8

Simple and Compound Interest

Interest is the fee paid for borrowed money. We *receive* interest when we let others use our money (for example, by depositing money in a savings account or making a loan). We *pay* interest when we use other people's money (such as when we borrow from a bank or a friend). Are you a "receiver" or a "payer"?

In this chapter we will study simple and compound interest. **Simple interest** is interest that is calculated on the balance owed but not on previous interest. **Compound interest**, on the other hand, is interest calculated on any balance owed including previous interest. Interest for loans is generally calculated using simple interest, while interest for savings accounts is generally calculated using compound interest.

The concepts of this chapter are used in many upcoming topics of the text. So hopefully you have *interest* in mastering the stuff in this chapter.

Unit Objectives

Unit 8.1 Computing simple interest and maturity value

- **a** Computing simple interest and maturity value—loans stated in months or years
- **b** Counting days and determining maturity date—loans stated in days
- **c** Computing simple interest—loans stated in days

Unit 8.2 Solving for principal, rate, and time

- **a** Solving for *P* (principal) and *T* (time)
- **b** Solving for *R* (rate)

Unit 8.3 Compound interest

- **a** Understanding how compound interest differs from simple interest
- **b** Computing compound interest for different compounding periods

Unit 8.1 Computing simple interest and maturity value

Wendy Chapman just graduated from college with a degree in accounting and decided to open her own accounting office (she can finally start earning money instead of paying it on college). On July 10, 2005, Wendy borrowed $12,000 from her Aunt Nelda for office furniture and other start-up costs. She agreed to repay Aunt Nelda in 1 year, together with interest at 9%.

The original amount Wendy borrowed—$12,000—is the **principal**. The percent that Wendy pays for the use of the money—9%—is the **rate of interest** (or simply the interest rate). The length of time—1 year—is called the **time** or **term**. The date on which the loan is to be repaid—July 10, 2006—is called the **due date** or **maturity date**. The total amount Wendy must repay (which we will calculate later) consists of principal ($12,000) and **interest** ($1,080); the total amount ($13,080) is called the **maturity value**.

Banks provide a valuable service as money brokers. They borrow from some people (through savings accounts, etc.) and loan that same money to others (at a higher rate). Some of these loans are simple interest loans.

a Computing simple interest and maturity value—loans stated in months or years

To calculate interest, we first multiply the principal by the annual rate of interest; this gives us interest per year. We then multiply the result by time (in years).

= simple interest formula

$$I = PRT$$

I = Dollar amount of interest P = Principal R = Annual rate of interest T = Time (in years)

TIP what is *PRT*?

Remember, when symbols are written side by side, it means to multiply, so *PRT* means $P \times R \times T$. Also, don't forget R, the interest rate, is the *annual* rate; and T is expressed in *years* (or a fraction of a year).

Example 1 On July 10, 2005, Wendy Chapman borrowed $12,000 from her Aunt Nelda. If Wendy agreed to pay a 9% annual rate of interest, calculate the dollar amount of interest she must pay if the loan is for **(a)** 1 year, **(b)** 5 months, and **(c)** 15 months.

a. 1 year: $I = PRT = \$12{,}000 \times 9\% \times 1 = \$1{,}080$

b. 5 months: $I = PRT = \$12{,}000 \times 9\% \times \frac{5}{12} = \$450$

c. 15 months: $I = PRT = \$12{,}000 \times 9\% \times \frac{15}{12} = \$1{,}350$

We can do the arithmetic of Example 1 with a calculator:

Keystrokes (for most calculators)

12,000	×	9	%	=						**1,080.00**
12,000	×	9	%	×	5	÷	12	=		**450.00**
12,000	×	9	%	×	15	÷	12	=		**1,350.00**

To find the maturity value, we simply add interest to the principal.

= maturity value formula
$M = P + I$
M = Maturity value P = Principal I = Dollar amount of interest

Example 2 Refer to Example 1. Calculate the maturity value if the 9% $12,000 loan is for **(a)** 1 year, **(b)** 5 months, and **(c)** 15 months.

a. 1 year: $M = P + I$ = $12,000 + $1,080 = $13,080
b. 5 months: $M = P + I$ = $12,000 + $450 = $12,450
c. 15 months: $M = P + I$ = $12,000 + $1,350 = $13,350

Wendy must pay a total of $13,080 if the loan is repaid in 1 year (July 10, 2006), $12,450 if the loan is repaid in 5 months (December 10, 2005), and $13,350 if the loan is repaid in 15 months (October 10, 2006).

b Counting days and determining maturity date—loans stated in days

In Examples 1 and 2, the term was stated in months or years. Short-term bank loans often have a term stated in days (such as 90 or 180 days) rather than months. Before calculating the amount of interest for these loans, we must know how to count days. One method is to look at a regular calendar and start counting: the day *after* the date of the loan is day 1, and so on. However, that method can be time-consuming and it is easy to make a mistake along the way. We will, instead, use a **day-of-the-year calendar**, shown as Appendix D; pay special attention to the entertaining footnote. In the day-of-the-year calendar, each day is numbered; for example, July 10 is day 191 (it is the 191st day of the year). The next example shows how to use a day-of-the-year calendar.

Example 3 Find **(a)** 90 days from September 10, 2006; **(b)** 180 days from September 10, 2006; and **(c)** 180 days from September 10, 2007.

a.	Sep. 10	→	Day	253	
				+90	
	Dec. 9	←		343	
b.	Sep. 10	→	Day	253	
				+180	
				433	*(This is greater than 365, so we must subtract 365)*
				-365	
	Mar. 9	←		68	
c.	Sep. 10	→	Day	253	
				+180	
				433	*(This is greater than 365, so we must subtract 365)*
				-365	
	Mar. 8	←		68	*(Because this is a leap year, March 8 is day 68)*

In parts (b) and (c) of Example 3, we found that the final date was the 68th day of the year. For a *non-leap year*, the 68th day is March 9. With a *leap year*, like 2008, there is an extra day in February so March 9 is day 69; March 8 is day number 68.

An optional method for counting days is known as the *days-in-a-month method*. With this method, we remember how many days there are in each month; the method is shown in Appendix D, page D-2. While a day-of-the-year calendar is often easier to use, understanding the days-in-a-month method is important because we may not always have a day-of-the-year calendar with us. Here is how we could do Example 3, part (c), using the days-in-a-month method:

180 days from September 10, 2007?

Days left in September: 30 - 10 =	20	*September has 30 days; not charged interest for first 10 days*
Days in October	+ 31	
Subtotal	51	
Days in November	+ 30	
Subtotal	81	
Days in December	+ 31	
Subtotal	112	
Days in January	+ 31	
Subtotal	143	
Days in February (leap year)	+ 29	
Subtotal	172	
Days in March	+ 8	*We need 8 more days to total 180*
Total	180	
Date is **March 8**		

In the next example, we'll figure out how many days between two dates. For some of us, there are quite a few days between dates (oops, wrong kind of date).

