \begin{cases} \dfrac{\log_e \cos x}{\sqrt[4]{1+x^2} - 1} & x > 0 \\[2ex] \dfrac{e^{\sin 4x} - 1}{\log_e (1 + \tan 2x)} & x < 0 \end{cases}$$

find $f(0)$

cont at $x = 0$

$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)\,g(x)$
cont $f(a^-)=f(a)=f(a^+)$	cont	$f(a^-)+g(a^-)=f(a)+g(a)$ $=f(a^+)+g(a^+)$ cont	$f(a^-)\,g(a^-)=f(a)\,g(a)$ $=f(a^+)\,g(a^+)$ cont
cont $f(a^-)=f(a)$ $=f(a^+)$	discont $g(a^-)\neq g(a)$	$f(a^-)+g(a^-)\neq f(a)+g(a)$ discont	$\overbrace{f(a^-)}^{\text{zero}}\,g(a^-)=\overbrace{f(a)}^{\text{zero}}\,g(a)$ $=\overbrace{f(a^+)}^{\text{zero}}\,g(a^+)$ discont or cont when $f(a)=0$
discont	discont	$\overbrace{f(a^-)}^{5}+\overbrace{g(a^-)}^{6}=\overbrace{f(a)}^{4}+\overbrace{g(a)}^{7}=\overbrace{f(a^+)}^{3}+\overbrace{g(a^+)}^{8}$ cont/disc	$\overbrace{f(a^-)}^{3}\,\overbrace{g(a^-)}^{2}=\overbrace{f(a^+)}^{2}\,\overbrace{g(a^+)}^{3}=\overbrace{f(a)}^{3}\,\overbrace{g(a)}^{2}$ cont/disc

discuss continuity of $\quad h(x) = -[x] + x$

at every integer: discont + cont = discont

$\{x\}$

discuss contin of

$f(x)+g(x)$

$$f(x) = \begin{cases} |x+1| & x \le 0 \\ x & x > 0 \end{cases} \qquad g(x) = \begin{cases} |x|+1 & x \le 1 \\ -|x-2| & x > 1 \end{cases}$$

at $x=0$ disc + cont = disc

at $x=1$ cont + disc = cont

$\cos\left(\text{odd}\ \frac{\pi}{2}\right) = 0$

Prove that $f(x) = [x] \cos\left(\dfrac{(2x-1)\pi}{2}\right)$ is cont for $x \in R$.

at $x \in I$ discont × cont = discont

$x = n+1$ discont × cont = cont

$(2x-1)\frac{\pi}{2} = n\pi + \frac{\pi}{2}$

$2x-1 = 2n+1$

$2x = 2n+2$

$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ is cont at $x=0$, find $f(0)$?

$f(x) = \frac{1-\tan x}{4x-\pi}$ $x \in \left[0, \frac{\pi}{2}\right]$. if $f(x)$ is contin in $x \in \left[0, \frac{\pi}{4}\right]$ find $f\left(\frac{\pi}{4}\right)$

$f(x) = \left(\tan\left(\frac{\pi}{4} + \log_e x\right)\right)^{\log_x e}$ cont at $x=1$ find $f(1)$?

$e^{g(x)(f(x)-1)}$

$= e^{\lim_{x\to 1} \frac{1}{\log x}\left(\tan\left(\frac{\pi}{4}+\log x\right)-1\right)} = e^{\frac{\sec^2(\pi/4+\log x)\cancel{/x}}{\cancel{1/x}}}$

$= e^{\sec^2 \pi/4} = e^2$

$$f(x) = \begin{cases} (1+3x)^{1/x} & x \neq 0 \\ e^3 & x = 0 \end{cases}$$

Discuss cont of $f(x)$ at $x=0$ and $x=1$

$f(x) = [x] + |1-x| \qquad -1 \leq x \leq 3$

$f(x) = \operatorname{sgn} x + [x]$ at $x = 0 \Rightarrow$ discont

$f(x) = [x]\{x\}$

discuss cont of $f(x) = \begin{cases} \dfrac{x-1}{e^{\frac{1}{x-1}}+1} & x \neq 1 \\ 0 & x = 1 \end{cases}$ at $x=1$

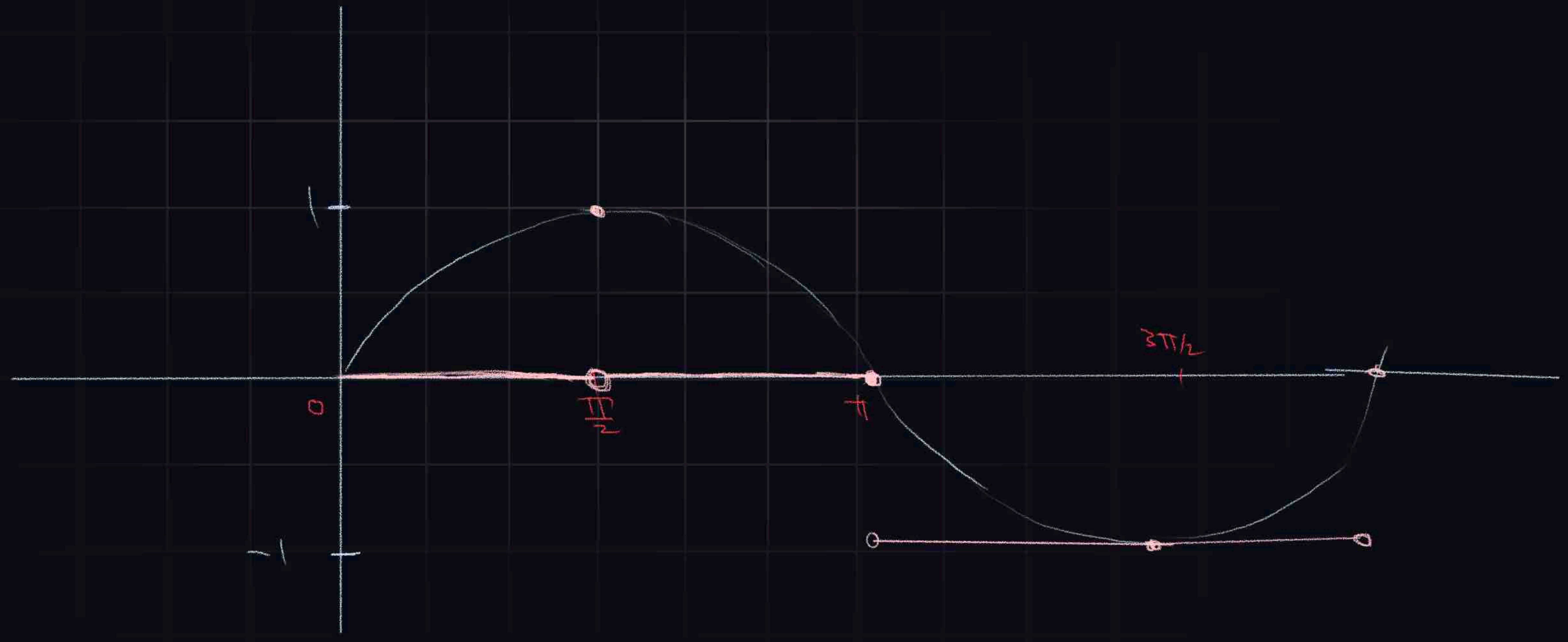
Discuss cont of $f(x) = \sqrt{2\,\mathrm{sgn}\,x + 1}$

$f(x) = \begin{cases} \dfrac{\log_e(px^2+qx+r)}{x^2} & -1 \le x < 0 \\ 1 & x = 0 \\ \dfrac{\sin(e^{x^2}-1)}{x^2} & 0 < x \le 1 \end{cases}$ find p, q, r

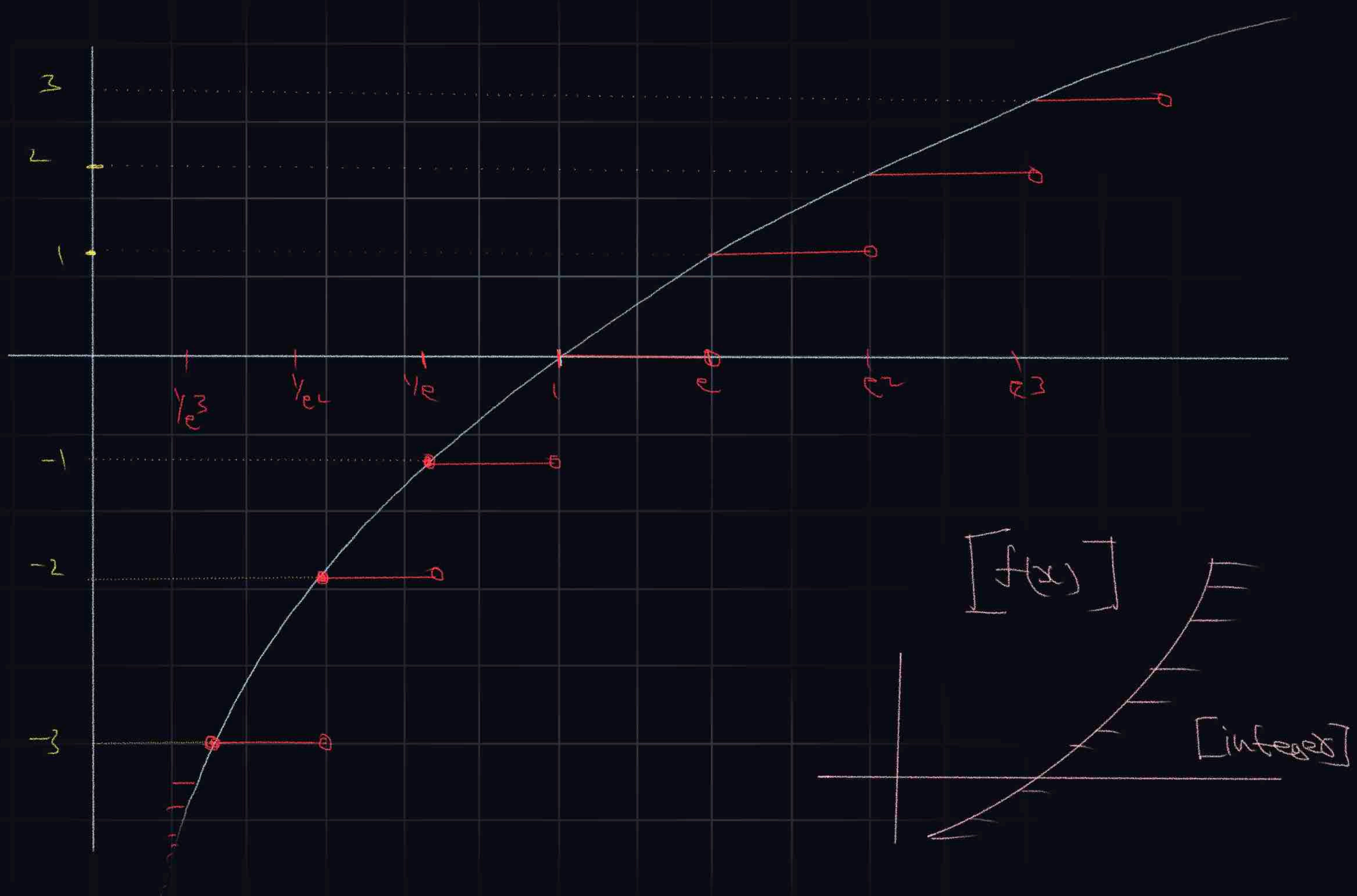
Continuity of $[f(x)]$: → graph / → rule

Discuss cont of $f(x) = [\sin x]$ $x \in (0, 2\pi)$

[heights]



discuss cont of $f(x) = [\log_e x]$?



