Chapter - 2

Inverse trigonometry functions:

previous year Questions:

1. Find the value of (CBSE : 2024)

$$\sin^{-1}(-1/2) + \cos^{-1}(-\sqrt{3}/2) + \cot^{-1}(\tan \tfrac{4\pi}{3}) \rightarrow (1)$$

Soln

$$\sin^{-1}(-1/2) = -\sin^{-1}(1/2)$$

$\because \sin^{-1}(-x) = -\sin^{-1}(x)$

$$= -\pi/6.$$

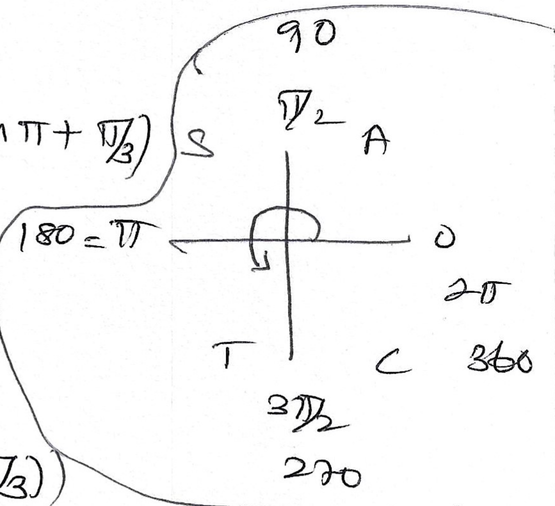
$$\cos^{-1}(-\sqrt{3}/2) = \pi - \cos^{-1}(\sqrt{3}/2)$$

$$= \pi - \pi/6.$$

$$= 5\pi/6.$$

$$\cot^{-1}(\tan 4\pi/3) = \cot^{-1}(\tan \pi + \pi/3)$$

$$\frac{4 \times \cancel{180}^{60}}{\cancel{3}} = 240$$



$$= \cot^{-1}(\tan \pi/3)$$

$$= \cot^{-1}(\sqrt{3})$$

$$= \pi/6.$$

$$(1) \Rightarrow -\pi/6 + 5\pi/6 + \pi/6 = -\pi/6 + \frac{6\pi}{6} = \frac{5\pi}{6}$$

2. Find the domain of the function (CBSE-24)
$f(x) = \sin^{-1}(x^2 - 4)$ and hence find its range.

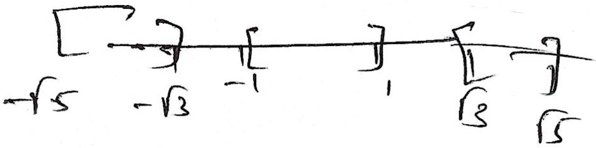
Soln W.K.T if $y = \sin^{-1}x$ then.

Domain : $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

Given

$-1 \leq x^2 - 4 \leq 1$



$-1 + 4 \leq x^2 \leq 1 + 4$

$3 \leq x^2 \leq 5$

$\pm\sqrt{3} \leq |x| \leq \pm\sqrt{5}$

$-\sqrt{5} \leq x \leq -\sqrt{3}$ & $\sqrt{3} \leq x \leq \sqrt{5}$

Domain:
i.e) $\boxed{[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]}$

Range: $[-\pi/2, \pi/2]$.

~~Set-3~~ - 2024 → Set

3) Find the value of $\sin^{-1}(-1/2) + \cos^{-1}(-\sqrt{3}/2) + \cot^{-1}(\tan 4\pi/3)$

Soln Ans: $(-\pi/6 + \frac{5\pi}{6} + \pi/6] = \boxed{\frac{5\pi}{6}}$ ans:

$\sin^{-1}(-1/2) = -\sin^{-1}(1/2)$

$= -\pi/6 \rightarrow (i)$

$\cos^{-1}(-\sqrt{3}/2) = \pi - \pi/6 = 5\pi/6 \rightarrow (ii)$

$\cot^{-1}(\tan(4\frac{\pi}{3})) = \cot^{-1}(\tan(\pi + \pi/3))$

$= \cot^{-1}(\tan \pi/3)$

$= \cot^{-1}(\sqrt{3}) = \pi/6. \rightarrow (iii)$

4) Find the domain of $f(x) = \cos^{-1}(1-x^2)$. Also find its range.