Example 4 Find the number of days between each set of dates: **(a)** July 24 to November 22, **(b)** July 24 to March 13 of the following year (non-leap year), and **(c)** July 24 to March 13 (leap year).

a. Nov. 22 ⟶ Day 326 (*Last day is minuend, on top*)
July 24 ⟶ Day -205
121 days

b. Number of days left in first year: 365 - 205 (day number for July 24) 160
Number of days in next year: Mar. 13 ⟶ +72
232 days

c. Number of days left in first year: 365 - 205 (day number for July 24) 160
Number of days in next year: Mar. 13 ⟶ 72 + 1 (for leap year) ⟶ +73
233 days

In part (b) of Example 4 (non-leap year), March 13 is day 72. But with a leap year in part (c), there is an extra day in February, making March 13 day 73, not day 72.

Here is how we could do Example 4, part (c), using the days-in-a-month method:

Days between July 24 and March 13 (a leap year)?

Days in July: 31 - 24 =	7	*July has 31 days; not charged interest for first 24 days*
Days in August	31	
Days in September	30	
Days in October	31	
Days in November	30	
Days in December	31	
Days in January	31	
Days in February (leap year)	29	
Days in March	+ 13	
Total	**233 days**	

C Computing simple interest—loans stated in days

The **Truth in Lending Act**, also known as **Regulation Z**, applies to **consumer loans**. The regulation does *not* set maximum interest rates; however many states set limits. It does require lenders to notify the borrower of two things: how much extra money the borrower is paying (known as **finance charges**) as a result of borrowing the money and the **annual percentage rate (APR)** the borrower is paying, accurate to $\frac{1}{8}$ of 1%. The law does not apply to business loans, loans over $25,000 (unless they are secured by real estate), most public utility fees, and student loan programs. Apparently, the government figures that businesspeople and—get this—students are bright enough to figure their own APR.

Prior to 1969, when the Truth in Lending Act became effective, lenders generally used a 360-day year for calculating interest. Without calculators and computers, calculations were easier using a 360-day year than a 365-day year. In calculating an APR for Truth in Lending purposes, lenders are required to use a 365-day year. Many lenders use a 360-day year for business loans (remember, business loans are exempt from the Truth in Lending Act).

Although we will not emphasize the following terminology, some people and some textbooks refer to interest based on a 360-day year as **ordinary interest** (or **banker's interest**) and interest based on a 365-day year as **exact interest**.

Example 5 Calculate interest on a 90-day $5,000 loan at 11%, using **(a)** a 360-day year and **(b)** a 365-day year.

a. 360-day year: $I = PRT = \$5{,}000 \times 11\% \times \frac{90}{360} = \$137.50$

b. 365-day year: $I = PRT = \$5{,}000 \times 11\% \times \frac{90}{365} = \$135.62$

As you can see from Example 5, a 360-day year benefits the lender and a 365-day year benefits the borrower.

TIP **use estimating to determine if an answer is reasonable**

It is easy to make a mistake when lengthy calculations are involved (none of us ever makes mistakes though, do we?). Estimating can be helpful in detecting errors. Using a rate of 10% and a term of 1 year provides a good reference point to estimate interest. In Example 5, $5,000 × 10% interest for 1 year is $500 (we simply move the decimal point one place to the left). The loan of Example 5 is for about $\frac{1}{4}$ of a year; $\frac{1}{4}$ of $500 is about $125. And the rate is 11%, not 10%, so the amount would be slightly greater than $125. The two answers of Example 5, $137.50 and $135.62, seem reasonable.

While some loan agreements require the borrower to pay a **prepayment penalty** if the loan is paid off early, most loans give the borrower the right to prepay part or all of the loan without penalty. Most lenders rely on what is called the **U.S. Rule** to calculate interest. With the U.S. Rule, interest is calculated to the date payment is received and on the basis of a 365-day year.

Example 6 Refer to Example 5, in which you get a 90-day $5,000 loan at 11%. You are able to pay the loan off early, in 65 days. Calculate interest using the U.S. Rule.

$$I = PRT = \$5{,}000 \times 11\% \times \frac{65}{365} = \$97.95$$

Interest is $97.95. You saved $37.67 ($135.62 - $97.95) by paying off the loan early.

When a borrower elects to repay a single-payment loan with **partial payments**, interest is calculated first; the remainder of each partial payment is treated as principal and reduces the loan balance.

partial payments: calculating interest, principal, and remaining balance

Step 1 Calculate interest: $I = PRT$
Step 2 The remainder of the payment is principal: Principal = Total paid - Interest portion
Step 3 New balance = Previous balance - Principal portion of payment

Note: For the final payment, principal is the previous balance (so the balance will end up zero).

Example 7 Refer to Example 6, in which you get a 90-day $5,000 loan at 11% interest. Suppose you have some extra cash and pay $2,000 on day 21 (21 days after getting the loan); on day 65 (65 days after getting the loan) you pay off the loan. Calculate the amount of interest and principal for each payment as well as the total amount of your final payment.

Day number	Total payment	Interest	Principal	Balance
0	—	—	—	$5,000.00
21	$2,000.00	$31.64	$1,968.36	$3,031.64
65	$3,071.84	$40.20	$3,031.64	$ 0.00
Totals	$5,071.84	$71.84	$5,000.00	—

Procedure for payment on day 21
$I = PRT = \$5{,}000.00 \times 11\% \times \frac{21}{365} = \$31.64$
Principal = $2,000.00 - $31.64 = $1,968.36
Balance = $5,000.00 - $1,968.36 = $3,031.64

Procedure for payment on day 65
$I = PRT = \$3{,}031.64 \times 11\% \times \frac{44}{365} = \$40.20$ *(65 days - 21 days = 44 days)*
Principal = $3,031.64 (previous balance, so balance will be $0.00)
Total payment = $40.20 interest + $3,031.64 principal = $3,071.84

TIP double the interest

In Example 7, when calculating interest for the payment on day 65, you may have been tempted to calculate interest for 65 days. Remember, however, we calculated interest for the first 21 days as part of the first payment; you don't want to be charged interest again for the first 21 days (once is enough!).

Notice that in Example 7 you paid total interest of $71.84 compared to interest of $97.95 in Example 6. You may wonder why you saved some interest since both loans were paid off on day 65. The reason is that by paying $2,000 on day 21, the balance decreased, and interest for the last 44 days was figured on that reduced balance.

Well, that does it for this unit. Let's do the U-Try-It exercises to see if you understand the *principal* points of this unit. Take your *time*; do the problems at your own *rate*.

U-Try-It (Unit 8.1)

1. Suppose you borrow $8,000 for 18 months at 11% simple interest. Find: **(a)** interest and **(b)** maturity value.
2. Find 180 days from August 5.
3. How many days are there between May 22 and October 14?
4. You get a 90-day $15,000 business loan from your bank at 9.25% interest. Calculate interest assuming the bank uses **(a)** a 365-day year and **(b)** a 360-day year.

5. You get a 180-day \$20,000 loan from your credit union at 10.5% interest. You have some extra cash and pay \$8,000 on day 40 (40 days after getting the loan); on day 115 (115 days after getting the loan) you pay off the loan. Find the missing numbers (use a 365-day year).