$[f(x)]$

if $f(x)$ = increasing : $f(x)$ = integer

if $f(x)$ = decreasing : $f(x)$ = integer

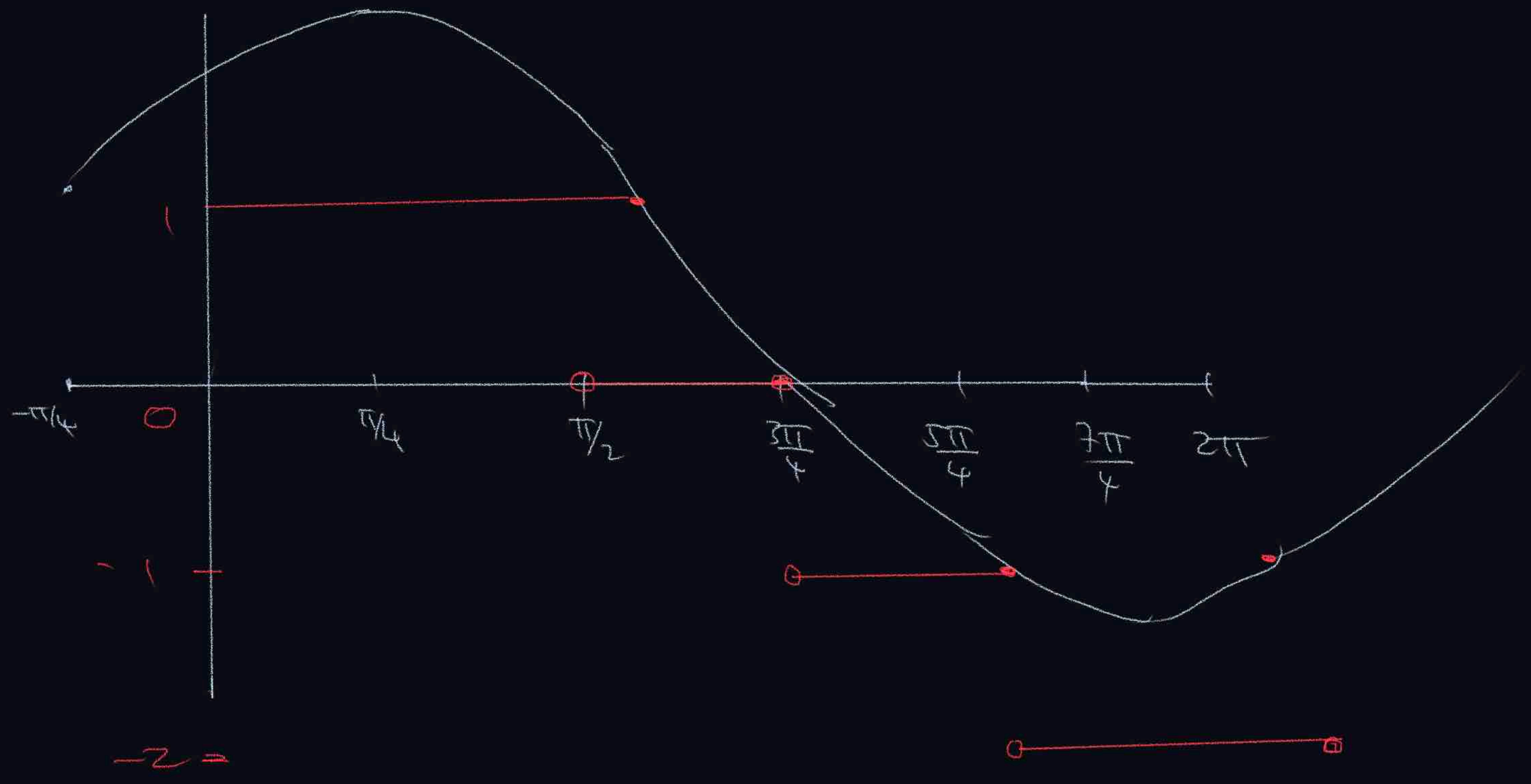
if $f(x)$ = non monotonic : $f(x)$ graph

$$f(x) = [2\cos x] \qquad x \in [0, 2\pi]$$

$$f(x) = [x/3] \qquad x \in [0, 30]$$

$$f(x) = \left[\frac{2}{1+x^2}\right], \quad x \geq 0$$

$f(x) = [\sin x + \cos x]$ $x \in [0, 2\pi]$



$f(x) = \left[\dfrac{(x-2)^3}{a} \right] \sin(x-2) + \cos(x-2)$, is cont in $x \in [4, 6]$

find values of a.

$f(x) = \operatorname{sgn}(x^3 - x)$

$sgn(f(x))$ is discont when $f(x)=0$

i) $sgn(2\cos x-1)$

ii) $sgn(x^2-2x+3)$

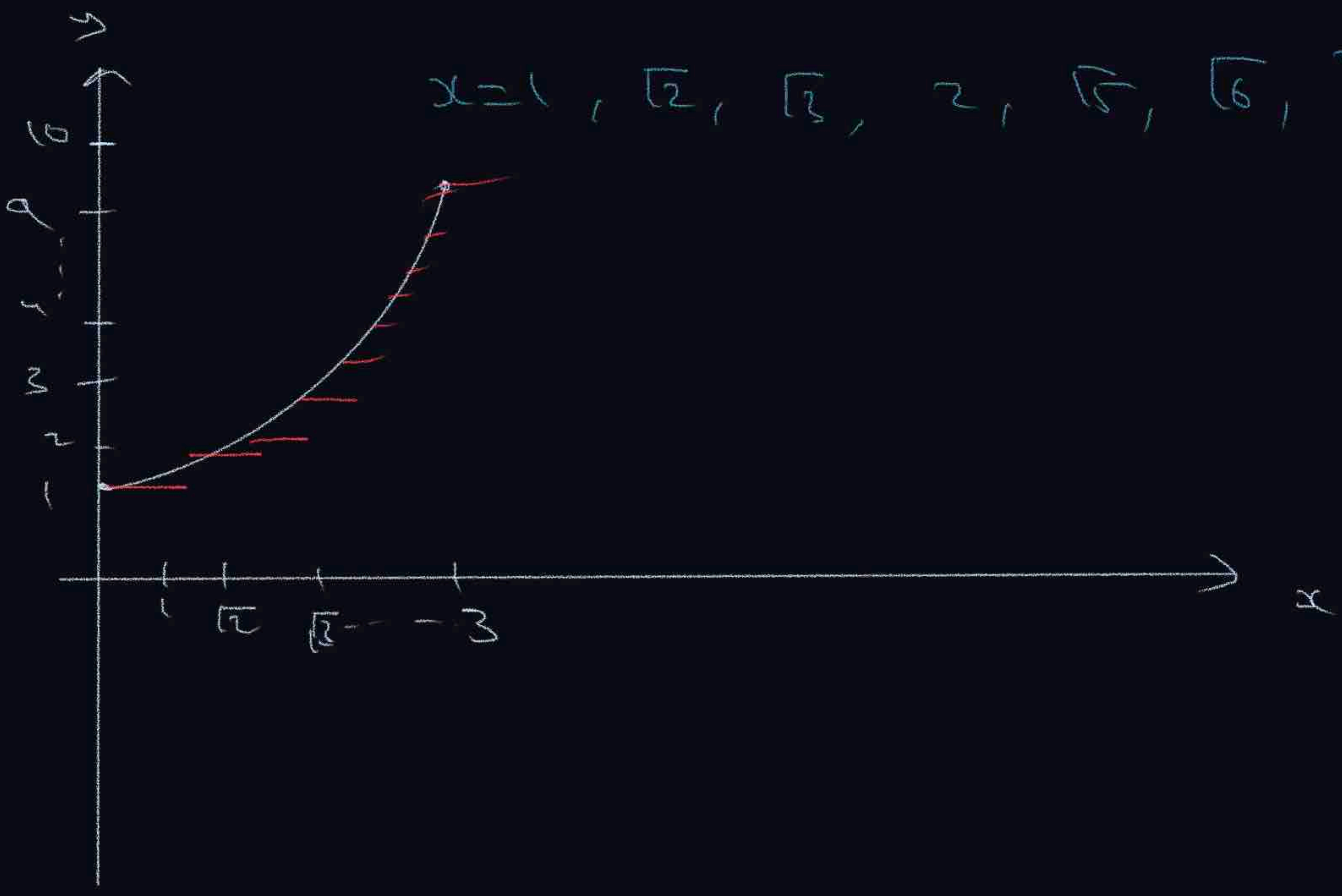
$f(x)=sgn(2\sin x+a)$ is continuous for all real x, th

$f(x)=|x|\,sgn(x^3-x)$

Discuss cont : $f(x) = |x| \, \mathrm{sgn}(x^3 - x)$

discuss cont of $f(x) = [x^2+1]$, $x \in [1,3]$

$x = 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3$



$$f(x) = \begin{cases} \operatorname{sgn}(x-2) \times [\log_e x] & 1 \le x \le 3 \\ \{x^2\} & 3 < x \le 3.5 \end{cases}$$

Discuss continuity of $f(x)$.

Critical Points: $x = \underbrace{1, 3, 3.5}_{\text{intervals}}, \underbrace{2}_{\text{Signum}}, \underbrace{\sqrt{10}, \sqrt{11}, \sqrt{12}}_{\{x^2\}}, \underbrace{e}_{[\log x]}$

$x^2 = \text{Integer}$ $\quad x^2 \in (9, 12.25]$

$x \in (3, \sqrt{12.25}]$

$x = \sqrt{10}, \sqrt{11}, \sqrt{12}$

discont: $e, 3, \sqrt{10}\ \sqrt{11}, \sqrt{12}$

$$f(x) = \begin{cases} [\sin \pi x] & 0 \le x < 1 \\ \operatorname{sgn}\left(x - \frac{5}{4}\right) x \left\{x - \frac{2}{3}\right\} & 1 \le x \le 2 \end{cases}$$

Critical: $0, \frac{1}{2}, 1, \frac{5}{4}, \frac{5}{3}, 2$

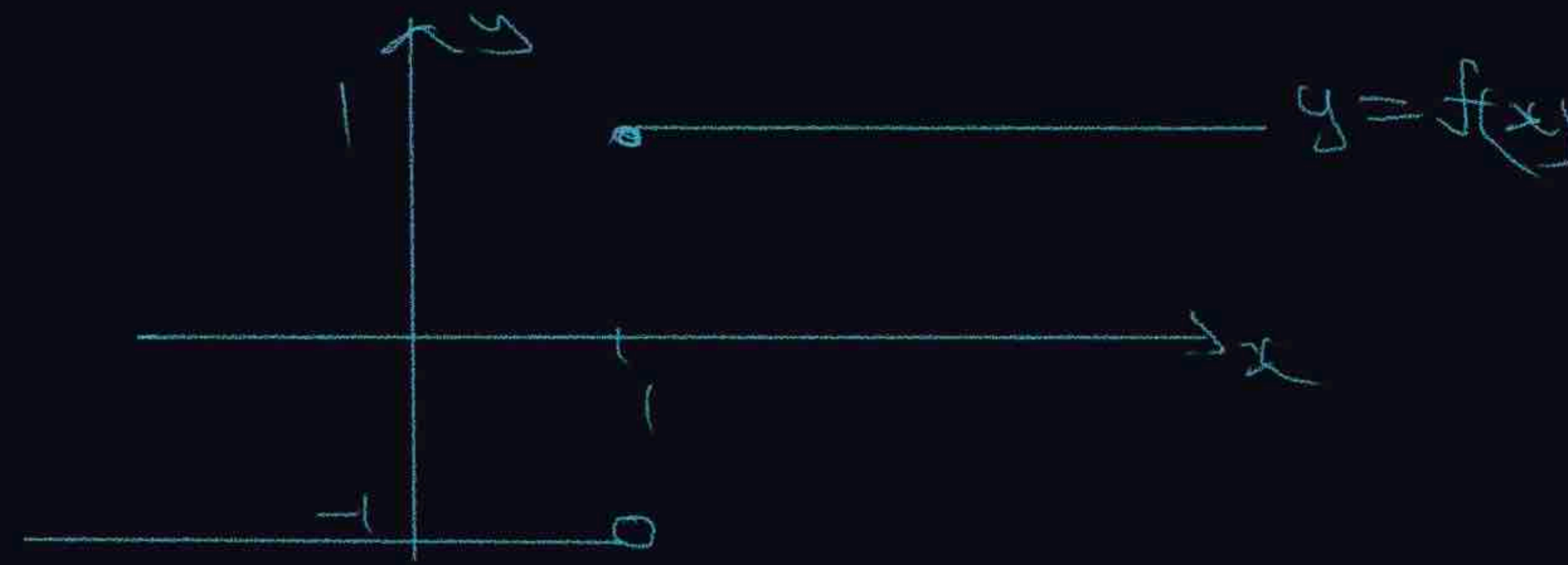
Discont: $\frac{1}{2}, 1, \frac{5}{4}, \frac{5}{3}$