Soln

Domain $[-1, 1]$

Range : $[0, \pi]$

$-1 \le \theta \le 1$

$-1 \le 1-x^2 \le 1$

$-1-1 \le -x^2 \le 1-1$

$-2 \le -x^2 \le 0$

$\div(-)$ $2 \geq x^2 \geq 0.$

$0 \le x^2 \le 2$

$0 \le x \le \pm 2$

$0 \le x \le -2$ & $0 \le x \le +2$

$-2 \geq x \geq 0.$; $0 \le x \le 2$

$0 \le x \le -2$ $x \in [0, 2]$.

(4)' Find the domain of $f(x) = \cot x + \cot^{-1} x$

Soln W.K.T

Domain of $\cot^{-1} x$: R

" " $\cot x$: R.

Domain of $f(x) = R - \{n\pi : n \in Z\}$

5) Find the domain of the function $f(x) = \sin^{-1}(2x-3)$ (4)

Soln

W.K.T if $y = \sin^{-1}(x)$ then.

Domain : $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

To find: Domain of $\sin^{-1}(2x-3)$.

$-1 \le 2x-3 \le 1$

$-1+3 \le 2x \le 1+3$

$2 \le 2x \le 4$

$\div 2 \Rightarrow \boxed{1 \le x \le 2.}$

$\Rightarrow \quad x \in [1, 2].$

6) Find the domain of $f(x) = \sin^{-1}(-x^2)$

Soln

W.K.T Domain : $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

$-1 \le -x^2 \le 1$

x Pling by (-) $\Rightarrow \quad 1 \ge x^2 \ge -1$

$\Rightarrow \quad -1 \le x^2 \le 1$

$\Rightarrow$ −1 0 1

Case (i)

$x^2 = -1$

$x = \pm i$

not possible

Case (ii)

$x^2 \le 1 \Rightarrow x = \pm 1$

$x^2 - 1 \le 0$

$(x-1)(x+1) \le 0$

$\Rightarrow \boxed{x = 1 ; x = -1}$

Domain

$\therefore \boxed{-1 \le x \le 1}$

7) Find the domain of $f(x) = \sin^{-1}x + \cos x$.

Soln

Domain of $\sin^{-1}x = [-1, 1]$

Domain of $\cos x = R$.

$\therefore$ Domain of $f(x) = \sin^{-1}x + \cos x$

is $[-1, 1] \cap R = [-1, 1]$.

8) Find the domain of the function

$f(x) = \sin^{-1}\sqrt{x-1}$.

Soln

W.K.T

Domain : $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

$-1 \leq \sqrt{x-1} \leq 1$

$0 \leq \sqrt{x-1} \leq 1$

$0 \leq x-1 \leq 1$

$1 \leq x \leq 2$

Domain $\Rightarrow \boxed{x \in [1, 2]}$

9) Find the domain of the following.

(i) $f(x) = \sin^{-1} x^2$

Soln

W.K.T Domain: $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

$-1 \leq x^2 \leq 1$.

$x^2 \leq 1$

$x^2 \leq \pm 1$

$x = -1$; $x = 1$

Domain: $\boxed{-1 \leq x \leq 1}$ $x \in [-1, 1]$.

10) $f(x) = \sin^{-1} x + \sin x$.

Domain of $\sin^{-1}$: $[-1, 1]$

" " $\sin x$: R.

domain of $f(x) = [-1, 1] \cap R = [-1, 1]$

$x \in [-1, 1]$

$\boxed{-1 \leq x \leq 1}$

11) $f(x) = \cos^{-1} x + \cos x$

Domain of $\cos^{-1} x$ $= [-1, 1]$

" " $\cos x = R$

domain of $f(x) = [-1, 1] \cap R$
$= [-1, 1]$.