Day number	Total payment	Interest	Principal	Balance
0	—	—	—	\$20,000.00
40	\$8,000.00			
115				
Totals				—

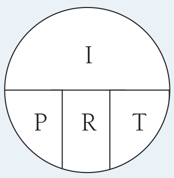
Answers: (If you have a different answer, check the solution in Appendix A.)
1a. \$1,320 **1b.** \$9,320 **2.** Feb. 1 **3.** 145 days **4a.** \$342.12 **4b.** \$346.88 **5.** Payment on day 40: \$230.14, \$7,769.86, \$12,230.14. Payment on day 115: \$12,494.01, \$263.87, \$12,230.14, \$0.00 Totals: \$20,494.01, \$494.01, \$20,000.00

Unit 8.2 Solving for principal, rate, and time

In Unit 8.1, we used the simple interest formula $I = PRT$ to solve for I. We can also solve for the other variables (P, R, and T). It will be easier if we have a formula especially designed for the variable in question. We can create separate formulas by using the Golden Rule of Equation Solving: *Do unto one side as you do unto the other.* For example, to find P, we can divide both sides of the formula $I = PRT$ by RT, getting $\frac{I}{RT} = P$. Or, we can use the following memory aid:

TIP **memory aid**

As a memory aid, some people like to place the symbols I, P, R, and T in a circle (notice that the I is alone at the top). The formula for each of the variables is found by covering the appropriate letter. Covering P with your finger, for example, shows I over RT.



$$I = PRT$$

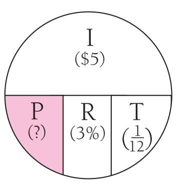
$$P = \frac{I}{RT} \qquad R = \frac{I}{PT} \qquad T = \frac{I}{PR}$$

I = Dollar amount of interest P = Principal
R = Annual rate of interest T = Time (in years)

Now we will solve a few problems for the variables P, R, and T. Because there are many more applications in solving for R than for P and T, we will solve for R last.

a Solving for *P* (principal) and *T* (time)

Example 1 You open a checking account. You are paid 3% interest on the average balance but are charged a \$5 monthly charge. Assuming that interest is paid monthly (regardless of the number of days in the month), calculate the average balance you must maintain to offset the \$5 monthly charge.



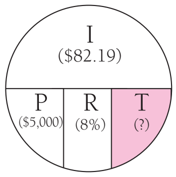
$$P = \frac{I}{RT} = \frac{\$5}{3\% \times \frac{1}{12}} = \frac{\$5}{.03 \times 1 \div 12} = \frac{\$5}{.0025} = \$2{,}000$$

Check answer: $I = PRT = \$2000 \times 3\% \times \frac{1}{12} = \$5.00$

You must maintain an average balance of \$2,000.

Example 2 You decide to pay off an 8% \$5,000 loan early. The bank tells you that you owe a total of \$82.19 interest. Assuming that the bank uses a 365-day year, for how many days are you being charged interest?

Remember, when we solve for T, we are finding the *portion of a year*, not the number of days.



$$T = \frac{I}{PR} = \frac{\$82.19}{\$5{,}000 \times 8\%} = \frac{\$82.19}{\$400.00} = .205475$$

You are being charged for .205475 of a year. Because there are 365 days in a year:

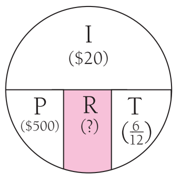
365 days × .205475 = 75 days. *Check answer*: $I = PRT = \$5{,}000 \times 8\% \times \frac{75}{365} = \$82.19$

You are being charged interest for 75 days.

b Solving for *R* (rate)

Now we will solve for *R*. Remember, *R* can be considered to be an APR (annual percentage rate).

Example 3 You borrow \$500 from your uncle and agree to repay the \$500 plus \$20 interest in 6 months. What interest rate are you paying?



$$R = \frac{I}{PT} = \frac{\$20}{\$500 \times \frac{6}{12}} = \frac{\$20}{\$250} = .08 = 8\%$$

You are paying an annual rate of 8%.

The Truth in Lending Act requires lenders to treat certain loan fees (such as credit report fees or "set-up" fees) as finance charges for purposes of calculating an APR. This is consistent with the concept that an APR considers the "amount and timing of value received and the amount and timing of payments made."

Example 4 You get a 90-day \$3,000 consumer loan at 8%. You are required to pay a document preparation fee of \$50. Calculate your APR. Express the rate with two decimal places.

- Principal (*P*) for APR purposes is the amount of money you have use of: \$3,000 - \$50 fee = \$2,950
- Interest (*I*) for APR purposes is total finance charges:

$I = PRT = \$3{,}000 \times 8\% \times \frac{90}{365} =$	\$ 59.18
Document preparation fee	+ 50.00
Total finance charges	\$109.18

$$R = \frac{I}{PT} = \frac{\$109.18}{\$2{,}950 \times \frac{90}{365}} = \frac{\$109.18}{\$727.40} \approx .1501 \approx 15.01\%$$

The symbol ≈ means "approximately equal to"

You are really paying an annual rate (APR) of 15.01%, considerably higher than the 8% stated rate. Because the loan is a consumer loan, the lender must inform you in writing what the APR is before you sign the loan agreement.

Suppose, instead of getting the loan in Example 4, you can get a 12% loan with no fees. You would be better off getting the 12% loan, since the APR is only 12%, while the APR for the loan in Example 4 is 15.01%.

Some lenders use a 360-day year for business loans. In the next example, we will calculate an APR on a loan using a 360-day year.

Example 5 You get a 60-day $2,000 business loan at 10% interest. The lender uses a 360-day year. Calculate your APR.

$$I = PRT = \$2000 \times 10\% \times \frac{60}{360} = \$33.33$$

$$R = \frac{I}{PT} = \frac{\$33.33}{\$2{,}000 \times \frac{60}{365}} = \frac{\$33.33}{\$328.77} \approx .1014 \approx 10.14\%$$

Even if interest is calculated using a 360-day year, an APR always uses a 365-day year

You must pay $33.33 interest. You are really paying an annual rate (APR) of 10.14%. Because the loan is a business loan (not a consumer loan), the lender is not required to inform you of the APR. But that's no problem because you can calculate your own APR (right?).

Another way to calculate interest is known as the **discount method** (also referred to as the **bank discount method**). This method, not used as much now as in the past, figures interest on the *maturity value*; the **proceeds** (maturity value minus interest) are given to the borrower, who must repay the maturity value. The bank discount method uses a 360-day year.

bank discount method formulas

$$D = MRT$$

$$\text{Proceeds} = M - D$$

D = Bank discount (dollar amount of interest)
M = Maturity value
R = Annual rate
T = Time, in years (using a 360-day year)

Example 6 You get a loan using the discount method. You sign a note, agreeing to repay the lender $5,000 in 90 days. Assuming a discount rate of 12%, calculate **(a)** interest (discount), **(b)** proceeds you receive, and **(c)** the APR.

a. $D = MRT = \$5{,}000 \times 12\% \times \frac{90}{360} = \$150$

b. Proceeds $= M - D = \$5{,}000 - \$150 = \$4{,}850$

c. APR: $R = \frac{I}{PT} = \frac{\$150}{\$4{,}850 \times \frac{90}{365}} = \frac{\$150}{\$1{,}195.89} \approx .1254 \approx 12.54\%$

This is the amount of money you have use of

You will be given $4,850 and must pay back $5,000 in 90 days, resulting in an APR of 12.54%.

In Example 6, the APR (12.54%) is higher than the discount rate (12%). This is due to two factors: (1) interest for the loan of Example 6 is calculated on the maturity value ($5,000), rather than the amount you have use of ($4,850), and (2) the discount method uses a 360-day year, whereas the APR always uses a 365-day year.

That does it for this unit. Let's try the U-Try-It set to find out if it sunk in!