Limits at infinity:

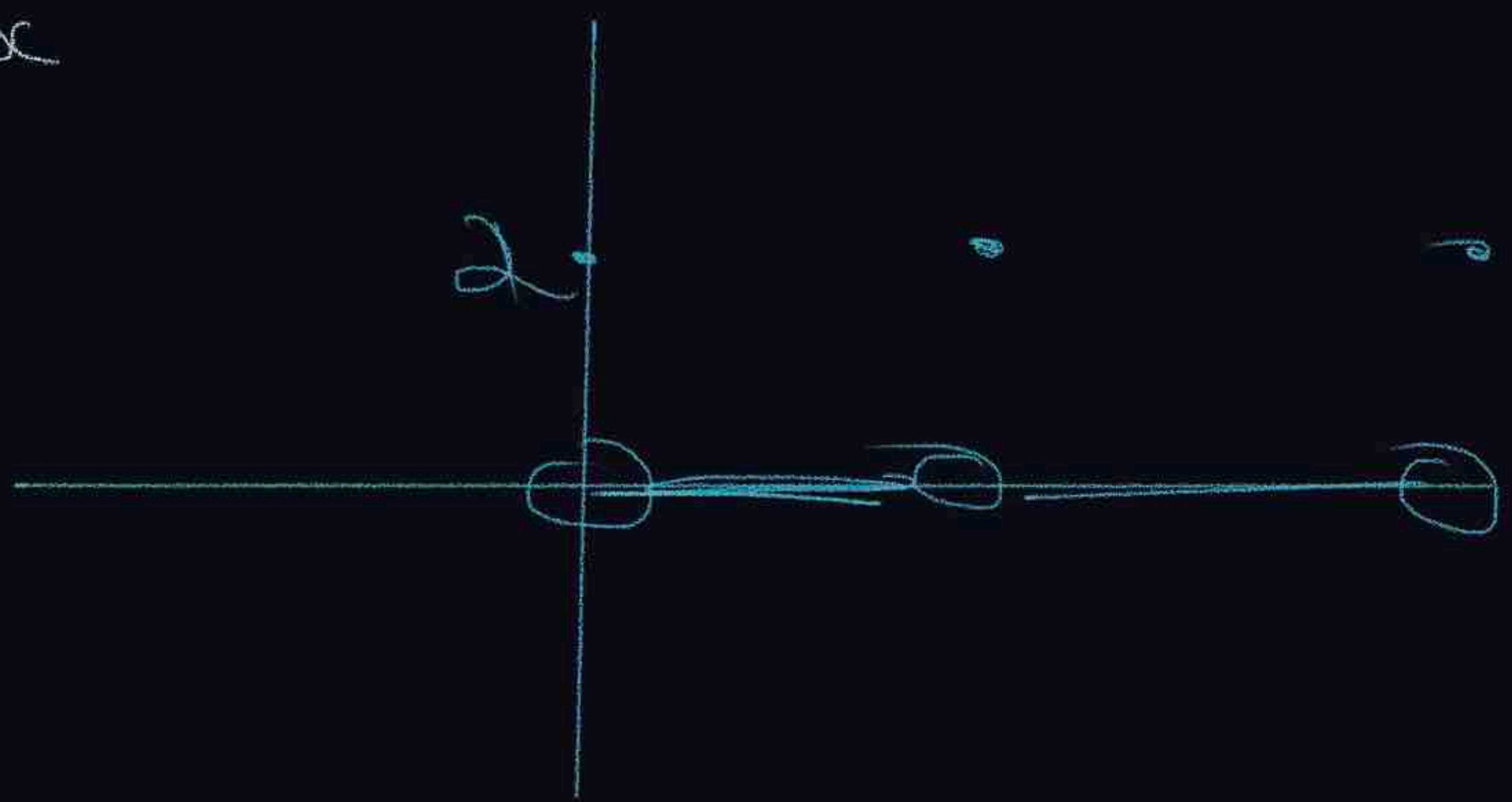
$$\lim_{n\to\infty} a^n = \begin{cases} 0 & 0 < a < 1 \\ 1 & a = 1 \\ \infty & a > 1 \end{cases}$$

$$\lim_{n\to\infty} a^{2n} = \begin{cases} 0 & -1 < a < 1 \\ 1 & a = \pm 1 \\ \infty & |a| > 1 \end{cases}$$

$$f(x) = \lim_{n\to\infty} \frac{x^{2n} - 1}{x^{2n} + 1}$$



$$f(x) = 1 + \lim_{n\to\infty} \cos^{2n} x$$

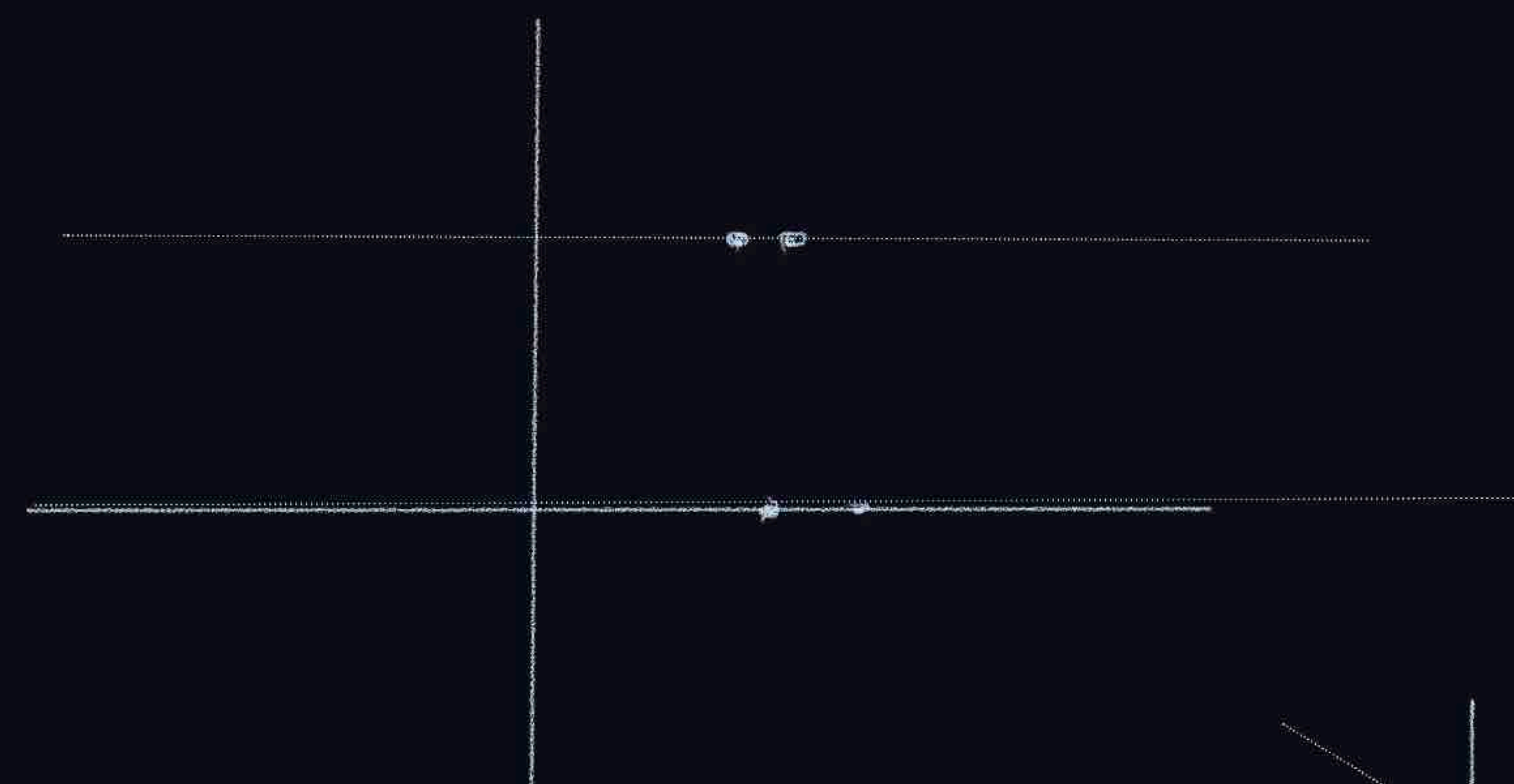


find value of 'a' if $f(x) = \lim_{n\to\infty} \frac{a x^{2n} + 2}{x^{2n} + a + 1}$ is cont at $x = 1$

function defined for rational and irrational domain :

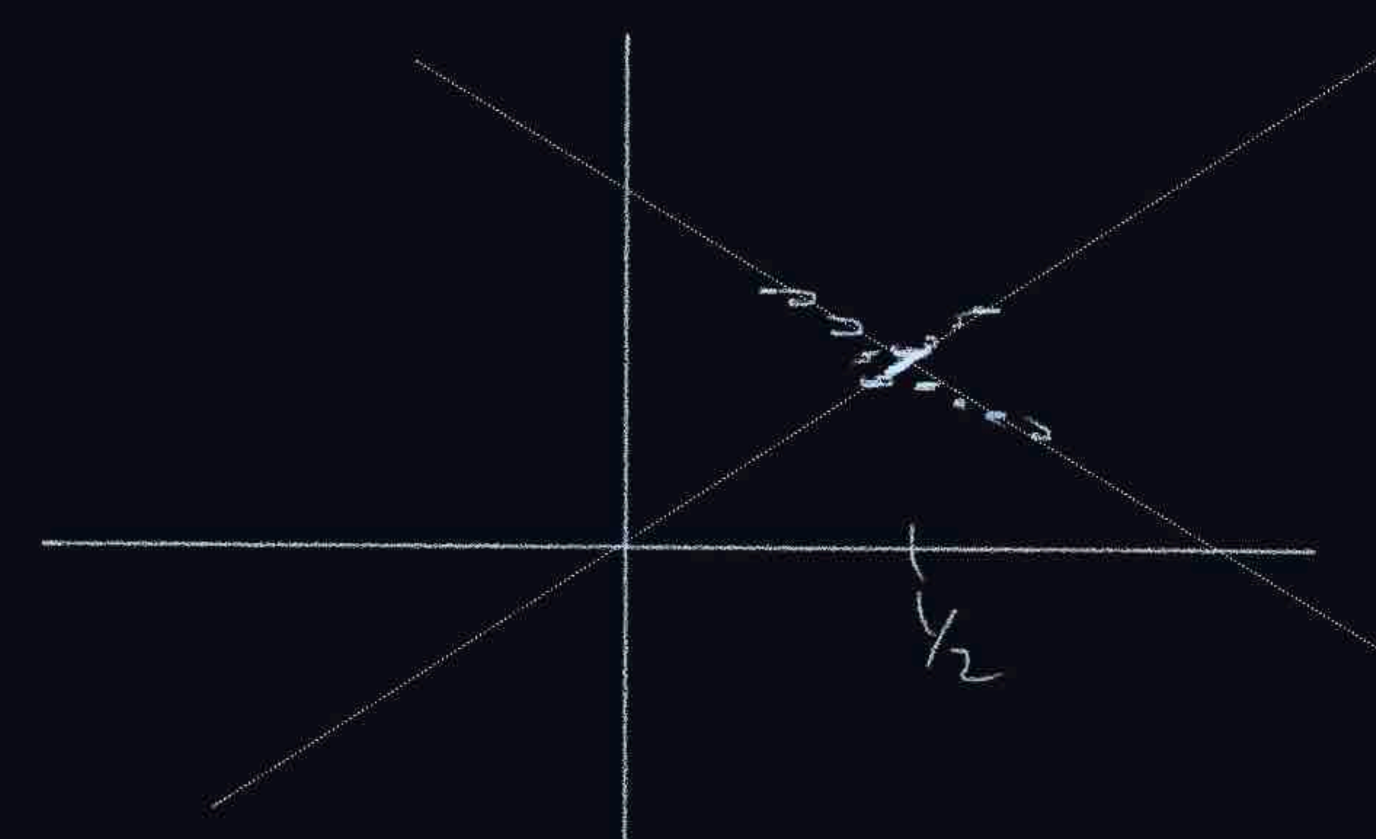
$$f(x) = \begin{cases} 1, & \text{if } x \in \text{Rational} \\ 0, & \text{if } x \in \text{irrational} \end{cases}$$

Discuss continuity.



$$f(x) = \begin{cases} x \quad (0.5) & \text{if } x \in \text{rational} \\ 1 - x \quad (0.5001) & \text{if } x \in \text{irrational} \end{cases}$$

cont at $x = 1/2$ only.