(11) $f(x) = \sin^{-1}\sqrt{x^2-1}$ (7)

Soln

W.K.T

Domain : $[-1, 1]$

Range : $[-\pi/2, \pi/2]$

$-1 \le \sqrt{x^2-1} \le 1$

Case i)

$-1 \le \sqrt{x^2-1}$

$\sqrt{x^2-1} \ge -1$

$x^2-1 \le 1$

$x^2 \le 2$

$\boxed{x \le \pm\sqrt{2}}$

case (ii)

$\sqrt{x^2-1} \le 1$

$x^2-1 \le 1$

$x^2 \le 1+1$

$x^2 \le 2$

$\boxed{x \le \pm\sqrt{2}}$

$\boxed{\therefore \text{Domain} : [-\sqrt{2}, -1] \cup [1, \sqrt{2}]}$

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(12) $f(x) = \sin^{-1}x + \sin^{-1}2x$

Soln

Domain of $\sin^{-1}x$: $[-1, 1]$

Domain of $\sin^{-1}2x$

$-1 \le 2x \le 1$

$\div 2, \quad -1/2 \le x \le 1/2.$

$-1 \quad -1/2 \quad 0 \quad 1/2 \quad 1$

$x \in [-1/2, 1/2]$

domain of $f(x) = [-1/2, 1/2] \cap [-1, 1]$

$x \in [-1/2, 1/2]$

(12) Find the domain of $\cos^{-1}[2x-1]$. (8)

Soln

W.K.T,

Domain of $\cos^{-1}x$: $[-1, 1]$

$\therefore -1 \le 2x-1 \le 1$

$-1+1 \le 2x \le 1+1$

$0 \le 2x \le 2$

$0 \le x \le 1$

domain of $f(x)$ $\therefore$ $x \in [0,1]$

(13) If $x, y, z \in [-1,1]$ such that

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$. Then find the value of

(i) $xy+yz+zx$.

(ii) $x(y+z)+y(z+x)+z(x+y)$

Soln

W.K.T

$x, y, z \in [-1,1]$.

$-1 \le x \le 1$; $-1 \le y \le 1$; $-1 \le z \le 1$

$0 \le \cos^{-1}x \le \pi$; $0 \le \cos^{-1}y \le \pi$; $0 \le \cos^{-1}z \le \pi$

$\therefore$ $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi+\pi+\pi$

$\therefore \cos^{-1}x = \pi$; $\cos^{-1}y = \pi$; $\cos^{-1}z = \pi$

$\Rightarrow \cos^{-1} x = \pi$; $\cos^{-1} y = \pi$; $\cos^{-1} z = \pi$

$\Rightarrow x = \cos\pi$; $y = \cos\pi$; $z = \cos\pi$

$\boxed{x=-1}$ $\boxed{y=-1}$ $\boxed{z=-1}$

$\therefore$ (i) $xy+yz+zx = (-1)(-1)+(-1)(-1)+(-1)(-1)$

$=3.$

(ii) $x(y+z)+y(z+x)+z(x+y)$

$=(-1)(-2)+(-1)(-2)+(-1)(-2)$

$= 2+2+2 = 6.$

(14) Find the domain of definition of $f(x)$

$f(x) = \cos^{-1}(x^2-4).$

Soln W.K.T

Domain: $[-1, 1]$

$-1 \le x^2-4 \le 1$

$-1+4 \le x^2 \le 1+4$

$3 \le x^2 \le 5$

$x^2 \geq 3$; $x^2 \le 5$

$\boxed{x \geq \pm\sqrt{3}}$ $\boxed{x \le \pm\sqrt{5}}$

$x \in [-\sqrt{5}, -\sqrt{3}]$; $x \in [\sqrt{3}, \sqrt{5}]$

$\boxed{\text{domain} : [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]}$

(17) Find the domain of (11)

(i) $\sec^{-1}(3x-1)$

(ii) $\sec^{-1}x - \tan^{-1}x$.

<u>Soln</u>

W.K.T

Domain of $\sec^{-1}x$: $R-(-1,1)$

$x \in [-\infty,-1] \cup [1,\infty)$

$3x-1 \geq 1$ or $3x-1 \leq -1$

$3x \geq 1+1$ or $3x \leq -1+1$

$3x \geq 2$ $\quad$ $3x \leq 0$

$\boxed{x \geq 2/3}$ $\quad$ $\boxed{x \leq 0}$

$\Rightarrow \boxed{x \in [2/3, \infty]}$ $\quad$ $\boxed{x \in (-\infty, 0]}$.

$\therefore$ $\boxed{\text{Domain: } (-\infty,0] \cup [2/3, \infty)}$.

(ii)

Domain of $\sec^{-1}x$

$x \in [-\infty,-1] \cup [1,\infty)$

Domain of $\tan^{-1}x$: R.