U-Try-It (Unit 8.2)

1. You pay your bank $157.50 interest for 6 months on a 9% loan. How much did you borrow?
2. You pay your bank $78.90 interest on an 8% $4,000 loan. If the bank uses a 365-day year, for how many days are you being charged interest?
3. You get a $5,000 business loan for 180 days at 10.5% interest. The lender charges you a $150 document preparation fee and uses a 360-day year for calculating interest. What is your APR, to the nearest hundredth of a percent?

Answers: (If you have a different answer, check the solution in Appendix A.)
1. $3,500 **2.** 90 days **3.** 17.25%

Unit 8.3 Compound interest

a Understanding how compound interest differs from simple interest

Simple interest is interest that's earned only on principal. Compound interest, on the other hand, is interest earned on principal *plus previous interest*. The next example illustrates the difference.

Example 1 Trish and Hannah each have \$100. Trish loans her \$100 to a friend. Her friend agrees to repay her in 3 years, together with 6% simple interest. Hannah deposits her \$100 in a savings account and leaves it there for 3 years to accumulate interest at 6%, compounded annually. Calculate the amount Trish and Hannah will have in 3 years.

Trish's balance at the end of year 3 (simple interest):

$I = PRT = \$100 \times 6\% \times 3 = \$18$ $M = P + I = \$100 + \$18 = \$118$

Hannah's balance at the end of each year (compound interest):

Yr. 1: $I = PRT = \$100 \times 6\% \times 1 = \$6$ $M = P + I = \$100 + \$6 = \$106$
Yr. 2: $I = PRT = \$106 \times 6\% \times 1 = \$6.36$ $M = P + I = \$106 + \$6.36 = \$112.36$
Yr. 3: $I = PRT = \$112.36 \times 6\% \times 1 = \$6.74$ $M = P + I = \$112.36 + \$6.74 = \$119.10$

Trish will end up with \$118. Hannah will end up with \$119.10. In figuring interest for Hannah, the year 1 ending balance (\$106) was used to calculate interest for year 2, and the year 2 ending balance (\$112.36) was used to calculate interest for year 3. This is how compound interest works; interest is earned on principal plus previous interest.

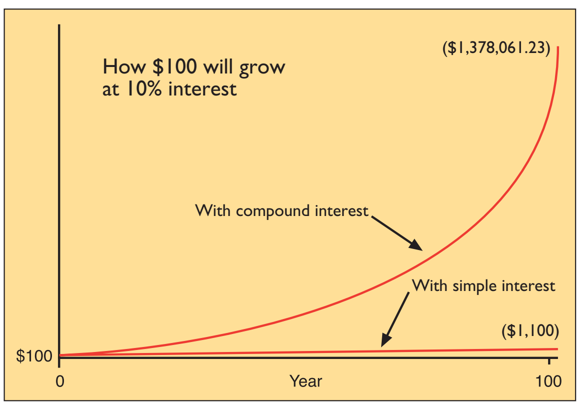
In Example 1, because of compounding, Hannah ends up with \$1.10 more than Trish. The amount may seem fairly insignificant. However, as the time period is extended, the difference becomes substantial. Illustration 8-1 compares simple interest with compound interest over a 100-year period, using a \$100 amount at 10% interest. As you can see, compounding makes quite a difference (\$1,378,061.23 balance instead of \$1,100). Notice that the balance using simple interest is represented by a straight line, while the balance using compound interest is represented by an *accelerated curve* (due to earning interest on interest).

b Computing compound interest for different compounding periods

In Example 1, Hannah earns interest of 6% compounded annually (each year). Interest is often compounded more often than once a year, such as:

- *Semiannually*, where interest is calculated twice a year (each 6 months)
- *Quarterly*, where interest is calculated four times a year (each 3 months)
- *Monthly*, where interest is calculated 12 times a year (each month)
- *Daily*, where interest is calculated each day

Illustration 8-1 Magic of Compound Interest



In the next example, we will calculate Hannah's balance at the end of 6 months if she earns 6% compounded *semiannually*.

Example 2 Hannah deposits \$100 in a savings account earning 6% compounded semiannually. What will her balance be in 6 months?

Let's use the simple interest formula: $I = PRT$. Remember, T is 6 months, or $\frac{1}{2}$ of a year.

$I = PRT = \$100 \times 6\% \times \frac{1}{2} = \$3$ $\qquad$ $M = P + I = \$100 + \$3 = \$103$

In 6 months Hannah's balance will be \$103.

In Example 2, we found interest by multiplying principal by 6% and then by $\frac{1}{2}$: $\$100 \times 6\% \times \frac{1}{2} = \$3$. We would get the same result multiplying principal by $\frac{1}{2}$ of 6%, which is 3%: $\$100 \times 3\% = \$3$. This 3% rate is referred to as the **interest rate per period**, or the **periodic rate.** The periodic rate is found by dividing the annual rate (often called the **nominal rate**) by the number of compounding periods per year.

= periodic rate formula

$$\text{Periodic rate} = \frac{\text{Annual Rate}}{\text{Periods per year}}$$

Example 3 Find the periodic rate for **(a)** 6% compounded semiannually, **(b)** 7.5% compounded quarterly, and **(c)** 8.25% compounded monthly.

a. $\frac{6}{2} = 3(\%)$ $\qquad$ **b.** $\frac{7.5}{4} = 1.875(\%)$ $\qquad$ **c.** $\frac{8.25}{12} = 0.6875(\%)$

For an interest rate of 6% compounded semiannually, a person will earn 3% each period (6 months); for 7.5% compounded quarterly, a person will earn 1.875% each period (3 months); and for 8.25% compounded monthly, a person will earn 0.6875% each period (1 month).

TIP don't round periodic rate

Don't make the common mistake of rounding a preiodic rate. In Example 3(c), the periodic rate for 8.25% compounded monthly is 0.6875%, *not* 0.69%. In some cases the periodic rate may be a repeating decimal. For example, the periodic rate for 7% compounded monthly is $0.58\overline{3}$ (with the 3*s* continuing forever). If the periodic rate is used in calculations, be sure to use as many decimal places for the rate as your calculator will allow (such as 0.58333333%).

In the next example, we will use a periodic rate to find an ending balance. As you will see, it is easier than using the simple interest formula.

Example 4 Hannah deposits \$100 in a savings account earning 6% compounded semiannually and leaves it there for 3 years. Find the ending balance using the periodic rate of 3%.

	Interest	Balance
Beginning	—	\$100.00
6 months	\$100.00 × 3% = \$3.00	\$103.00
12 months	\$103.00 × 3% = \$3.09	\$106.09
18 months	\$106.09 × 3% = \$3.18	\$109.27
24 months	\$109.27 × 3% = \$3.28	\$112.55
30 months	\$112.55 × 3% = \$3.38	\$115.93
36 months	\$115.93 × 3% = \$3.48	\$119.41

Notice, the dollar amount of interest increases each period as the balance increases. That's because of compounding.

The arithmetic of Example 4 can be done on a calculator by increasing the balance 3% each 6 months.

Keystrokes (for most calculators)

100 [+] 3 [%] [=]	103.00
[+] 3 [%] [=]	106.09
[+] 3 [%] [=]	109.27
[+] 3 [%] [=]	112.55
[+] 3 [%] [=]	115.93
[+] 3 [%] [=]	**119.41**

Let's compare the results of Examples 1 and 4.