Continuity of Composite Functions:

$f(x) = \frac{1}{1-x}$ disc at $x=1$

$ff(x) = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}} = \frac{x-1}{x}$ discont at $x=0$ and $x=1$

$ff(x) = f$ (not defined)

$fff(x) =$

$$f(x) = \frac{x+1}{x-1} \qquad g(x) = \frac{1}{x-2}$$

dissuss cont of

$fg(x)$

$$f(x) = \begin{cases} x-2 & x \le 0 \\ 4-x^2 & x > 0 \end{cases}$$

cont of $ff(x)$

$$f(x) = \begin{cases} 1+x & 0 \le x \le 2 \\ 3-x & 2 < x \le 3 \end{cases}$$

$g(x) = f\,f(x)$

discont at $x = 1, 2$

$$f(x) = \frac{1}{1 - e^{-\frac{(x-1)}{(x-2)}}}$$

discont at $x = 1, 2$

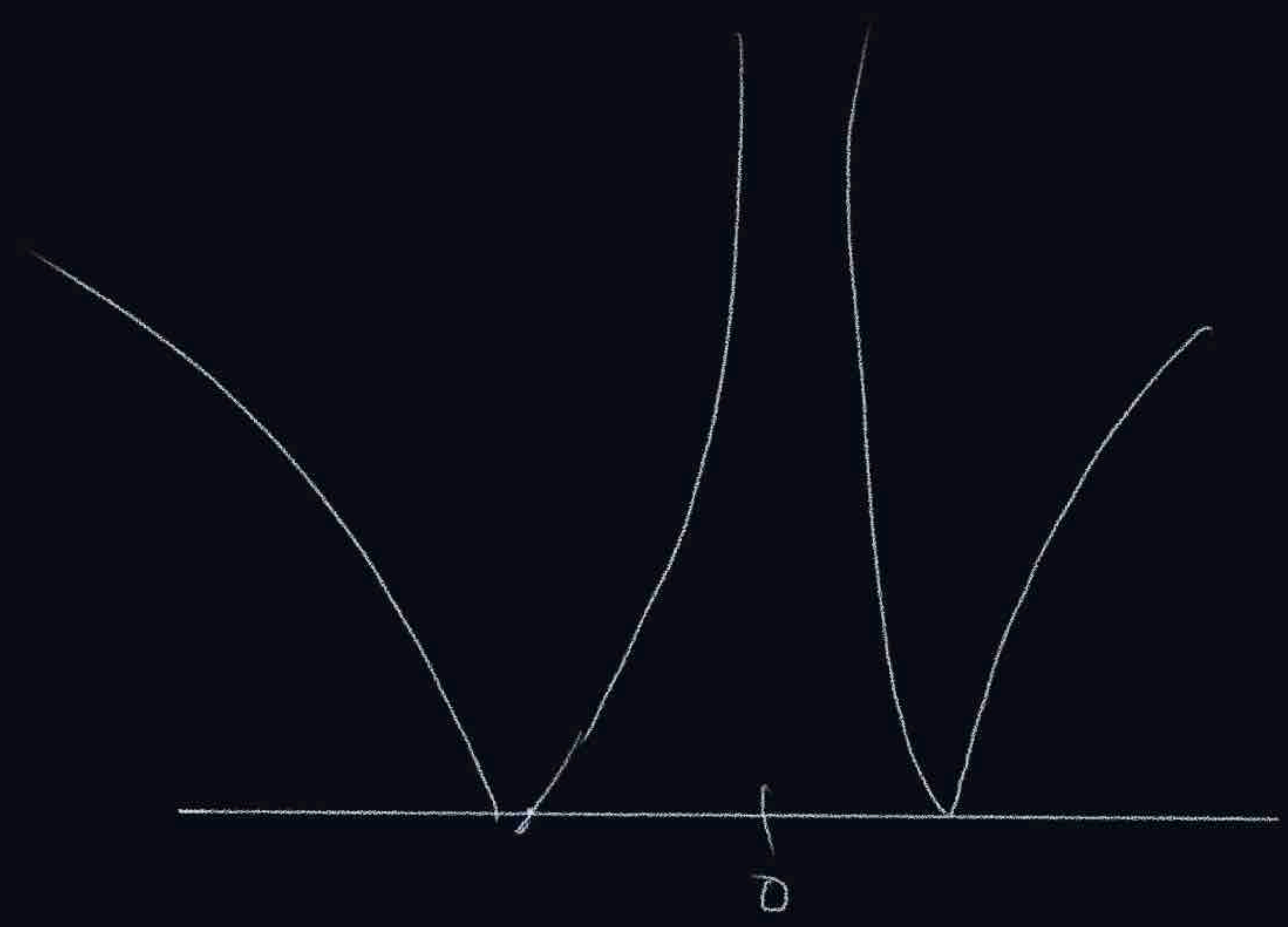
$$f(x) = \frac{1}{t^2 + t - 2} \qquad t = \frac{1}{x-1}$$

discuss

discont of $f(x)$.

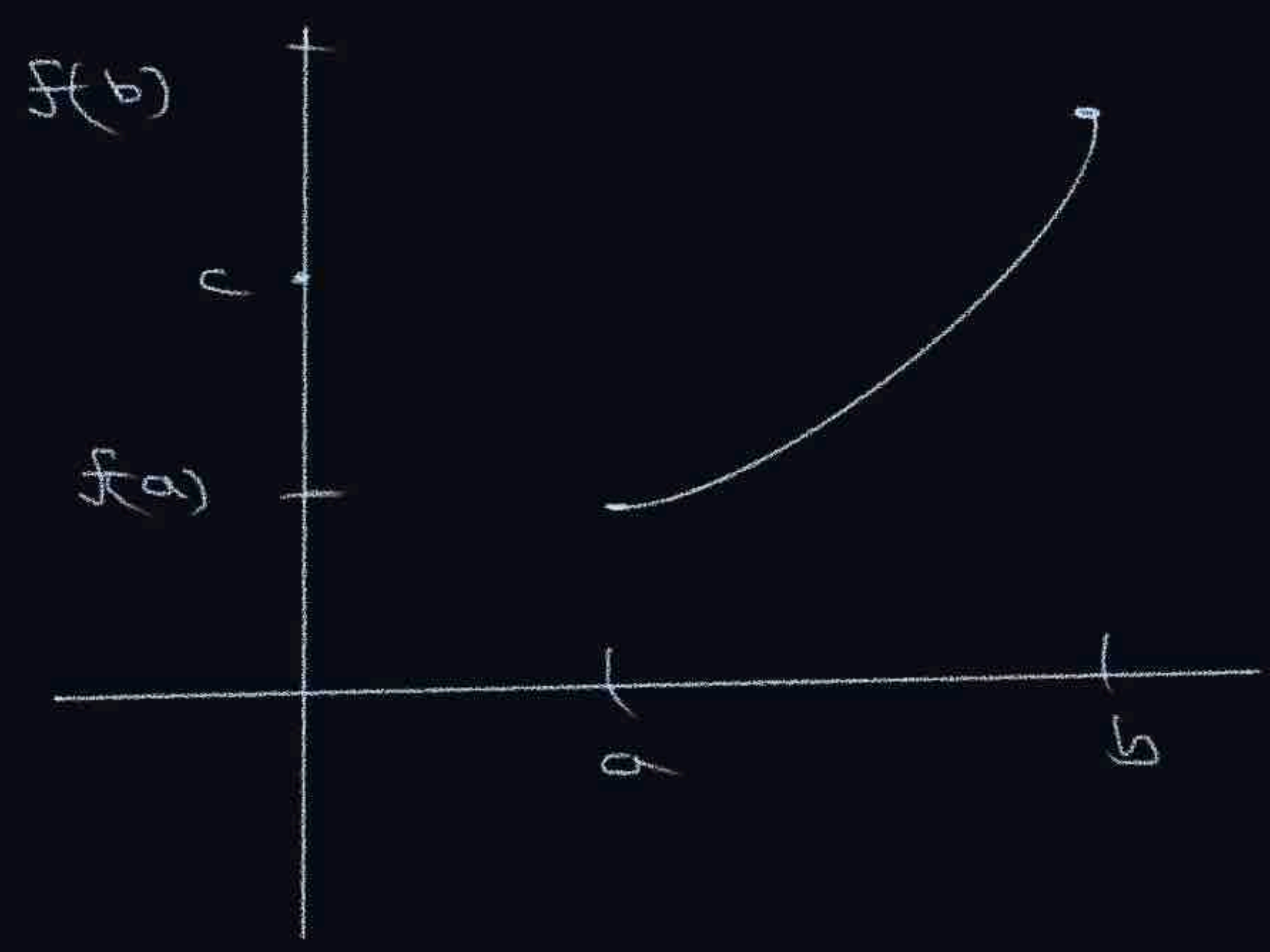
$$f(x) = \log|x| \ \operatorname{sgn}(x^2-1)$$

$$f(x) = \begin{cases} \log(-x) & x<-1 \\ 0 & x=-1 \\ -\log|x| & -1<x<1 \\ 0 & x=1 \\ \log(x) & x>1 \end{cases}$$



$\sqrt{\ }$

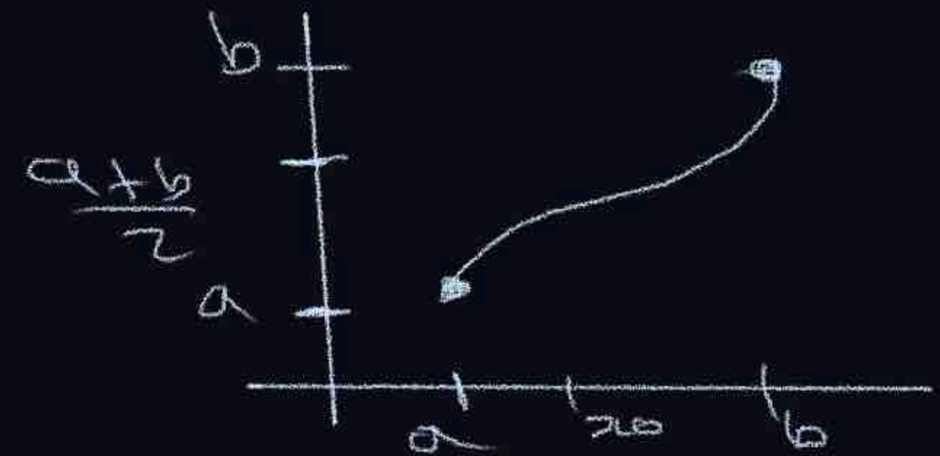
Intermediate value theorem:



