CIRCLES

LONG ANSWER QUESTIONS (7 Marks)

***1. Find equation and centre of the circle passing through the points $(3,4'),(3,2)$ & $(1,4)$

(May-12,16 A.P&T.S)

***2. Find the value of 'c' if the points $(2,0),(0,l),(4,5)$ and (O,c) are concyclic. '

(Mar-07,15-T.S,May-07)

***3. Find the equation of a circle which passes through $(2,-3)$ and $(-4,5)$ and having the centre on 4x+3y+1=0

(May-05) (May-06)

***4. Find the equation of a circle which passes through the points $(4, 1), (6, 5)$ and having centre on

4x+3y –24 =0. (Mar-08, 12,16 A.P)

***5. Find the equation of the circle whose center lies on X-axis and passing through the points

$(-2, 3)$ and $(4, 5)$ (Mar-10, 15 - A.P, May-09)

***6. Show that the circles $x^2 + y^2 -4x -6y -12 = 0$ and $x^2 +y^2 + 6x +18y + 26 = O$ touch each other also find the point of contact and common tangent at this point of contact.

(Mar-02,05,08)(Mar-13)

***7. Show that the circles $x^2+y^2-6x-2y+1= 0$; $x^2+y^2 +2x-8y+13=0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.

(May-06,16A.P,Mar-09,10,11,16A.P)

***8. Find the equations of the pair of direct common tangents to the circles

$x^2 +y^2 +22x -4y -100 = 0$ and $x^2 +y^2 –22x +4y +100 = 0$. (Mar-07, 15-T.S)

***9. Find the equations of transverse common tangents of the circles.
$x^2 + y^2 - 4x - 10y + 28 = 0$; $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is $S_1^2 = S.S_{11}$ (Mar-06,June-05)

***10. Find the equations of the circles with radius $\sqrt{13}$ units and touching 2x -3y + 1= 0 at (1, 1).

***11. Prove that the equation to the pair of tangents drawn from the point (x_1, y_1) to the circle

$S \equiv x^2 + y^2 + 2gx + 2fx + c = 0$ is $S_1^2 = S.S_{11}$ (Mar-03

**12. Show that the four points $(-6,0), (-2,2), (-2,-8)$ and $(1, 1)$ are concyclic. (Mar-05,06)

**13. Find the equations of the circles which touches the x-axis at a distance of 3 units from the origin

and making an intercept of length 6 units on the y-axis.

**14. If the polar of points on the circle $x^2 +y^2 = a^2$ with respect to $x^2 +y^2 = b^2$ touches the circle

$x^2 +y^2 = c^2$ then show that a, b, c are in G.P.

*15.	Show that the points $(1,2), (3,-4), (5,-6)$ and $(19,8)$ are concyclic and find the equation of the circle on which they lie. (May-15 T.S)

*16.	Show that the points $(9, 1), (7,9), (-2,12), (6,10)$ are concyclic and find the equation of the circle on which they lie. (May-08)

*17.	Find the equation of the circle passing through the points $(5, 7), (8,1), (1,3)$ (June-10)

*18.	Show that four common tangents can be drawn for the circles given by

$x^2 + y^2 - 14x + 6y + 33 = 0$, $x^2 + y^2 + 30x - 2y + 1 = 0$ and find the internal and external center of similitudes.

*19.	Find the equation of the circle circumscribing the triangle formed by the lines $2x+y=4, x+y=6$ and $x+2y=5$

*20	If θ_1, θ_2 are the angles of inclination of tangents through a point P to the circle $x^2 + y^2 = a^2$ then find the locus of P when $\cot\theta_1 + \cot\theta_2 = k$

*21.	Find the equation of the circle passing through the three points $(1,2), (3,-4), (5,-6)$ (Mar-16 TS)

*22.	Find the pair of tangents drawn from $(1,3)$ to the circle $x^2 + y^2 - 2x + 4y - 11 = 0$ and also find the angle between them. (Mar- 2016 TS)

SHORT ANSWER QUESTIONS (4 Marks)

***1.	Find the length of the chord intercepted by the circle $x^2 + y^2 - x + 3y - 22 = 0$ on the line

$y = x - 3$ (Mar-13, May-11,16 A.P)

***2.	Find the length of the chord intercepted by the circle $x^2 + y^2 - 8x - 2y - 8 = 0$ on the line

$x + y + 1 = 0$ (Mar-16 T.)

***3.	Show that the tangent at $(-1, 2)$ of the circle $x^2 + y^2 - 4x - 8y + 7 = 0$ touches the circle

$x^2 + y^2 + 4x + 6y = 0$ and also find its point of tangency. (June-10)

***4.	Find equations of tangents to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ which are parallel to

$x + y - 8 = 0$.