Interest rate	What $100 grows to in 3 years
6% simple interest (Example 1)	$118.00
6% compounded annually (Example 1)	$119.10
6% compounded semiannually (Example 4)	$119.41

As you can see, the more often interest is calculated, the more benefit there is to the person receiving the interest.

That finishes this chapter. Congratulations! As mentioned, the concepts of this chapter are important to many upcoming topics; we will calculate simple interest and use compounding many times. Hopefully, that excites you. Now, let's make sure we've got the concepts of this last unit mastered by doing the U-Try-It problems.

U-Try-It
(Unit 8.3)

1. David Christopher deposits $500 in a savings account earning 5% compounded annually. What will the balance be in 4 years?
2. John Travis deposits $1,200 in a savings account earning 4.5% compounded quarterly. What will the balance be in 1 year?

Answers: (If you have a different answer, check the solution in Appendix A.)
1. $607.75 **2.** $1,254.92

Chapter in a Nutshell

Objectives	Examples
Unit 8.1 Computing simple interest and maturity value	
a Computing simple interest and maturity value—loans stated in months or years	$5,000 at 8% for 18 months: $I = PRT = \$5{,}000 \times 8\% \times \frac{18}{12} = \$600$ $M = P + I = \$5{,}000 + \$600 = \$5{,}600$
b Counting days and determining maturity date—loans stated in days	180 days from Apr. 23: Apr. 23 → Day 113 +180 Oct. 20 ← 293 90 days from Nov. 17: Nov. 17 → Day 321 + 90 411 - 365 Feb. 15 ← 46 Days between Mar. 12 and Oct. 28: Oct. 28 → Day 301 Mar. 12 → Day - 71 230 days Days between Nov. 13 and Apr. 22 (leap year): First Year: 365 - 317 48 Next Year: 112 + 1 (leap year) 113 161 days

Chapter in a Nutshell (continued)

Objectives | **Examples**

c Computing simple interest—loans stated in days

$8,000 at 9% for 90 days, using: **(a)** 365-day year and **(b)** 360-day year

a. $I = PRT = \$8{,}000 \times 9\% \times \frac{90}{365} = \$177.53$

b. $I = PRT = \$8{,}000 \times 9\% \times \frac{90}{360} = \$180.00$

$10,000 at 7% for 180 days; $4,800 partial payment on day 52; balance paid on day 115

Day	Total payment	Interest	Principal	Balance
0	—	—	—	$10,000.00
52	$4,800.00	$99.73	$4700.27	$5,299.73
115	$5,363.76	$64.03	$5,299.73	$0.00
Totals	$10,163.76	$163.76	$10,000.00	—

Calculations for payment on day 52

$I = PRT = \$10{,}000 \times 7\% \times \frac{52}{365} = \$99.73$

Principal = $4,800 - $99.73 = $4,700.27

Balance = $10,000 - $4,700.27 = $5,299.73

Calculations for payment on day 115

$I = PRT = \$5{,}299.73 \times 7\% \times \frac{63}{365} = \$64.03$ *(115 days - 52 days = 63 days)*

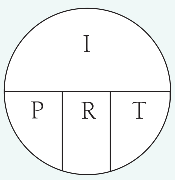
Principal = $5,299.73 (previous balance)

Total payment = $64.03 + $5,299.73 = $5,363.76

Unit 8.2 Solving for principal, rate, and time

a Solving for *P* (principal) and *T* (time)

$3,000 loan for 73 days at 9.75% interest



$I = PRT = \$3{,}000 \times 9.75\% \times \frac{73}{365} = \$58.50$

$P = \frac{I}{RT} = \frac{\$58.50}{9.75\% \times \frac{73}{365}} = \$3{,}000$

$T = \frac{I}{PR} = \frac{\$58.50}{\$3{,}000 \times 9.75\%} = .20$ 365 days × .20 = 73 days

b Solving for *R* (rate)

$6,500 business loan for 90 days at 13% interest with $250 set-up fee. If lender uses a 360-day year to calculate interest, what is APR?

Principal for APR purposes: $6,500 - $250 = $6,250

Interest for APR purposes:

$I = PRT = \$6{,}500 \times 13\% \times \frac{90}{360}$ =	$211.25
Set-up fee	+250.00
Total finance charges	$461.25

$R = \frac{I}{PT} = \frac{\$461.25}{\$6{,}250 \times \frac{90}{365}} = .2993 = 29.93\%$

Use 365-day year for APR

Chapter in a Nutshell (concluded)

Objectives	Examples
b (continued)	Loan using the discount method; you agree to repay lender \$4,000 in 180 days using discount rate of 10%. APR? $D = MRT = \$4{,}000 \times 10\% \times \frac{180}{360} = \$200$ ← *Discount method uses a 360-day year* Proceeds $= M - D = \$4{,}000 - \$200 = \$3{,}800$ $R = \frac{I}{PT} = \frac{\$200}{\$3{,}800 \times \frac{180}{365}} \approx .1067 \approx 10.67\%$ *Use 365-day year for APR*

Unit 8.3 Compound interest

Objectives	Examples
a Understanding how compound interest differs from simple interest	\$5,000 for 3 years at 8% using **(a)** simple interest, and **(b)** interest compounded annually: **a.** $I = PRT = \$5{,}000 \times 8\% \times 3 = \$1{,}200$ $M = P + I = \$5{,}000 + \$1{,}200 = \$6{,}200$ **b.** Balance Yr. 1: $I = PRT = \$5{,}000 \times 8\% \times 1 = \$400$ $M = P + I = \$5{,}000 + \$400 = \$5{,}400$ Yr. 2: $I = PRT = \$5{,}400 \times 8\% \times 1 = \$432$ $M = P + I = \$5{,}400 + \$432 = \$5{,}832$ Yr. 3: $I = PRT = \$5{,}832 \times 8\% \times 1 = \$466.56$ $M = P + I = \$5{,}832 + \$466.56 = \$6{,}298.56$
b Computing compound interest for different compounding periods	Periodic rate for 9.75% compounded quarterly: $\frac{9.75}{4} = 2.4375(\%)$ Deposit \$500 for 1 year at 9.75% compounded quarterly. Ending balance?

	Interest	Balance
Beginning	—	\$500.00
3 months	\$500.00 × 2.4375% = \$12.19	\$512.19
6 months	\$512.19 × 2.4375% = \$12.48	\$524.67
9 months	\$524.67 × 2.4375% = \$12.79	\$537.46
12 months	\$537.46 × 2.4375% = \$13.10	\$550.56

Critical Thinking

1. Suppose your business borrows some money. Who benefits from calculating interest using a 360-day year—you or the lender—and why?
2. In Unit 8.2, formulas for *P*, *R*, and *T* are derived from the formula $I = PRT$. Using equation-solving skills, show how these three formulas are derived.
3. If you get a loan with a front-end fee, why is the APR greater than the stated annual rate?
4. If you get a loan using the discount method, why is the APR greater than the stated annual rate?
5. Explain why you would rather earn compound interest than simple interest.
6. Explain why you are better off earning 6% interest compounded quarterly than 6% interest compounded semiannually.

Chapter Review Problems

Unit 8.1 Computing simple interest and maturity value

For Problems 1–7, consider a loan of Sterling George. Sterling borrowed $10,000 on October 1, 2005, for 1 year at 8% interest.

