**Fundamental Units**

Those physical quantities which are independent to each other are called fundamental quantities and their units are called fundamental units.

| S.No. | Fundamental Quantities | Fundamental Units | Symbol |
|---|---|---|---|
| 1. | Length | metre | m |
| 2. | Mass | kilogram | kg |
| 3. | Time | second | S |
| 4. | Temperature | kelvin | kg |
| 5 | Electric current | ampere | A |
| 6 | Luminous intensity | candela | cd |
| 7 | Amount of substance | mole | mol |

**Supplementary Fundamental Units**

Radian and steradian are two supplementary fundamental units. It measures plane angle and solid angle respectively.

| S.No. | Supplementary Fundamental Quantities | Supplementary Unit | Symbol |
|---|---|---|---|
| 1 | Plane angle | radian | rad |
| 2 | Solid angle | steradian | Sr |

**Derived Units**

Those physical quantities which are derived from fundamental quantities are called derived quantities and their units are called derived units.
e.g., velocity, acceleration, force, work etc.

**Definitions of Fundamental Units**

The seven fundamental units of SI have been defined as under.

1. **1 kilogram** A cylindrical prototype mass made of platinum and iridium alloys of height 39 mm and diameter 39 mm. It is mass of $5.0188 \times 10^{25}$ atoms of carbon-12.
2. **1 metre** 1 metre is the distance that contains 1650763.73 wavelength of orange-red light of Kr-86.
3. **1 second** 1 second is the time in which cesium atom vibrates 9192631770 times in an atomic clock.
4. **1 kelvin** 1 kelvin is the (1/273.16) part of the thermodynamics temperature of the triple point of water.
5. **1 candela** 1 candela is (1/60) luminous intensity of an ideal source by an area of cm' when source is at melting point of platinum (1760°C).
6. **1 ampere** 1 ampere is the electric current which it maintained in two straight parallel conductor of infinite length and of negligible cross-section area placed one metre apart in vacuum will produce between them a force $2 \times 10^{-7}$ N per metre length.
7. **1 mole** 1 mole is the amount of substance of a system which contains a many elementary entities (may be atoms, molecules, ions, electrons or group of particles, as this and atoms in 0.012 kg of carbon isotope ${}_6C^{12}$.

**Some Practical Units**

1. 1 fermi $=10^{-15}$ m
2. 1 X-ray unit = $10^{-13}$ m
3. 1 astronomical unit = $1.49 \times 10^{11}$ m (average distance between sun and earth)
4. 1 light year = $9.46 \times 10^{15}$ m
5. 1 parsec = $3.08 \times 10^{16}$ m = 3.26 light year

## Dimensional Formula of Some Physical Quantities

| S.No. | Physical Quantity | Dimensional Formula | MKS Unit |
|---|---|---|---|
| 1 | Area | $[L^2]$ | metre$^2$ |
| 2 | Volume | $[L^3]$ | metre$^3$ |
| 3 | Velocity | $[LT^{-1}]$ | $ms^{-1}$ |
| 4 | Acceleration | $[LT^{-2}]$ | $ms^{-2}$ |
| 5 | Force | $[MLT^{-2}]$ | newton (N) |
| 6 | Work or energy | $[ML^2T^{-2}]$ | joule (J) |
| 7 | Power | $[ML^2T^{-3}]$ | J $s^{-1}$ or watt |
| 8 | Pressure or stress | $[ML^{-1}T^{-2}]$ | $Nm^{-2}$ |
| 9 | Linear momentum or Impulse | $[MLT^{-1}]$ | kg $ms^{-1}$ |
| 10 | Density | $[ML^{-3}]$ | kg $m^{-3}$ |
| 11 | Strain | Dimensionless | Unitless |
| 12 | Modulus of elasticity | $[ML^{-1}T^{-2}]$ | $Nm^{-2}$ |
| 13 | Surface tension | $[MT^{-2}]$ | $Nm^{-1}$ |
| 14 | Velocity gradient | $T^{-1}$ | second$^{-1}$ |
| 15 | Coefficient of velocity | $[ML^{-1}T^{-1}]$ | kg $m^{-1}s^{-1}$ |
| 16 | Gravitational constant | $[M^{-1}L^3T^{-2}]$ | $Nm^2/kg^2$ |
| 17 | Moment of inertia | $[ML^2]$ | kg $m^2$ |
| 18 | Angular velocity | $[T^{-1}]$ | rad/s |
| 19 | Angular acceleration | $[T^{-2}]$ | rad/$S^2$ |
| 20 | Angular momentum | $[ML^2T^{-1}]$ | kg $m^2S^{-1}$ |
| 21 | Specific heat | $L^2T^{-2}\theta^{-1}$ | kcal $kg^{-1}K^{-1}$ |
| 22 | Latent heat | $[L^2T^{-2}]$ | kcal/kg |
| 23 | Planck's constant | $ML^2T^{-1}$ | $J^{-s}$ |

**Applications of Dimensions**

1. To check the accuracy of physical equations.
2. To change a physical quantity from one system of units to another system of units.
3. To obtain a relation between different physical quantities.

**Significant Figures**

In the measured value of a physical quantity, the number of digits about the correctness of which we are sure plus the next doubtful digit, are called the significant figures.

**Rules for Finding Significant Figures**

1. All non-zeros digits are significant figures, e.g., 4362 m has 4 significant figures.
2. All zeros occuring between non-zero digits are significant figures, e.g., 1005 has 4 significant figures.
3. All zeros to the right of the last non-zero digit are not significant, e.g., 6250 has only 3 significant figures.
4. In a digit less than one, all zeros to the right of the decimal point and to the left of a non-zero digit are not significant, e.g., 0.00325 has only 3 significant figures.
5. All zeros to the right of a non-zero digit in the decimal part are significant, e.g., 1.4750 has 5 significant figures.