$\therefore$ Domain of $f(x) = \sec^{-1}x - \tan^{-1}x$

$x \in (-\infty,-1] \cup [1,\infty) \cap R$

$\boxed{\therefore x \in (-\infty,-1] \cup [1,\infty)}$.

15 Find the domain of $f(x) = 2\cos^{-1} 2x + \sin^{-1}$ (10)

Soln W.K.T

Domain of $\sin^{-1} x$: $[-1, 1]$

" " $\cos^{-1} x$: $[-1, 1]$

$\boxed{-1 \le x \le 1}$; $-1 \le 2x \le 1$

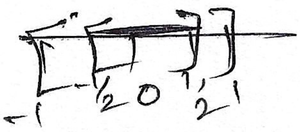
$-\frac{1}{2} \le x \le \frac{1}{2}$.

$x \in [-1, 1]$

$x \in [-\frac{1}{2}, \frac{1}{2}]$

Domain of $f(x) = [-\frac{1}{2}, \frac{1}{2}] \cap [-1, 1]$

$= [-\frac{1}{2}, \frac{1}{2}]$



16 Find the domain of $\sec^{-1}(2x+1)$

Soln W.K.T

domain : $R - (-1, 1)$

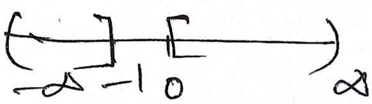
$x \in (-\infty, -1] \cup [1, \infty)$.

$\therefore$ $2x+1 \ge 1$ (or) $2x+1 \le -1$

$2x \ge 1-1$ or $2x \le -1-1$

$2x \ge 0$

$2x \le -2$



$\boxed{x \ge 0}$

$\boxed{x \le -1}$

domain $\therefore$ $\boxed{x \in (-\infty, -1] \cup [0, \infty)}$.

(17) which is greater, $\tan 1$ or $\tan^{-1} 1$? (12)

Soln

W.K.T

$\tan^{-1} 1 = \pi/4$ and $1 > \pi/4$

$1 > \pi/4$

$\tan 1 > \tan \pi/4$ [$\because \tan\theta$ is an increasing function]

$\tan 1 > 1$

$\tan 1 > 1 > \pi/4$ $\because [1 > \pi/4]$

$\Rightarrow \tan 1 > \pi/4$ $\boxed{\tan^{-1} 1 = \pi/4}$

$\tan 1 > \tan^{-1} 1.$

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(18) Find the minimum value of n for which

$\tan^{-1} \frac{n}{\pi} > \frac{\pi}{4}$, $n \in N$.

Soln We have

$\tan^{-1} \frac{n}{\pi} > \frac{\pi}{4}$ [$\because \frac{\pi}{4} = \tan^{-1} 1$]

$\tan(\tan^{-1} \frac{n}{\pi}) > \tan(\tan^{-1} 1)$. [$\because \tan\theta$ is an increasing fun]

$\frac{n}{\pi} > 1$

$n > \pi \cong 3.14$

$n = 4, 5, 6 \ldots$

Hence the minimum value of n is $\underline{\underline{4}}$

19) Find the principal value of

(i) $\sin^{-1}(-1/2)$ → $\boxed{\text{CBSE : 2011.}}$

Soln

$\theta \in [-\pi/2, \pi/2]$.

$\sin^{-1}(-1/2) = y$

$-1/2 = \sin y$.

$\sin(-\pi/6) = \sin y$

$\boxed{y = -\pi/6.} \in [-\pi/2, \pi/2]$.

(ii) $\sin^{-1}[\cos(\sin^{-1}(\sqrt{3}/2))]$.

$= \sin^{-1}[\cos[\pi/3]]$

$= \sin^{-1}[1/2]$

$= \pi/6$.

(iii) $\cos^{-1}(-1/2)$. ⇒ Even

$\theta \in [0, \pi]$ ⇒ $\cos\theta = -1/2$.

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (*).