1. What is the principal amount? **$10,000**
2. What is the term? **1 year**
3. What is the maturity date? **October 1, 2006**
4. What is the dollar amount of interest? $I = PRT = \$10{,}000 \times 8\% \times 1 =$ **$800**
5. What is the maturity value? $M = P + I = \$10{,}000 + \$800 =$ **$10,800**
6. If Sterling borrowed the money for only 8 months, what is the total amount he will owe?

   $I = PRT = \$10{,}000 \times 8\% \times \frac{8}{12} = \$533.33$ $M = P + I = \$10{,}000 + \$533.33 =$ **$10,533.33**
7. If Sterling borrowed the money for 14 months, what is the total amount he will owe?

   $I = PRT = \$10{,}000 \times 8\% \times \frac{14}{12} = \$933.33$ $M = P + I = \$10{,}000 + \$933.33 =$ **$10,933.33**
8. In the simple interest formula $I = PRT$, I stands for the interest rate. (T or F)

   False. I stands for the dollar amount of interest; R stands for interest rate.
9. In the simple interest formula $I = PRT$, T stands for time, in months. (T or F) **False**. T stands for time, in *years*.

For Problems 10–12, calculate the number of days for which interest should be charged.

	Date of loan	Date of payment	Number of days
10.	Jan. 11, 2006	Oct. 28, 2006	**290 days**
11.	July 13, 2006	Feb. 21, 2007	**223 days**
12.	Dec. 18, 2007	Mar. 23, 2008 (leap year)	**96 days**

10. Oct. 28 → Day 301
Jan. 11 → Day - 11
290

11. Number of days left in first year: 365 - 194 (day number for July 13) = 171
Number of days in next year: Feb. 21 → + 52
223

12. Number of days left in first year: 365 - 352 (day number for Dec. 18) = 13
Number of days in next year: Mar. 23 → 82 + 1 (for leap year) = + 83
96

For Problems 13–15, calculate the maturity date.

	Date of loan	Term	Maturity date
13.	May 15, 2006	60 days	135 + 60 = 195 → **July 14**
14.	Aug. 2, 2006	180 days	214 + 180 = 394; 394 - 365 = 29 → **Jan. 29**
15.	Jan. 18, 2008	90 days	18 + 90 = 108 → **Apr. 17 (leap year)**

For Problems 16 and 17, we will calculate interest on a 13% 90-day $15,000 loan.

16. Calculate interest, assuming the lender uses a 360-day year.

    $I = PRT = \$15{,}000 \times 13\% \times \frac{90}{360} =$ **$487.50**
17. Calculate interest, assuming the lender uses a 365-day year.

    $I = PRT = \$15{,}000 \times 13\% \times \frac{90}{365} =$ **$480.82**

18. The Truth in Lending Act sets the maximum interest rate lenders can charge. (T or F) **False**

19. The Truth in Lending Act applies to all loans. (T or F) **False**; the law does not apply to business loans, loans over $25,000 (unless they are secured by real estate), most public utility fees, and student loan programs.

20. In calculating an APR for Truth in Lending purposes, lenders are required to use a 365-day year. (T or F) **True**

For Problems 21–24, consider a loan of Mary Patterson. Mary borrowed $25,000 at 11.5% interest for 120 days. The lender uses a 365-day year.

21. How much interest will Mary owe on the maturity date? $I = PRT = \$25{,}000 \times 11.5\% \times \frac{120}{365} =$ **$945.21**

22. Assume Mary pays the loan off early, in 89 days. How much interest will she owe?

$I = PRT = \$25{,}000 \times 11.5\% \times \frac{89}{365} =$ **$701.03**

23. Assume Mary has some extra cash and instead pays $8,000 on day 24 (24 days after getting the loan), then the balance on day 89 (89 days after getting the loan). Fill in the blanks.

Day number	Total payment	Interest	Principal	Balance
0	—	—	—	$25,000.00
24	$8,000.00	**$189.04**	**$7,810.96**	**$17,189.04**
89	**$17,541.06**	**$352.02**	**$17,189.04**	$0.00
Totals	**$25,541.06**	**$541.06**	**$25,000.00**	—

Procedure for payment on day 24
$I = PRT = \$25{,}000.00 \times 11.5\% \times \frac{24}{365} =$ **$189.04**
Principal = $8,000.00 - $189.04 = **$7,810.96**
Balance = $25,000.00 - $7,810.96 = **$17,189.04**

Procedure for payment on day 89
$I = PRT = \$17{,}189.04 \times 11.5\% \times \frac{65}{365} =$ **$352.02** *(89 days - 24 days = 65 days)*
Principal = **$17,189.04** (previous balance)
Total payment = $352.02 + $17,189.04 = **$17,541.06**

24. How much interest does Mary pay under each situation: Problem 21, Problem 22, and Problem 23.

Problem 21: **$945.21** Problem 22: **$701.03** Problem 23: **$541.06**

Unit 8.2 Solving for principal, rate, and time

For problems in this unit, if the answer is a percent, express the answer to the nearest hundredth of a percent.

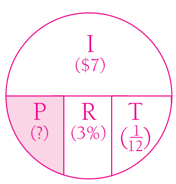
25. From memory, or by modifying the formula $I = PRT$, write a formula designed to solve for **(a)** P, **(b)** R, and **(c)** T.

$P = \frac{I}{RT}$ $R = \frac{I}{PT}$ $T = \frac{I}{PR}$

For Problems 26–29, find the missing value.

	I	P	R	T
26.	**$320.83**	$5,000	11%	7 months
27.	$63.75	**$4,500**	8.5%	2 months
28.	2,964.75	$35,400	**16.75%**	6 months
29.	$275	$2,000	11%	**1.25 yrs = 15 months**

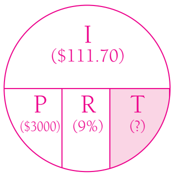
30. You open a checking account. You are paid 3% interest on the average balance but are charged a $7 monthly charge. Assuming that interest is paid monthly (regardless of the number of days in the month), calculate the average daily balance you must maintain to offset the $7 monthly charge.



$P = \frac{I}{RT} = \frac{\$7}{3\% \times \frac{1}{12}} = \frac{\$7}{.03 \times 1 \div 12} = \frac{\$7}{.0025} =$ **$2,800**

Check answer: $I = PRT = \$2{,}800 \times 3\% \times \frac{1}{12} = \$7.00$

31. You decide to pay off a 9% \$3,000 loan early. The bank tells you that you owe \$111.70 interest. Assuming that the bank uses a 365-day year, for how many days are you being charged interest?

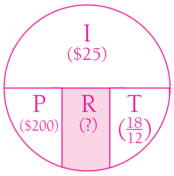


$$T = \frac{I}{PR} = \frac{\$111.70}{\$3{,}000 \times 9\%} = \frac{\$111.70}{\$270} \approx .4137037$$

365 days × .4137037 = **151 days**

Check answer: $I = PRT = \$3{,}000 \times 9\% \times \frac{151}{365} = \$111.70$

32. You borrow \$200 from your aunt and agree to repay her \$225 (\$200 principal + \$25 interest) in 18 months. What interest rate are you paying?



$$R = \frac{I}{PT} = \frac{\$25}{\$200 \times \frac{18}{12}} = \frac{\$25}{\$300} \approx .0833 \approx \mathbf{8.33\%}$$

33. You get a 180-day \$5,000 consumer loan at 9%. You are required to pay a \$100 setup fee at the time you get the loan. What is your APR?