$c \in (f(a), f(b))$

$f(x_0) = c \quad x_0 \in (a, b)$

Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$

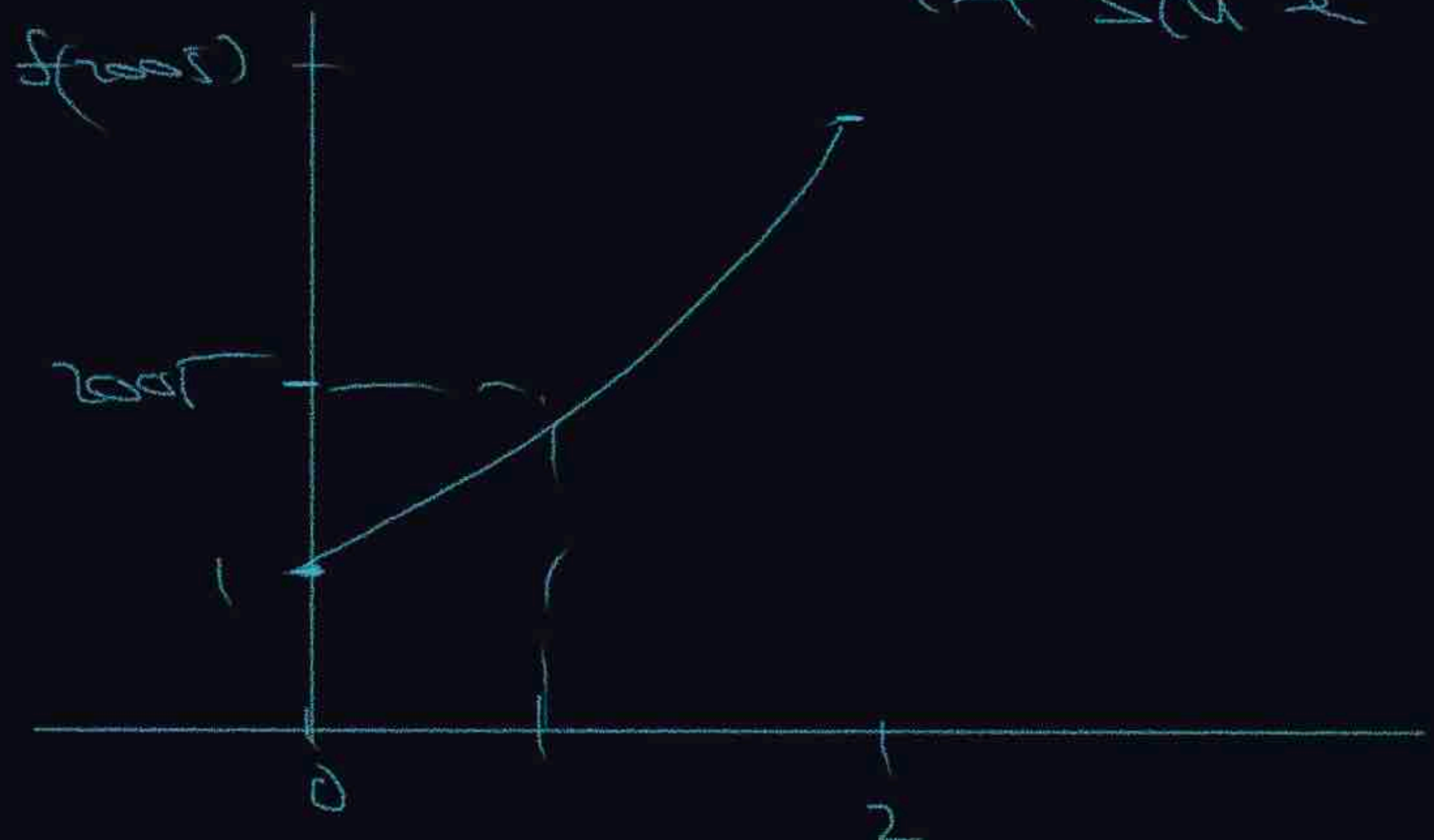


Using intermediate value theorem, prove that there exists a number $x \in [0, 2]$ such that

$$x^{2005} + \frac{1}{1+\sin^2 x} = 2005.$$

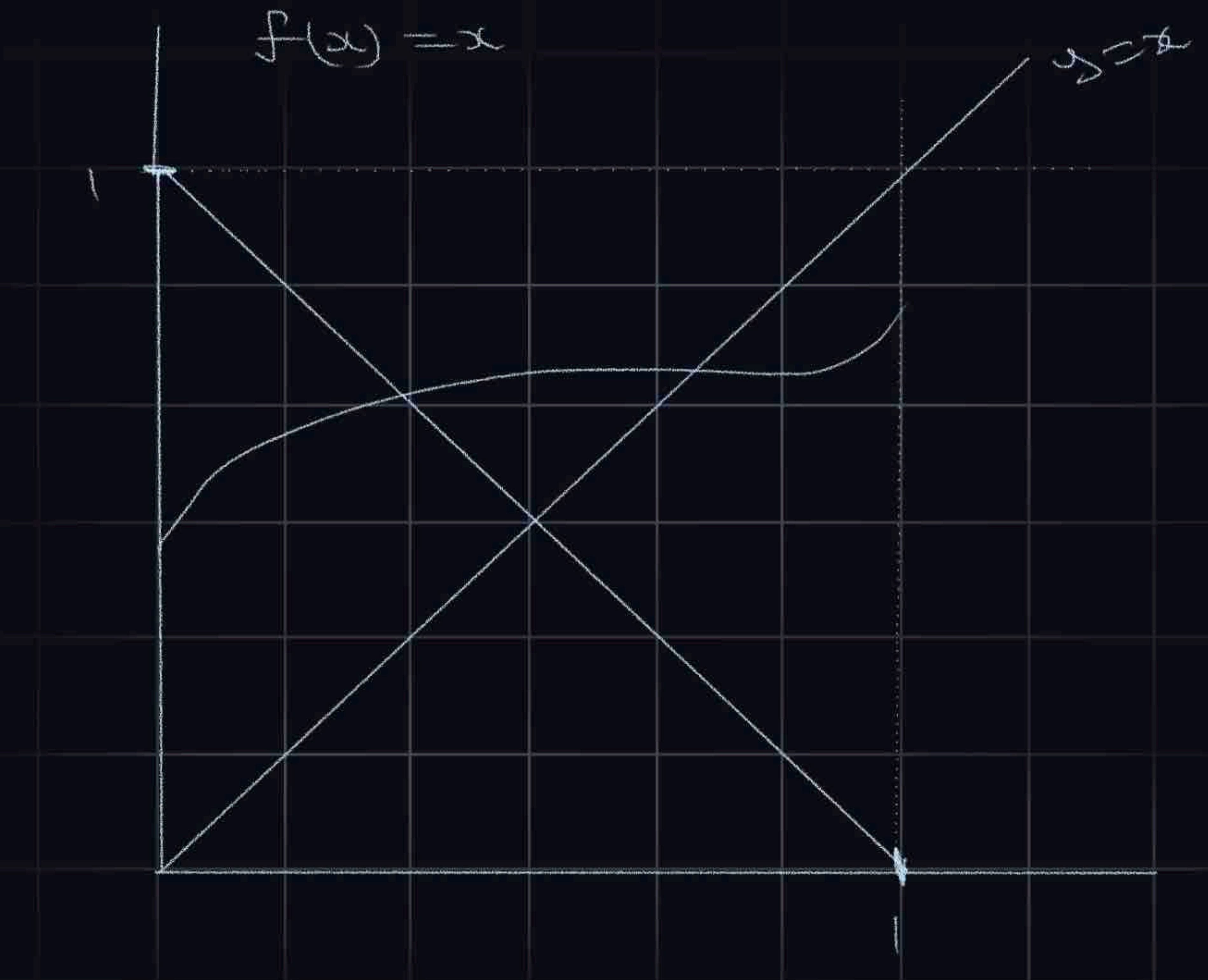
$$f(2) = 2^{2005} + \frac{1}{1+\sin^2 2}$$

$$f(x) = x^{2005} + \frac{1}{1+\sin^2 x}$$



$f : [0,1] \longrightarrow [0,1]$ Then prove that for some $c \in [0,1]$

i) $f(c) = c$ ii) $f(c) = 1-c$



$g(x) = f(x) - x$

$g(0) = f(0) \geq 0$

$g(1) = f(1) - 1 \leq 0$

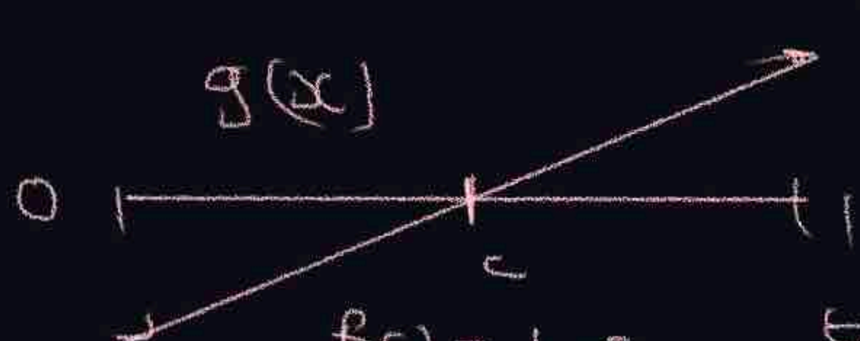
$g(c) = 0$ at $c \in (0,1)$

$f(c) = c$

$g(x) = f(x) + x - 1$

$g(0) = f(0) - 1 \leq 0$

$g(1) = f(1) \geq 0$



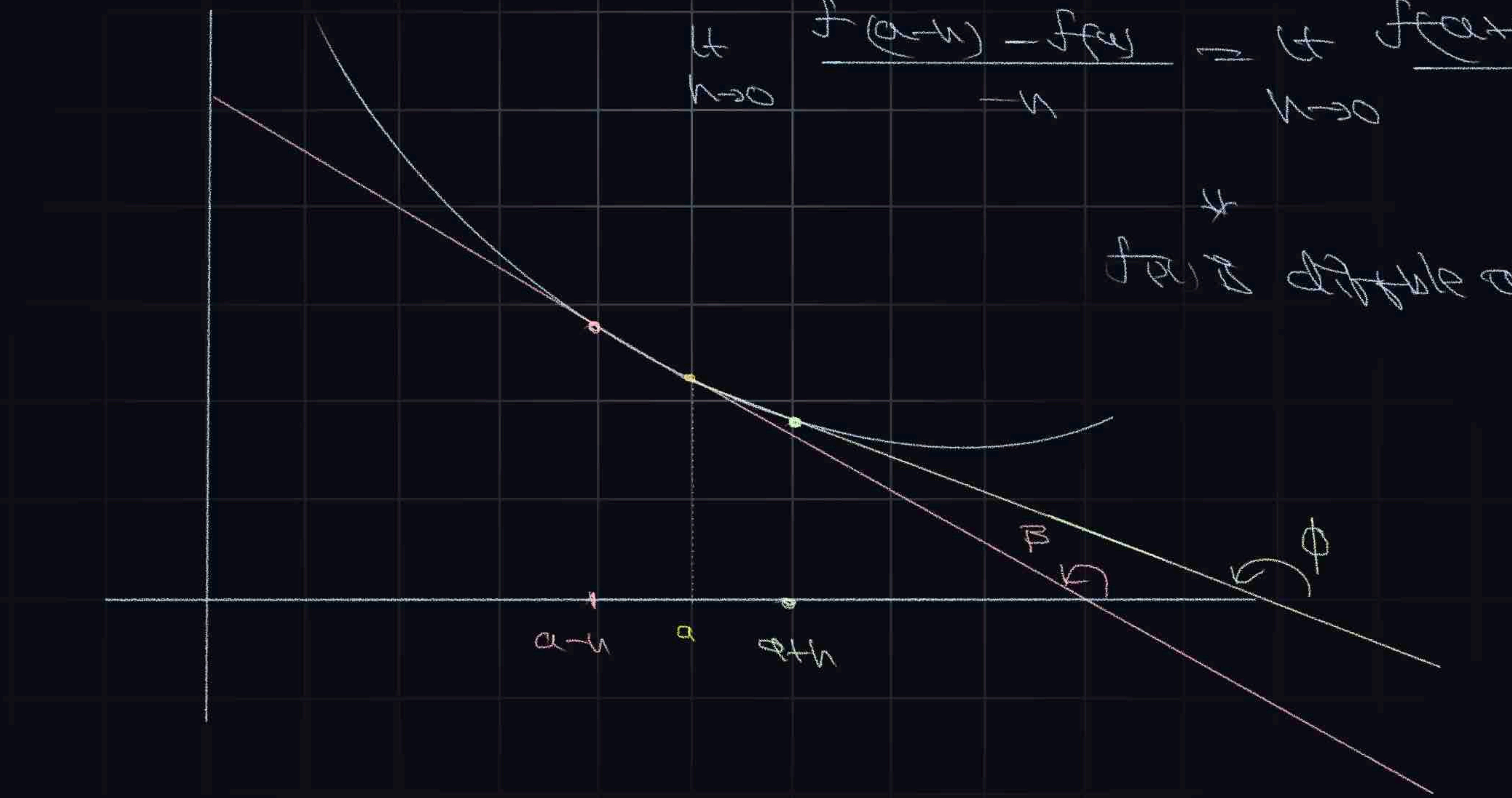
$f(c) = 1 - c \Leftarrow g(c) = 0 \quad c \in (0,1)$