(iv) HW $\cos^{-1}(-\sqrt{3}/2)$

Ans: $\frac{5\pi}{6}$

(v) $\cos^{-1}(-1/\sqrt{2})$

Ans; $\frac{3\pi}{4}$.

vi) $\cos^{-1}\frac{1}{2} + 2\sin^{-1}1/2$ CBSE : 2012

$= \frac{\pi}{3} + \cancel{2}\cdot\frac{\pi}{\cancel{6}3}$

$= \pi/3 + \pi/3 = \frac{2\pi}{3}$.

vii) $\cos^{-1}(\frac{1}{2}) - 2\sin^{-1}(-1/2)$ CBSE : 2012

$= \frac{\pi}{3} + \cancel{2}\cdot\frac{\pi}{\cancel{6}3} = \frac{2\pi}{3}$

(viii)

$\tan^{-1}(\sin(-\pi/2))$ (Exemplar)

Soln W.K.T $\sin(-\pi/2) = -\sin\pi/2 = -1$

$\tan^{-1}(-1) = -\tan^{-1}(1)$

$= -\pi/4$.

ix) $\cot[\sin^{-1}\{\cos(\tan^{-1}1)\}]$. Exemplar

Soln W.K.T $\tan^{-1}1 = \pi/4$

$= \cot[\sin^{-1}[\cos(\pi/4)]]$

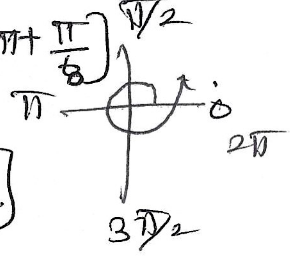
$= \cot[\sin^{-1}[\frac{1}{\sqrt{2}}]]$

$= \cot[\pi/4]$

$= 1$

19) $\tan^{-1}(\tan \frac{5\pi}{6}) + \cos^{-1}(\cos \frac{13\pi}{6})]$ $\Rightarrow$ $\boxed{\text{Exemplar}}$ 18

$= \tan^{-1}[\tan(\pi - \frac{\pi}{6}) + \cos^{-1}(\cos(2\pi + \frac{\pi}{6}))]$



$= \tan^{-1}[(-\tan\frac{\pi}{6}) + \cos^{-1}(\cos\frac{\pi}{6})]$

$= \tan^{-1}[\tan(-\pi/6)] + \pi/6$

$= -\pi/6 + \pi/6$

$= 0$

$\frac{5\pi}{6} = \frac{5 \times \cancel{180}^{30}}{\cancel{6}}$

$= 150.$

$\frac{13 \times \cancel{180}^{30}}{\cancel{6}} = 390$

$\underline{360}$

30

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20) $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\pi/2)]$.

Soln

$= -\tan^{-1}(\frac{1}{\sqrt{3}}) + \cot^{-1}(1/\sqrt{3}) + \tan^{-1}(-\sin\pi/2)$

$= -\frac{\pi}{6} + \frac{\pi}{3} + \tan^{-1}(-1)$

$= -\pi/6 + \pi/3 - \tan^{-1}(1)$

$= -\frac{\pi}{6} + \pi/3 - \pi/4$

$= \frac{-2\pi + 4\pi - 3\pi}{12}$

$= \frac{-\pi}{12}$

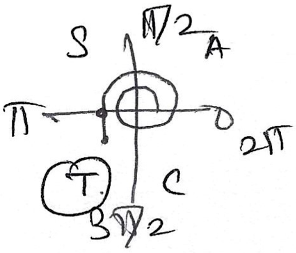
(21) Evaluate $\sin^{-1}[\sin(-600^\circ)]$ (66).

NCRT. (Exemplar)

Soln

$$\sin^{-1}[\sin(-600^\circ) = \sin^{-1}[\sin(-\cancel{600}^{10} \times \frac{\pi}{\cancel{180}_3})]$$

$$= \sin^{-1}[\sin(-\frac{10\pi}{3})].$$



$$= \sin^{-1}[-\sin(\frac{10\pi}{3})]$$

$$= \sin^{-1}[-\sin(3\pi+\frac{\pi}{3})]$$

$\therefore \boxed{\sin(3\pi+\pi/3) = -\sin\pi/3}$

$$= \sin^{-1}[-(-\sin\pi/3)]$$

$$= \sin^{-1}[+\sqrt{3}/2]$$

$$= +\sin^{-1}(\sqrt{3}/2)$$

$$= +\pi/3$$

$$= \pi/3.$$

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(22) $\cos^{-1}(\cos(-680^\circ)$

$= \cos^{-1}[\cos 680^\circ)$

even function $\therefore \cos(-x) = \cos x$

$$= \cos^{-1}[\cos(\cancel{680}^{34} \times \frac{\pi}{\cancel{180}_9}]$$

$$= \cos^{-1}[\cos\frac{34\pi}{9}]$$

$$= \cos^{-1}[\cos[4\pi - \frac{2\pi}{9}]$$

$\frac{2 \times \cancel{180}^{20}}{\cancel{9}}$

$$= \cancel{\cos^{-1}}[\cancel{\cos}\frac{2\pi}{9}]$$

$$= 2\pi/9. //$$

23)

Value based questions.