Principal (*P*) for APR purposes is the amount of money you have use of: \$5,000 - \$100 fee = \$4,900
Interest (*I*) for APR purposes is total finance charges:

$I = PRT = \$5{,}000 \times 9\% \times \frac{180}{365} =$	\$221.92
Set-up fee	+ 100.00
Total finance charges	\$321.92

$$R = \frac{I}{PT} = \frac{\$321.92}{\$4{,}900 \times \frac{180}{365}} \approx \frac{\$321.92}{\$2{,}416.44} \approx .1332 \approx \mathbf{13.32\%}$$

34. You get a \$3,500 loan for 90 days. Interest of 13% is charged, using a 360-day year. What is the APR?

$I = PRT = \$3{,}500 \times 13\% \times \frac{90}{360} = \$113.75$

$$R = \frac{I}{PT} = \frac{\$113.75}{\$3{,}500 \times \frac{90}{365}} \approx \frac{\$113.75}{\$863.01} \approx .1318 \approx \mathbf{13.18\%}$$

↖ *Even though interest is calculated using a 360-day year, an APR always uses a 365-day year*

35. You get a loan using the discount method. You sign a note, agreeing to repay the lender \$2,000 in 60 days. Assuming a discount rate of 15%, determine the APR.

$D = MRT = \$2{,}000 \times 15\% \times \frac{60}{360} = \$50$

↖ *Remember, the discount method uses a 360-day year to calculate interest*

Proceeds = *M* - *D* = \$2,000 - \$50 = \$1,950 (this is money you have use of)

$$R = \frac{I}{PT} = \frac{\$50}{\$1{,}950 \times \frac{60}{365}} \approx \frac{\$50}{\$320.55} \approx .1560 \approx \mathbf{15.60\%}$$

↖ *Even though interest is calculated using a 360-day year, an APR always uses a 365-day year*

Unit 8.3 Compound interest

For Problems 36–38, calculate the periodic rate.

36. 8% compounded semiannually. $\frac{8}{2} = \mathbf{4(\%)}$

37. 7% compounded quarterly $\frac{7}{4} = \mathbf{1.75(\%)}$

38. 7.5% compounded monthly $\frac{7.5}{12} = \mathbf{.625(\%)}$

39. Jessica Gutierrez loans a friend \$700 at 5% simple interest for 3 years. What is the maturity value?

$I = PRT = \$700 \times 5\% \times 3 = \$105$ $M = P + I = \$700 + \$105 = \mathbf{\$805}$

40. Glenna Gardner deposits $700 in a savings account. The money is left on deposit for 3 years earning 5% compounded annually. Calculate the account balance at the end of 3 years.

	Interest	Balance
Beginning	—	$700.00
1 year	$700 × 5% = $35.00	$735.00
2 years	$735 × 5% = $36.75	$771.75
3 years	$771.75 × 5% = $38.59	**$810.34**

41. George Lavin deposits $700 in a savings account. The money is left on deposit for 3 years earning 5% compounded semiannually. Calculate the account balance at the end of 3 years. Do not round intermediate results, but write amounts to the nearest penny.

	Interest	Balance
Beginning	—	$700.00
6 months	$700 × 2.5% = $17.50	$717.50
12 months	$717.50 × 2.5% = $17.94	$735.44
18 months	$735.44 × 2.5% = $18.39	$753.82*
24 months	$753.82 × 2.5% = $18.85	$772.67
30 months	$772.67 × 2.5% = $19.32	$791.99
36 months	$791.99 × 2.5% = $19.80	**$811.79**

**Note*: Without rounding intermediate results, $735.4375 + $18.3859375 = $753.8234375

42. Refer to Problems 39–41. Who ended up with the most money, and why? **George Lavin (Problem 41) ended up with the most. The more often interest is compounded, the more interest is earned.**

Challenge problems

43. Bob Green purchased merchandise from a supplier and failed to pay the invoice amount ($285) by the last day of the credit period (August 23). Calculate the total amount Bob must pay on October 16 if the supplier charges 18% interest on past-due accounts.

Number of days: Oct. 16 ⟶ Day 289
Aug. 23 ⟶ Day -235
54

$I = PRT = \$285 \times 18\% \times \frac{54}{365} = \$7.59$

$M = P + I = \$285 + \$7.59 = \mathbf{\$292.59}$

44. Alyce Lee, a sporting goods retailer, purchased ski clothing from a supplier for $2,450. The seller offers a 4% discount if the invoice is paid within 10 days; if not paid within 10 days, the full amount must be paid within 30 days of the invoice date. Use the formula $R = \frac{I}{PT}$ to find the annual rate Alyce, in effect, is paying the supplier if she fails to pay the invoice at the end of the discount period. *Hint*: Alyce is, in effect, borrowing the net amount (amount after deducting the discount) for 20 days and must pay the difference as interest.

Invoice amount	$2,450
Discount: $2,450 × 4%	- 98
Net amount due	$2,352

If Alyce fails to pay the invoice within the discount period she is, in effect, borrowing $2,352 for 20 days and paying an extra $98 as interest, so:

$$R = \frac{I}{PT} = \frac{\$98}{\$2{,}352 \times \frac{20}{365}} = \frac{\$98}{\$128.88} \approx .7604 \approx \mathbf{76.04\%}$$

For Problems 45–48, do some calculations for delinquent property taxes.

45. You fail to pay your annual property taxes on the November 30, 2006, due date. If the tax was $845.23 and you are charged simple interest at 12%, calculate the amount of interest you must pay if you make payment on May 4, 2007.

Number of days left in first year: 365 - 334 (day number for Nov. 30) =		31
Number of days in next year:	May 4 ⟶	+ 124
		155

$I = PRT = \$845.23 \times 12\% \times \frac{155}{365} = \mathbf{\$43.07}$

46. In addition to the 12% simple interest, you are charged a one-time 6% penalty for failing to pay the tax on time. What is the one-time penalty?

$\$845.23 \times 6\% = \mathbf{\$50.71}$

47. What is the total amount you must pay on May 4, 2007?

$845.23 + $43.07 interest + $50.71 penalty = **$939.01**

48. Calculate your APR (including the 6% penalty).

$$R = \frac{I}{PT} = \frac{\$43.07 + \$50.71}{\$845.23 \times \frac{155}{365}} \approx \frac{\$93.78}{\$358.93} \approx .2613 \approx \mathbf{26.13\%}$$

49. The ad to the right states that $1,000 left on deposit for 5 years earning 8.75% compounded semiannually would result in the same balance as $1,000 earning 10.69% simple interest. Determine if the ad is correct. First, find the maturity value using 10.69% simple interest. Then, find the ending balance for 8.75% compounded semiannually.