If $f(x)$ is a continuous funct at $x=a$

$$\lim_{h\to 0} \frac{f(a-h) - f(a)}{-h} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

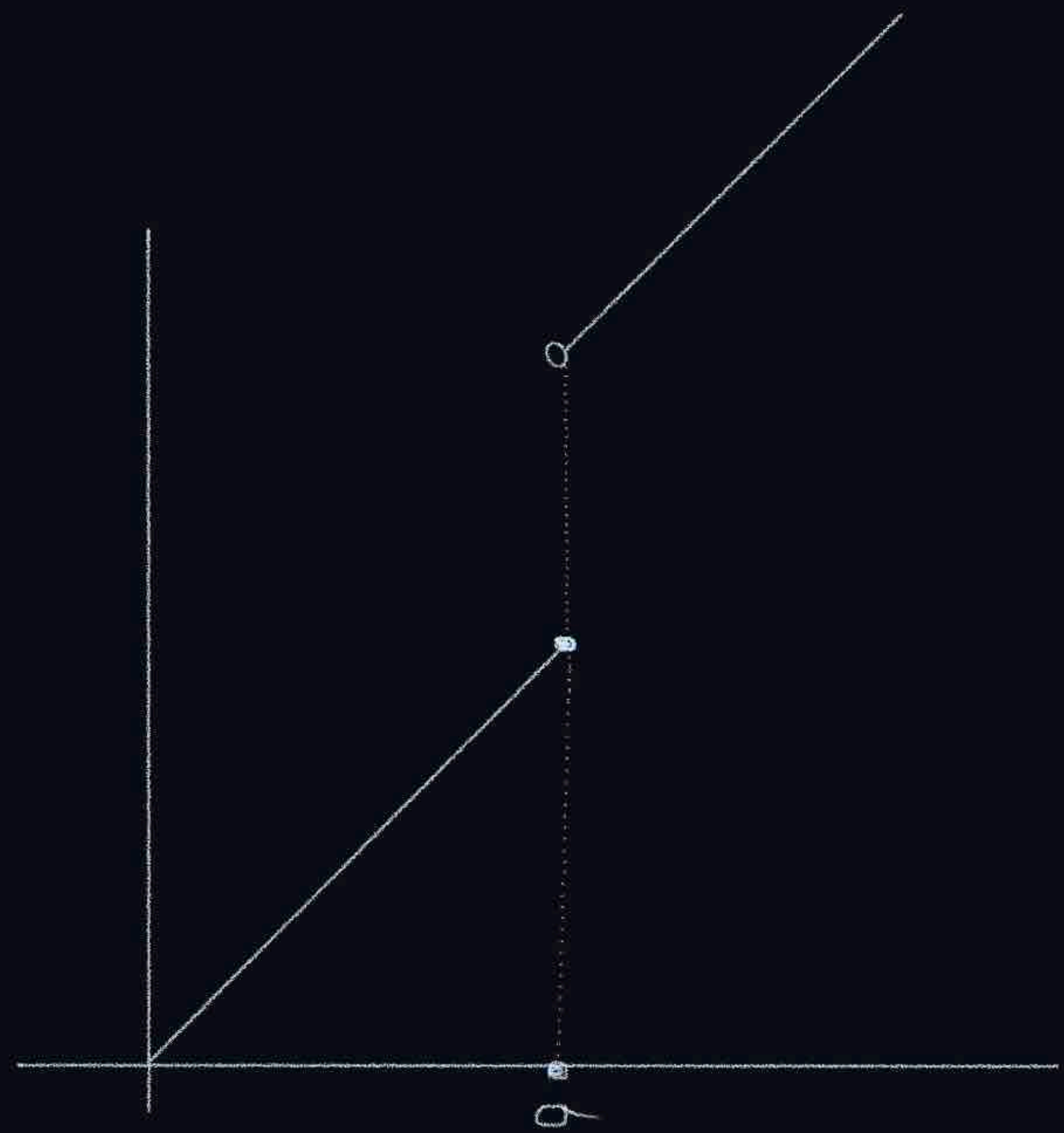
&

$f(x)$ is diffble at $x=a$

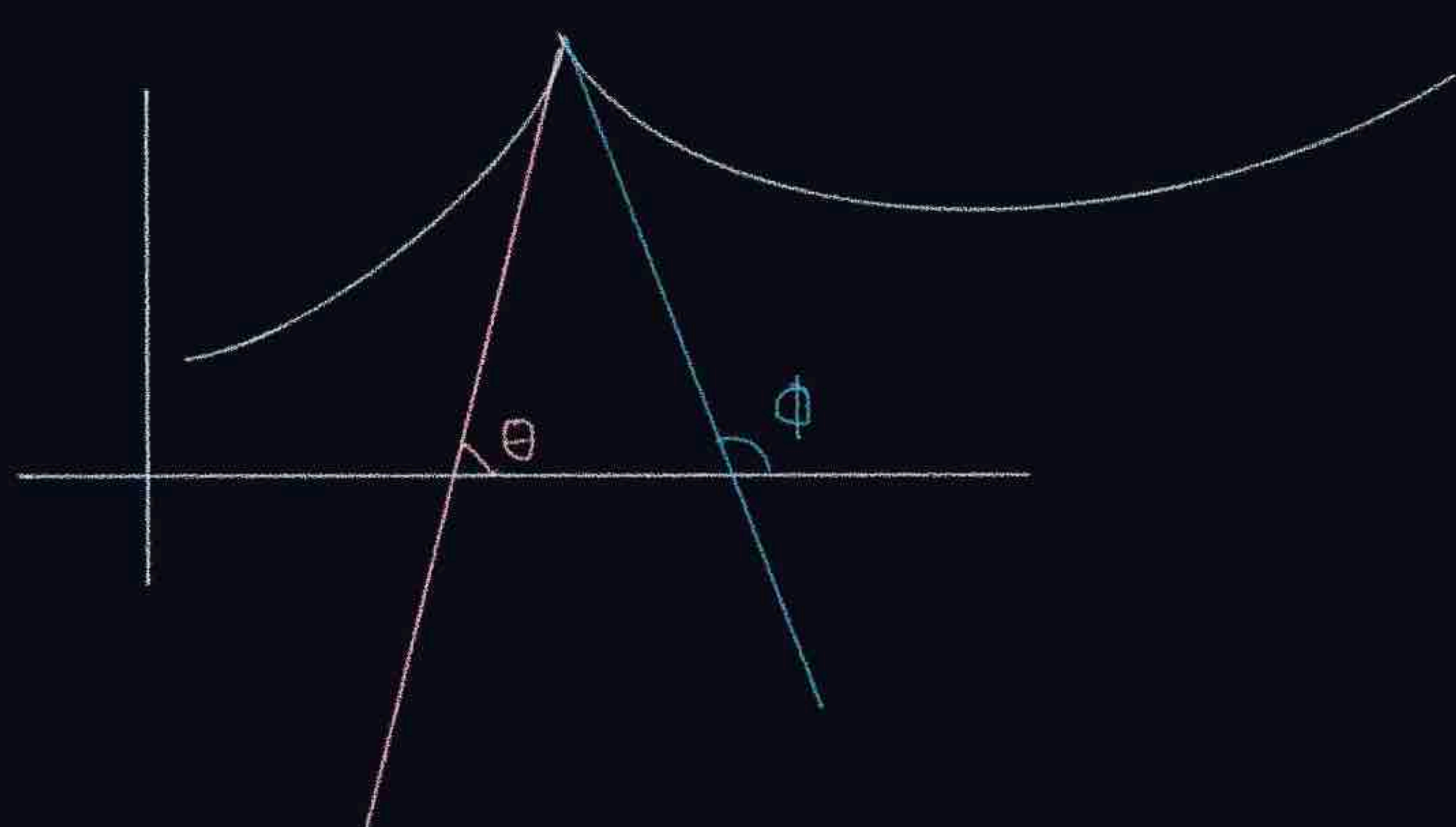


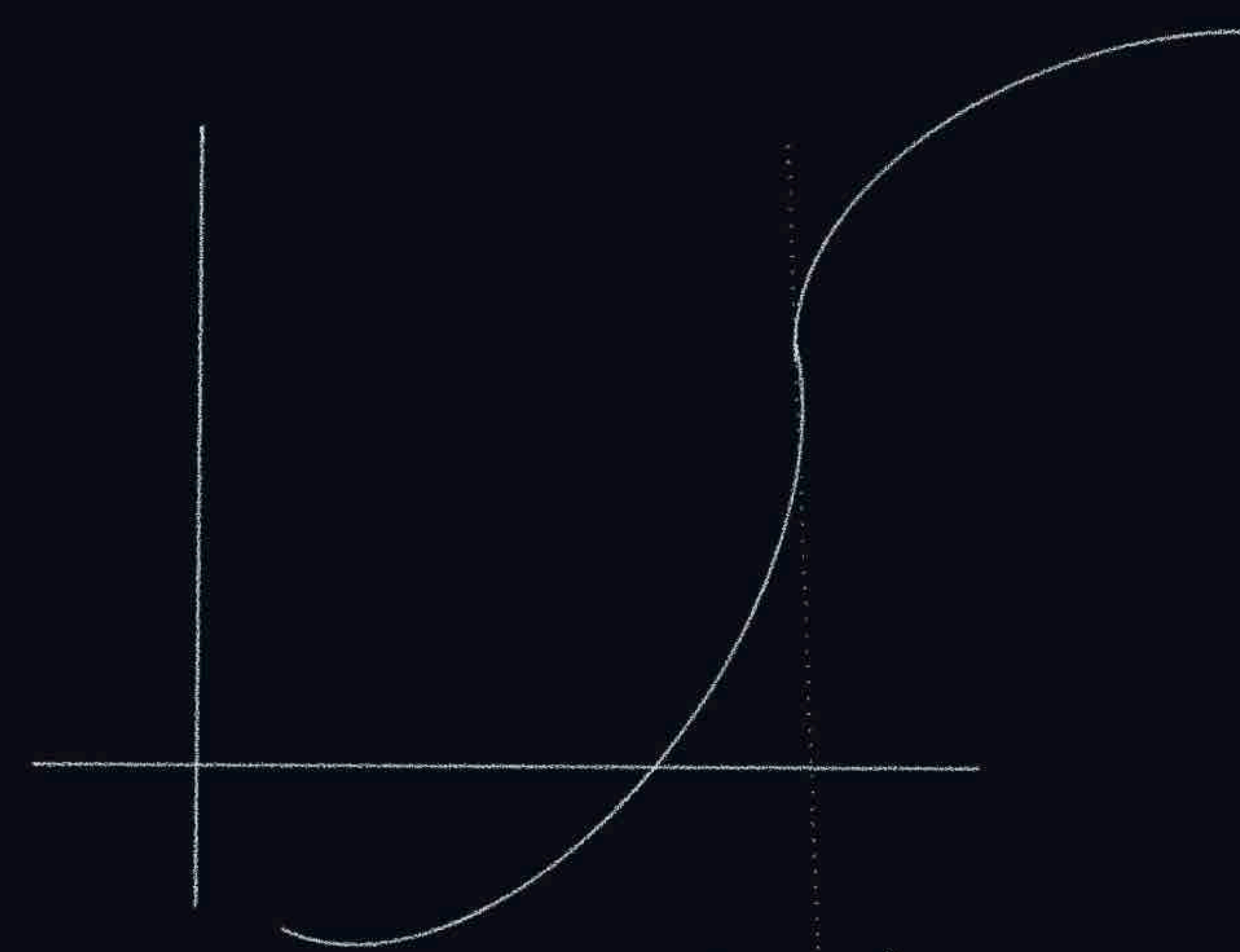
$$RHD = \tan\phi = \frac{f(a+h) - f(a)}{h}$$