1. Find the value of $\tan^{-1}(\tan \frac{3\pi}{5})$. What value do you learn from it?

Soln

W.k.T $\tan^{-1}(\tan x) = x \quad ; \quad x \in (-\pi/2, \pi/2)$

$$\tan^{-1}(\tan \tfrac{3\pi}{5}) \neq \tfrac{3\pi}{5}, \quad \tfrac{3\pi}{5} \notin (-\pi/2, \pi/2)$$

$$\therefore \tan^{-1}(\tan \tfrac{3\pi}{5}) = \tan^{-1}[\tan(\pi - \tfrac{2\pi}{5})]$$

$$= \tan^{-1}[-\tan \tfrac{2\pi}{5}]$$

$$= -\cancel{\tan^{-1}}[\cancel{\tan} \tfrac{2\pi}{5}]$$

$$= -\tfrac{2\pi}{5}.$$

Value: All that we see or feel from senses is not true. We must verify if other mean also, before conclusions.

24) Prove that $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}(a/b)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}(a/b)\right] = \frac{2b}{a}$.

Give one real life example, which does not satisfy the property of inverse function.

Proof:

$L.H.S = \tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}(a/b)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$

put $\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = \theta$

$\Rightarrow \left(\frac{a}{b}\right) =$

$= \cos 2\theta$

$\therefore \tan(\pi/4 + \theta) + \tan\left(\frac{\pi}{4} - \theta\right)$.

$$= \frac{(1+\tan\theta)}{(1-\tan\theta)} + \frac{(1-\tan\theta)}{1+\tan\theta}$$

$$= \frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{(1-\tan^2\theta)}$$

$$= \frac{1 + \cancel{2\tan\theta} + \tan^2\theta + 1 - \cancel{2\tan\theta} + \tan^2\theta}{(1-\tan^2\theta)}$$

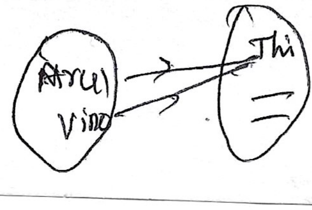
$$= \frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$$

$\because \cos 2x = \frac{1+\tan^2 x}{1-\tan^2 x}$

$= 2\cos 2\theta$

$= \frac{2}{a/b} = \frac{2b}{a} = R.H.S.$

Eg: f : Parents $\rightarrow$ children.



many-one, into

So inverse does not exist.

25) Solve for x, $\sin^{-1}(1-x) - 2\sin^{-1}x = \pi/2$. What value do you observe in real life scenario?

Soln

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \pi/2$$

$$\Rightarrow \sin^{-1}(1-x) = \pi/2 + 2\sin^{-1}x.$$

$$(1-x) = \sin\left(\pi/2 + 2\sin^{-1}x\right)$$

$$= \cos(2\sin^{-1}x).$$

$$(1-x) = 1 - 2\sin^2(\sin^{-1}x).$$

$$1-x = 1-2x^2 \qquad (\because \cos 2\theta = 1-2\sin^2\theta)$$

$$\Rightarrow x = 0, \tfrac{1}{2}.$$

If $x=0$, then.

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \sin^{-1}1 - 2\sin^{-1}0$$

$$= \pi/2$$

If $x = \tfrac{1}{2}$, then.

$$\sin^{-1}(1-\tfrac{1}{2}) - 2\sin^{-1}\tfrac{1}{2} = \sin^{-1}\tfrac{1}{2} - 2\sin^{-1}\tfrac{1}{2}$$

$$= \pi/6 - \pi/3.$$

$$= -\pi/6 \quad \text{where } -\pi/6 \neq \pi/2.$$

Hence, the required soln is $x=0$.

Value: In real life choose the correct path for reaching the goal.