10.69% simple interest
$I = PRT = \$1{,}000 \times 10.69\% \times 5 = \$534.50$
$M = P + I = \$1{,}000 + \$534.50 = \mathbf{\$1{,}534.50}$

8.75% compounded semiannually (let's use calculators)

Balance in 6 months: $1,000 + 4.375% =	$1,043.75
Balance in 12 months: + 4.375% =	$1,089.41
Balance in 18 months: + 4.375% =	$1,137.08
Balance in 24 months: + 4.375% =	$1,186.82
Balance in 30 months: + 4.375% =	$1,238.75
Balance in 36 months: + 4.375% =	$1,292.94
Balance in 42 months: + 4.375% =	$1,349.51
Balance in 48 months: + 4.375% =	$1,408.55
Balance in 54 months: + 4.375% =	$1,470.17
Balance in 60 months: + 4.375% =	**$1,534.49**

The ending balances are almost identical, showing that, for a 5-year period, 10.69% simple interest is equivalent to 8.75% compounded semiannually.

Practice Test

1. In the simple interest formula $I = PRT$, I stands for the interest rate. (T or F) **False**. I stands for the dollar amount of interest; R stands for interest rate.

2. Lynette Read borrowed $12,000 at 9.5% interest for 8 months. What is the maturity value?

$I = PRT = \$12{,}000 \times 9.5\% \times \frac{8}{12} = \$760$ $\qquad M = P + I = \$12{,}000 + \$760 = \mathbf{\$12{,}760}$

3. On June 22, 2005, Lo Nguyen borrowed some money for 120 days. What is the maturity date?

June 22 ⟶ Day 173 + 120 = 293 ⟶ **Oct. 20**

4. Buck Tanner gets a 9% $1,500 loan on December 23, 2007, to do some holiday shopping. If Buck repays the money on April 10, 2008 (a leap year), how much interest does he owe? Assume the lender uses a 365-day year.

Number of days left in first year: 365 - 357 (day number for Dec. 23) 8
Number of days in next year: Apr. 10 ⟶ 100 + 1 (for leap year) + 101
109

$I = PRT = \$1{,}500 \times 9\% \times \frac{109}{365} = \mathbf{\$40.32}$

5. You borrow $15,000 for 90 days at 9% interest. The lender uses a 365-day year. You make a payment of $3,000 on day 22 (22 days after getting the loan). Calculate your balance after the $3,000 payment is applied.

Day number	Total payment	Interest	Principal	Balance
0	—	—	—	$15,000.00
22	$3,000.00	$81.37	$2,918.63	**$12,081.37**

$I = PRT = \$15{,}000.00 \times 9\% \times \frac{22}{365} = \$81.37$
Principal = $3,000.00 - $81.37 = $2,918.63
Balance = $15,000.00 - $2,918.63 = **$12,081.37**

6. You get a 7% 90-day $3,000 loan. The lender uses a 360-day year and charges you a $100 set-up fee at the time you get the loan. What is your APR?

Principal (*P*) for APR purposes is the amount of money you have use of: $3,000 - $100 fee = $2,900.
Interest (*I*) for APR purposes is total finance charges:

$I = PRT = \$3{,}000 \times 7\% \times \frac{90}{360} =$	$ 52.50
Set-up fee	+100.00
Total finance charges	$152.50

$R = \frac{I}{PT} = \frac{\$152.50}{\$2{,}900 \times \frac{90}{365}} \approx \frac{\$152.50}{\$715.07} \approx .2133 \approx \mathbf{21.33\%}$

↖ *Even though interest is calculated using a 360-day year, an APR always uses a 365-day year*

7. You get a loan using the discount method. You sign a note, agreeing to repay the lender $30,000 in 180 days. Assuming a discount rate of 13.5%, determine the APR.

$D = MRT = \$30{,}000 \times 13.5\% \times \frac{180}{360} = \$2{,}025$

↖ *Remember, the discount method uses a 360-day year to calculate interest*

Proceeds = *M* - *D* = $30,000 - $2,025 = $27,975 (this is amount you have use of)

$R = \frac{I}{PT} = \frac{\$2{,}025}{\$27{,}975 \times \frac{180}{365}} \approx \frac{\$2{,}025}{\$13{,}795.89} \approx .1468 \approx \mathbf{14.68\%}$

8. Kyle Santini deposits $500 in a savings account. The money is left on deposit earning 6% compounded semiannually. Calculate the account balance at the end of 2 years.

	Interest	Balance
Beginning	—	$500.00
6 months	$500.00 × 3% = $15.00	$515.00
12 months	$515.00 × 3% = $15.45	$530.45
18 months	$530.45 × 3% = $15.91	$546.36
24 months	$546.36 × 3% = $16.39	**$562.75**

Quotable Quip

Compound interest is the eighth wonder of the world.
– Albert Einstein

Prices in the Good Old Days

People often talk about how cheap things were in the good old days. For example, in 1964 a loaf of bread would set you back 21 cents, a pound of butter 75 cents, a gallon of milk $1.06, a gallon of gasoline 30 cents, a new Ford auto $3,495, and an average home $13,050.

But, ignoring inflation makes these comparisons meaningless. Inflation from 1964 to 2004 has averaged 4.6% per year. Based on this rate, compounded annually here is what these same products would cost today:

Loaf of bread	$1.27
Pound of butter	$4.53
Gallon of milk	$6.41
Gallon of gasoline	$1.81
Average car	$21,121
Average home	$78,864

"Linda, my interest in you is compounding daily!"

Where Did Calendars Come From?

Our current calendar goes back to Julius Caesar, in 46 B.C. To get the calendar year to equal the astronomical year, Caesar ordered the calendar to have 365 days. He wanted the calendar to have 12 months, so days were added to various months to bring the total to 365. Because seasons do not repeat every 365 days, but actually 365 days, 5 hours, 48 minutes, and 46 seconds, the calendar ended about one-quarter of a day early. After every fourth year, it would have been a full day in error.

To make up for this difference, every fourth year had an extra day added to February. It was decided that any year evenly divisible by 4 was a leap year, which made the average length of the calendar exactly 365.25 days. However, that correction made the year 11 minutes, 14 seconds too long; after 128 years, the calendar was ending a full day later than the astronomical year.

In 1582, Pope Gregory XII stepped in and ordered yet another correction to the calendar. This change resulted in the Gregorian calendar, which we use today. The change stated that century years not evenly divisible by 400 would not be leap years. Thus, 1900 was not a leap year, but 2000 was. This made the average length of the calendar 365.244 days and reduced the calendar error to only 1 day in 3,322 years.

To obtain still greater accuracy, another change was made. Years evenly divisible by 4,000 are non-leap years. With this modification the Gregorian calendar's accuracy improves even more—our calendar will lose only a single day over a time span of 20,000 years.

Suggestions or Comments

If you have any comments or suggestions about the text, please call the publisher or author:

Publisher: 1-800-844-1856
Author: 1-800-6WEBBER

Quotable Quip

A banker is a fellow who lends you his umbrella when the sun is shining and wants it back the minute is starts to rain.
– Mark Twain

No-Interest Plans

Have you seen advertisements for products, like a $2,000 TV, with no interest for 12 months? Here's a tip. If you have the $2,000, don't pay it. Instead, buy the TV with the 12-month, no-interest plan and keep your $2,000 in an interest-bearing account. Pay the store just before the no-interest period expires. Assuming that your $2,000 earns 5% ($100), the TV will, in effect, cost you only $1,900.

Brainteaser

Suppose someone agrees to pay you 1¢ today, 2¢ tomorrow, 4¢ the third day, and keeps doubling the amount each day. How much would you receive on day 40?

Answer: $5,497,558,138.88