$$LHD = \tan\beta = \frac{f(a-h) - f(a)}{-h}$$



$$\frac{f(a+h)-f(a)}{h} \qquad \frac{f(a-h)-f(a)}{-h}$$



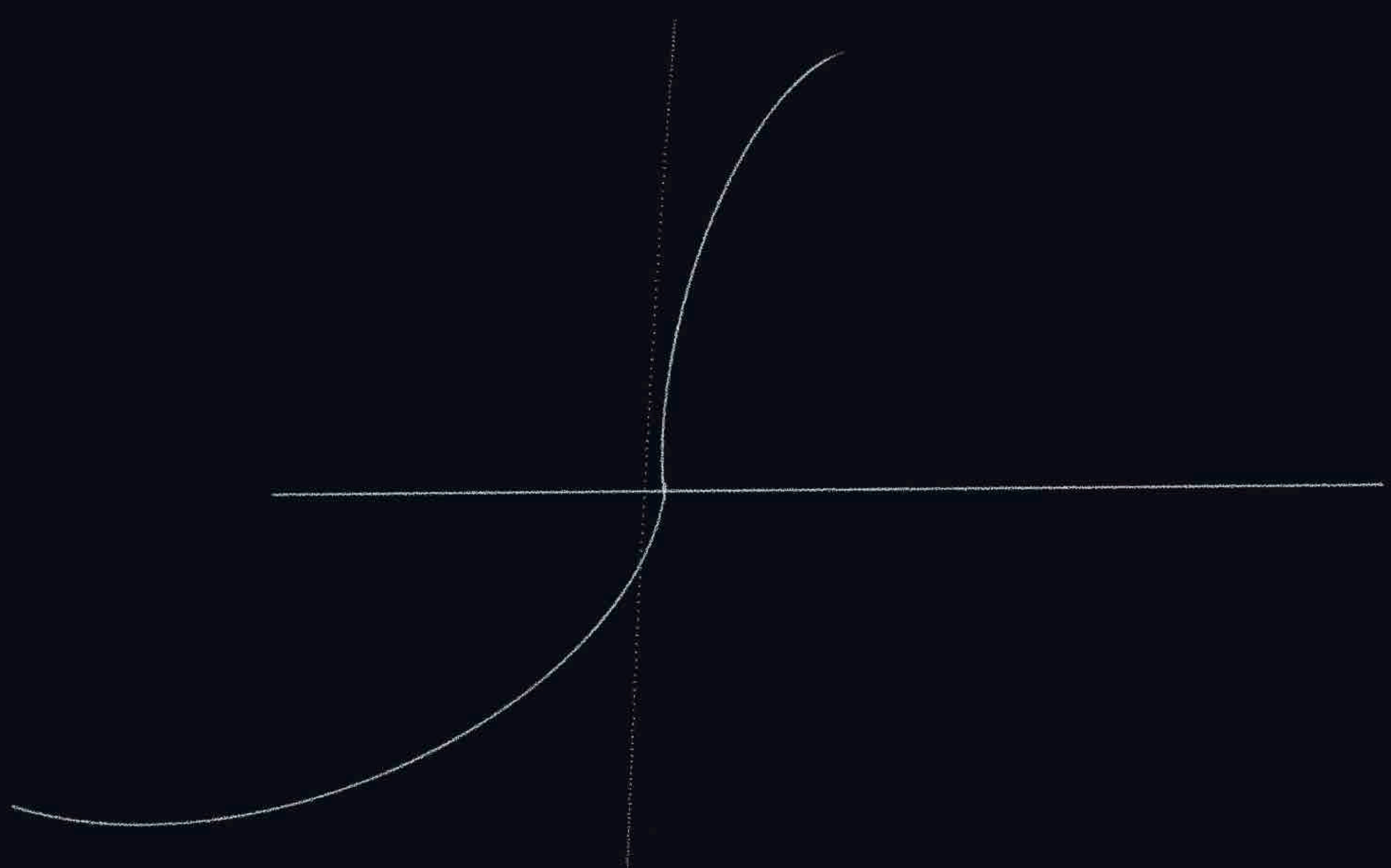


vertical tangent

discuss diffblty of $f(x) = (2x-5)^{3/5}$ at $x = 5/2$

$$\frac{f(\frac{5}{2}+h) - f(\frac{5}{2})}{h} = \frac{(2(\frac{5}{2}+h) - 5)^{3/5}}{h} = \frac{(2h)^{3/5}}{h} = \frac{3}{h^{2/5}}$$

$$= \infty$$



$$f(x) = \begin{cases} \dfrac{\sin x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$\frac{f(h)}{h} \qquad \frac{f(-h)}{-h}$$

$$f(x) = \begin{cases} x \sin \log_e x^2 & x \neq 0 \\ 0 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$\frac{h \sin \log h^2}{h} = \sin(\infty)$$

$$= (-1 \text{ to } 1)$$

$$f(x) = \begin{cases} (x-e)\, 2^{-2^{\frac{1}{e-x}}} & x \neq e \\ 0 & x = e \end{cases} \quad \text{at } x = e$$

$$\frac{f(e-h) - f(e)}{-h} = \frac{f(e-h)}{-h}$$

$$(-h)\, 2^{-2^{\frac{1}{h}}}$$

$$= (-h)\, 2^{-\infty} = 0$$

$$f(x) = \sqrt{1 - \sqrt{1 - x^2}} \quad \text{at } x = 0$$

$$f(x) = \begin{cases} (\sin^{-1}x)^2 \cdot \cos\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$f(x) = \begin{cases} \sqrt{x}\left(1 + x\sin\left(\frac{1}{x}\right)\right) & x > 0 \\ -\sqrt{-x}\left(1 + x\sin\left(\frac{1}{x}\right)\right) & x < 0 \\ 0 & x = 0 \end{cases} \quad \text{at } x = 0$$

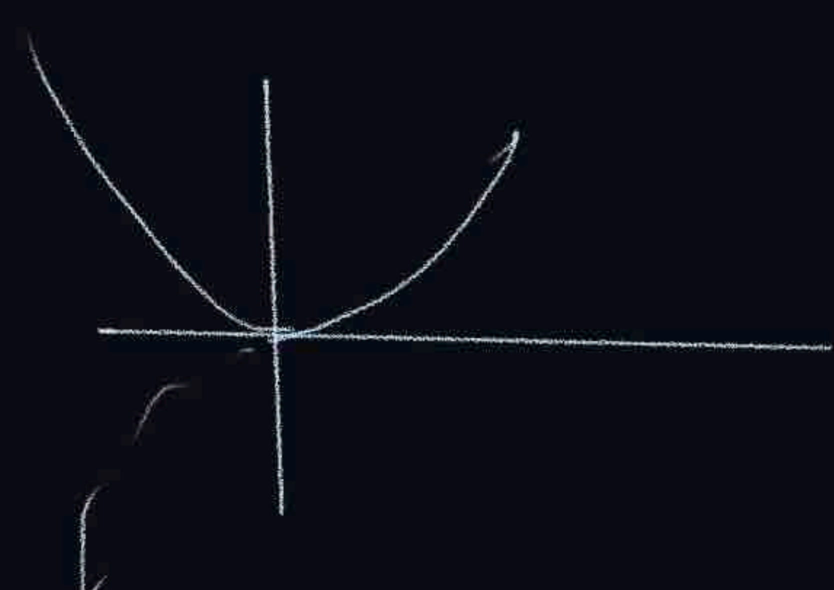
$$f(x) = \cos|x| = \begin{cases} \cos x & x<0 \\ \cos x & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} \sin x & x \leq 0 \\ -\sin x & x \geq 0 \end{cases}$$

$$f(x) = x|x| = \begin{cases} -x^2 & x<0 \\ x^2 & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -2x & x<0 \\ 2x & x \geq 0 \end{cases}$$

$$f(x) = |x^3| =$$



$$f(x) = \frac{x}{1+|x|}$$

$$f(x) = \begin{cases} x & x \le 1 \\ x^2+bx+c & x > 1 \end{cases}$$

find b and c if diffble at $x = 1$

$$f(x) = \begin{cases} a + \sin^{-1}(x+b) & x \ge 1 \\ x & x < 1 \end{cases}$$

find a and b if diffble at $x = 1$

$$f(x) = \begin{cases} ax(x-1)+b & x<1 \\ x-1 & 1 \le x \le 3 \\ px^2+qx+2 & x>3 \end{cases}$$

find p, q, a, b so that

i) f(x) cont for all $x \in R$

ii) $f'(1)$ doesn't exist

iii) $f'(x)$ is exists at $x =$

$2ax - a$

$2a - a = a$

$f'(1^-) \neq f'(1^+)$

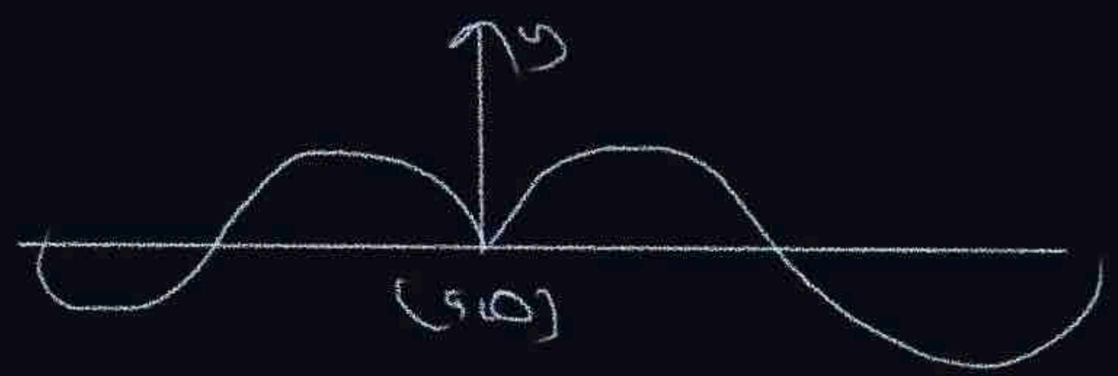
$a \neq 1$

$$f(x) = \begin{cases} a\sqrt{x+1} & 0 < x < 3 \\ bx + a & 3 \leq x < 5 \end{cases}$$

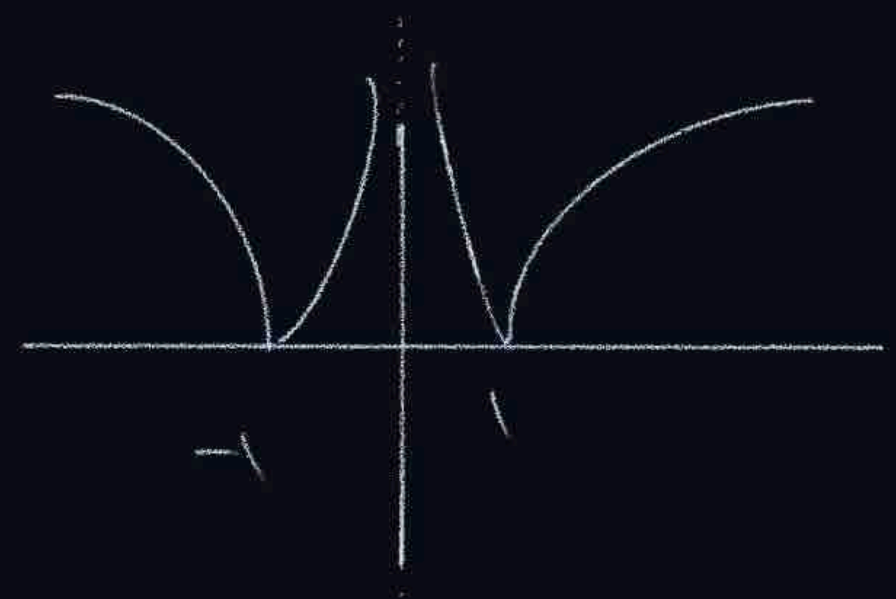
find a and b ?