**1. Absolute Error**

The difference between the true value and the measured value of a quantity is called absolute error.

If $a_1$ , $a_2$, $a_3$ ,..., $a_n$ are the measured values of any quantity a in an experiment performed n times, then the arithmetic mean of these values is called the true value ($a_m$) of the quantity.

$$a_m = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

The absolute error in measured values is given by

$\Delta a_1 = a_m - a_1$
$\Delta a_2 = a_m - a_1$

.............

$\Delta a_m = \Delta a_m - \Delta a_n$

2. Mean Absolute Error

The arithmetic mean of the magnitude of absolute errors in all the measurement is called mean absolute error.

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

3. Relative Error The ratio of mean absolute error to the true value is called relative

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{True value}} = \frac{\overline{\Delta a}}{a_m}$$

4. Percentage Error The relative error expressed in percentage is called percentage error.

$$\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100\%$$

**Uses of dimensional analysis**

The method of dimensional analysis is used to

(i) Convert a physical quantity from one system of units to another.

(ii) Check the dimensional correctness of a given equation.

(iii) Establish a relationship between different physical quantities in an equation

Given the value of G in cgs system is $6.67 \times 10^{-8} dyne\, cm^2\, g^{-2}$. Calculate its value in SI units.

| In cgs system | In SI system |
| --- | --- |
| $G_{cgs} = 6.67 \times 10^{-8}$ | $G = ?$ |
| $M_1 = 1g$ | $M_2 = 1\ kg$ |
| $L_1 = 1\ cm$ | $L_2 = 1m$ |
| $T_1 = 1s$ | $T_2 = 1s$ |

Given the value of G in cgs system is $6.67 \times 10^{-8} dyne\, cm^2\, g^{-2}$. Calculate its value in SI units.

| In cgs system | In SI system |
| --- | --- |
| $G_{cgs} = 6.67 \times 10^{-8}$ | $G = ?$ |
| $M_1 = 1g$ | $M_2 = 1\ kg$ |
| $L_1 = 1\ cm$ | $L_2 = 1m$ |
| $T_1 = 1s$ | $T_2 = 1s$ |

The dimensional formula for gravitational constant is $\left[M^{-1}L^{3}T^{-2}\right]$.

In *cgs* system, dimensional formula for G is $\left[M_1^x\ L_1^y\ T_1^z\right]$

In SI system, dimensional formula for G is $\left[M_2^x\ L_2^y\ T_2^z\right]$

Here $x = -1$, $y = 3$, $z = -2$

$$\therefore \quad G\left[M_2{}^x L_2{}^y T_2{}^z\right] = G_{cgs}\left[M_1{}^x L_1{}^y T_1{}^z\right]$$

$$\text{or} \quad G = G_{cgs}\left[\frac{M_1}{M_2}\right]^x \left[\frac{L_1}{L_2}\right]^y \left[\frac{T_1}{T_2}\right]^z$$

$$= 6.67 \times 10^{-8}\left[\frac{1\,g}{1\,kg}\right]^{-1}\left[\frac{1\,cm}{1\,m}\right]^{3}\left[\frac{1\,s}{1\,s}\right]^{-2}$$

$$= 6.67 \times 10^{-8}\left[\frac{1\,g}{1000\,g}\right]^{-1}\left[\frac{1\,cm}{100\,cm}\right]^{3}[1]^{-2}$$

$$= 6.67 \times 10^{-11}$$

$\therefore$ In SI units,

$$G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$$

### (ii) *To check the dimensional correctness of a given equation*

Let us take the equation of motion

$$s = ut + (½)at^2$$

Applying dimensions on both sides,

$$[L] = [LT^{-1}]\ [T] + [LT^{-2}]\ [T^2]$$

(½ is a constant having no dimension)

$$[L] = [L] + [L]$$

As the dimensions on both sides are the same, the equation is dimensionally correct.

### (iii) *To establish a relationship between the physical quantities in an equation*

Let us find an expression for the time period $T$ of a simple pendulum. The time period $T$ may depend upon (i) mass $m$ of the bob (ii) length $l$ of the pendulum and (iii) acceleration due to gravity $g$ at the place where the pendulum is suspended.

(i.e) $T \alpha m^x l^y g^z$

or $T = k\ m^x l^y g^z$ ...(1)

where $k$ is a dimensionless constant of propotionality. Rewriting equation (1) with dimensions,

$$[T^1] = [M^x]\ [L^y]\ [LT^{-2}]^z$$

$$[T^1] = [M^x L^{y+z} T^{-2z}]$$

Comparing the powers of M, L and T on both sides

$$x = 0,\ y + z = 0 \text{ and } -2z = 1$$

Solving for $x$, $y$ and $z$, $x = 0$, $y = ½$ and $z = -½$

From equation (1), $T = k\ m^0 l^{½} g^{-½}$

$$T = k\left[\frac{l}{g}\right]^{1/2} = k\sqrt{\frac{l}{g}}$$

Experimentally the value of $k$ is determined to be $2\pi$.

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}$$

### *1.9.2 Limitations of Dimensional Analysis*

(i) The value of dimensionless constants cannot be determined by this method.

(ii) This method cannot be applied to equations involving exponential and trigonometric functions.

(iii) It cannot be applied to an equation involving more than three physical quantities.

(iv) It can check only whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not. For example applying this technique $s = ut + \frac{1}{4}at^2$ is dimensionally correct whereas the correct relation is $s = ut + \frac{1}{2}at^2$.