diffble at $x = 3$

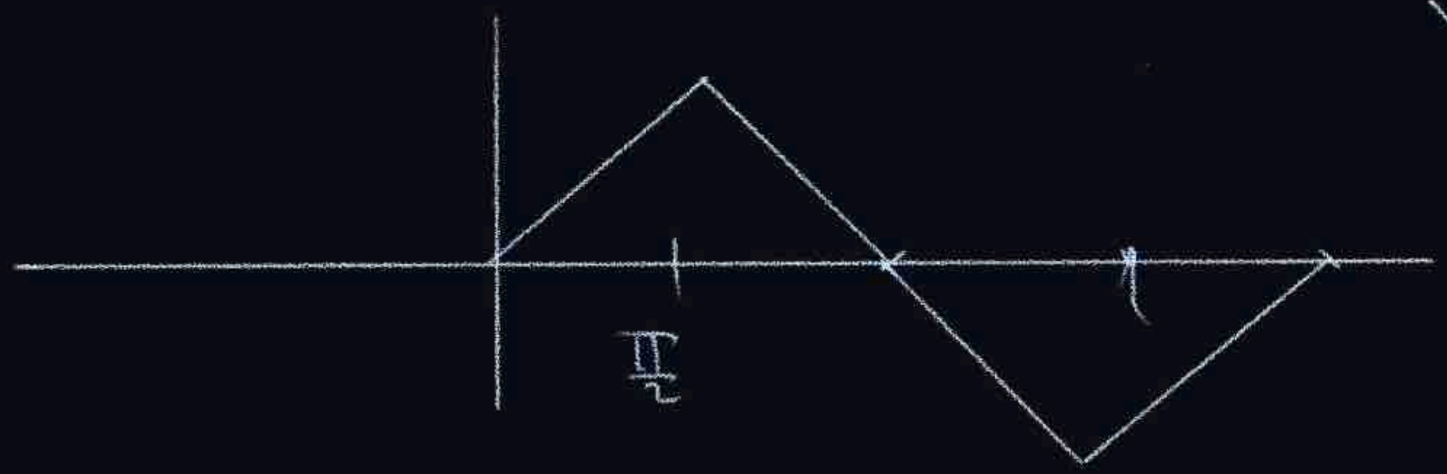
$f(x) = \sin|x|$



$f(x) = |\log|x||$ $x \neq 0$



$f(x) = \sin^{-1}(\sin x)$

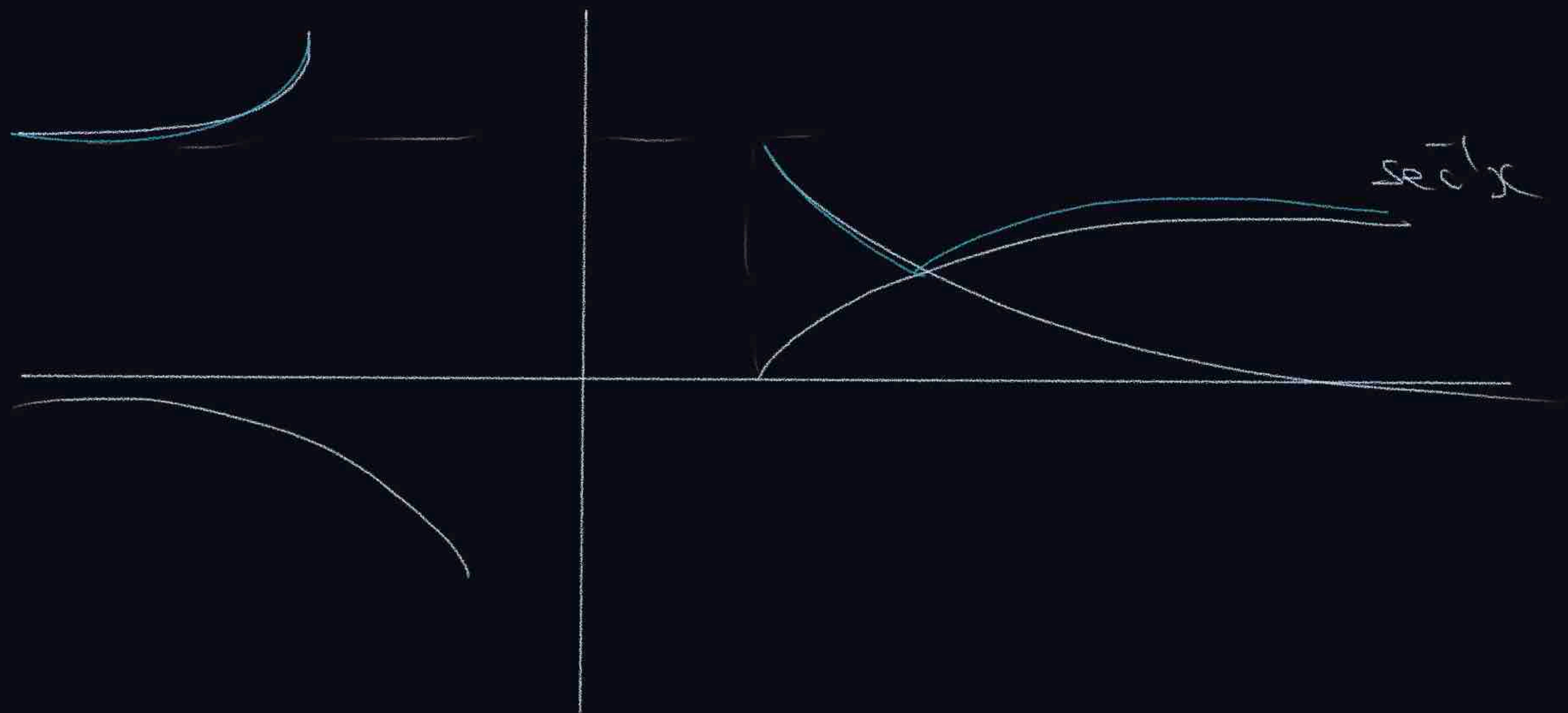


$(2n+1)\frac{\pi}{2}$

$f(x) = \sin^{-1}|\sin x|$

$f(x) = e^{-|x|}$

$f(x) = \max\{\sec^{-1}x, \operatorname{cosec}^{-1}x\}$



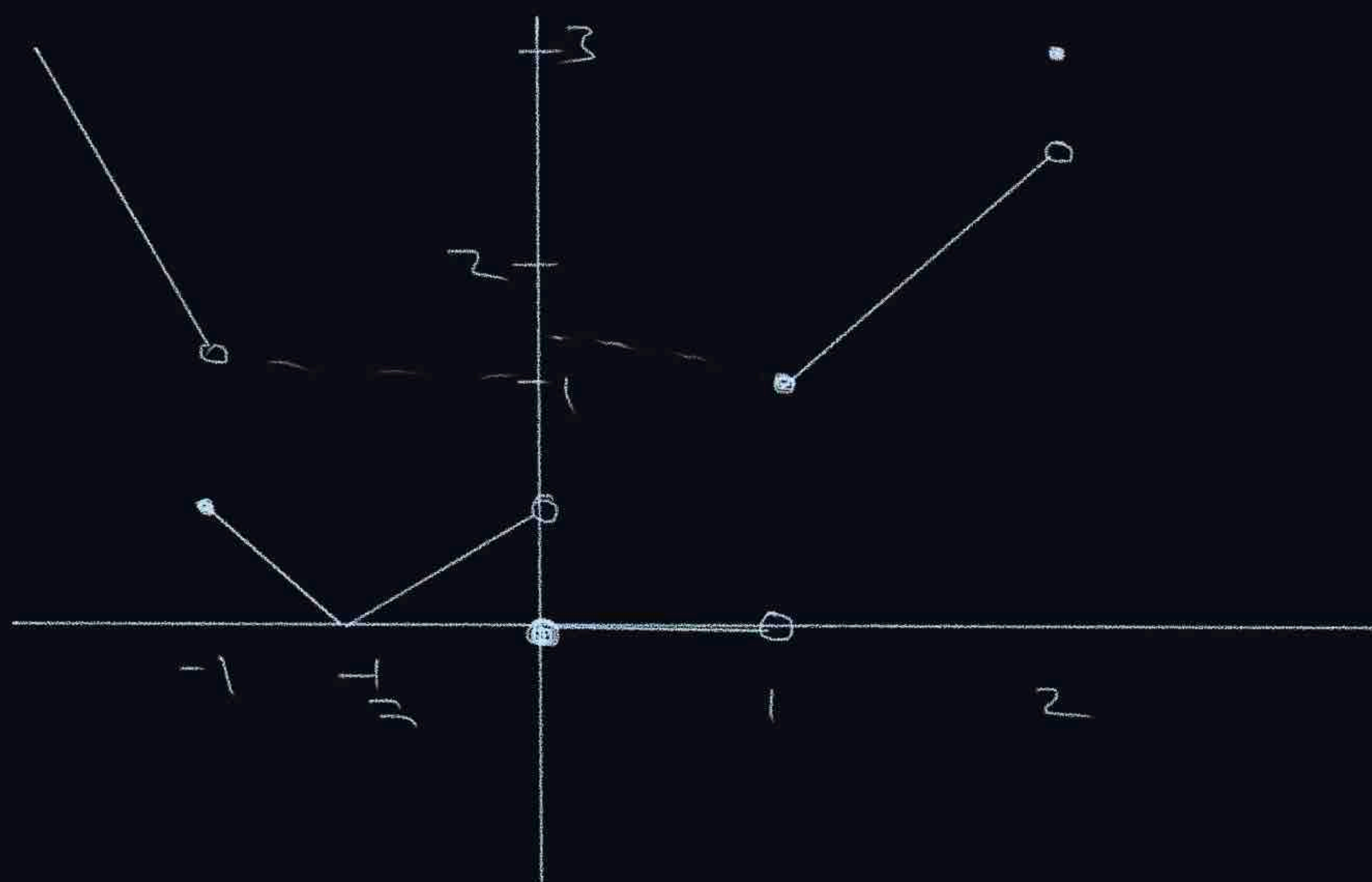
$f(x) = \max\{x^2 + 2ax + 1, b\}$

prove that

$a^2 > 1 - b$

$f(x)$ has two points of nondiffblt

$$f(x) = \left| \left(x + \frac{1}{2}\right) [x] \right| \qquad -2 \le x \le 2$$



$x = -1, -\frac{1}{2}, 0, 1, 2$

Differentiability at $x=a$ for cont funct $f(x)$ and $g(x)$

$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)\,g(x)$
Diffble	Diffble	$f'(a^+)+g'(a^+) = f'(a^-)+g'(a^-)$	$f'(a^+)\,g'(a^+) = f'(a^-)\,g'(a^-)$
Diffble	Not Diff	Not diffble	May or May not $f'(a^+)\,g'(a^+) = f'(a^-)\,g'(a^-)$ $0 \times g'(a^+) = 0 \times g'(a^-)$
Not diff	Not diff	May or May not $5+6 = 6+5$ $f'(a^+)+g'(a^+) = f'(a^+)+g'(a^-)$	May or may not $5(6) = 6(5)$ $f'(a^+)\,g'(a^+) = f'(a^-)\,g'(a^-)$

$f(x) = |x| + |x-1|$

$f(x) = [x] + |1-x| \quad x \in (-1,3)$

$f(x) = (x^2-1)|x^2-x-2| + \sin|x|$

$f(x) + g(x)$

diff + diff = diff

Not + diff = Not diff

Not + Not = $\boxed{\text{Check}}$

$h'(a^-) = h'(a^+)$

$f(x)\, g(x)$

diff diff = diff

Not diff = check if (diff = 0)

Not Not = check

$h'(a^-) = h'(a^+)$