***5.	If a point P is moving such that the lengths of the tangents drawn from P to the circles

$x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ are in the ratio 2:3 then find the equation of the locus of P.
(Mar-09)

***6.	If the chord of contact of a point 'p' with respect to the circle $x^2 + y^2 = a^2$ cut the circle at A and B such that $\lfloor AOB = 90^0$ then show that 'p' lies on the circle $x^2 + y^2 = 2a^2$

***7.	Find the angle between the tangents drawn from $(3,2)$ to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$
(Mar-12)

***8.	Find the locus of mid points of the chords of contact $x^2 + y^2 = a^2$ from the points lying on the line

$lx + my + n = 0$ (Mar-2002) (June-03)

9. If the abscissae of points A,B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and ordinates of A, B are roots of $y^2 + 2py - q^2 = 0$, then find the equation of a circle for which $\overline{AB}$ is a diameter. (Mar-14)

10. Find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at $(5, 5)$ with radius 5 units. (May-16 T.S)

11. Find the inverse point of $(-2, 3)$ with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

12. Show that the lines $2x + 3y + 11 = 0$ and $2x - 2y - 1 = 0$ are conjugate with respect to the circle $x^2 + y^2 + 4x + 6y + 12 = 0$

13. Show that the poles of the tangent of the circle $x^2 + y^2 = a^2$ w.r.to the circle $(x+a)^2 + y^2 = 2a^2$ lies on the curve $y^2 + 4ax = 0$

14. Find the pair of tangents drawn from P$(3, 2)$ to the circle $x^2 + y^2 - 6x + 4y - 2 = 0$ (Apr-01)

15. Show that $x + y + 1 = 0$ touches the circle $x^2 + y^2 - 3x + 7y + 14 == 0$ and find the point of contact. (May-09)

16. Find the equation of the cirlce with centre $(-2, 3)$ cutting a chord length 2 units on $3x + 4y + 4 = 0$. (Mar 2011)

17. Find the equation of the circle passing through $(O, 0)$ and making intercepts 4, 3 on X-axis and Y-axis respectively

18. Find the area of the triangle formed with the coordinate axes and the tangent drawn at the point $P(x_1, y_1)$ on the circle $x^2 + y^2 = a^2$

19. If P (x_1, y_1) is the mid point of a chord AB (other than the diameter) of the circle $x^2 + y^2 + 2gx + gfy + c = 0$ then the equation of the chord AB is $S_1 = S_{11}$.

20. Equation of the chord joining $P(\theta_1), Q(\theta_2)$ on the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ is $(x+g)\cos\left(\dfrac{\theta_1 + \theta_2}{2}\right) + (y+f)\sin\left(\dfrac{\theta_1 + \theta_2}{2}\right) = r\cos\left(\dfrac{\theta_1 - \theta_2}{2}\right)$ where r is radius of a circle.

21. Prove that the tangent at $(3, -2)$ of the circle $x^2 + y^2 = 13$ touches the circle $x^2 + y^2 + 2x - 10y - 26 = 0$ and find its point of contact.

22. Find the value of k, if $kx + 3y - 1 = 0, 2x + y + 5 = 0$ are conjugate lines with respect to circle $x^2 + y^2 - 2x - 4y - 4 == 0$. (May-15 T.S)

23. Find the equation of tangents of the circle $x^2 + y^2 == 10$ at the points whose abscissae are 1.

24. Find the equation of circle which touches $x^2 + y^2 - 4x + 6y - 12 == 0$ at $(-1, 1)$ internally with a radius of 2.

*25.   The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B. If $AB = 2\lambda$ then show that
$c^2 = (1 + m^2)(a^2 - \lambda^2)$

*26.   Find the condition that the tangents drawn from (0,0) to the circle $s = x^2 + y^2 + 2gx + 2fy + c = 0$
perpendicular to each other (May-15 TS)

VERY SHORT ANSWER QUESTIONS (2M)

1.   Find the centre and radius of the circle $\sqrt{1 + m^2}\left(x^2 + y^2\right) - 2cx - 2mcy = 0 \, (c > 0)$ (June-10)

2.   Find the values of a, b if $ax^2 + bxy + 3y^2 - 5x + 2y - 3 = 0$ represents a circle. Also find radius and centre of
the circle.

3.   If the center of the circle $x^2 + y^2 + ax + by - 12 = 0$ is (2,3), find the values of a,b and the radius
of the circle.   (May-07, 9, Mar-08)

4.   Find the values of g and f, if $x^2 + y^2 + 2gx + 2fy - 12 = 0$ represents the circle with centre
(2,3) and radius?

5.   Find the other end of the diameter of the circle $x^2 + y^2 - 8x - 8y + 27 = 0.$ if one end of it is (2,3).

(Mar-13)

6.   If $x^2 + y^2 - 4x + 6y + c = 0$ represents a circle with radius '6', then find the value of 'c'

(Mar-09)

7.   If the length of the tangent from (2,5) to the circle $x^2 + y^2 - 5x + 4y + k = 0$ is $\sqrt{37}$,then find 'k'
(May-06).

8.   Find the equation of the circle passing through (2, -1) and having the centre at (2, 3);(May-08)

9.   Find the equation of the circle passing through (3, 4) and having the centre at (-3, 4) (Mar-12)

10.   Find the equation of the circle whose centre is (-4, -3) and which passes through the origin.
(Mar-04, June-02)

11.   Obtain the parametric equation of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ (Mar-06, 10)

12.   Obtain the parametric equation of the circle $(x - 3)^2 + (y - 4)^2 = 8^2$ (Mar-11,16,A.P)

13.   Find the equation of the circle whose extremities of a diameter are (1,2) and (4,5)

14.   Find the polar of (1,2) with respect to $x^2 + y^2 = 7$

15.   Find the pole of $ax + by + c = 0 \, (c \neq 0)$ with respect to $x^2 + y^2 = r^2$ (Mar-16 A.P)

16.   Find the pole of $3x + 4y - 45 = 0$ wsith respect $x^2 + y^2 - 6x - .8y + 5 = 0$ (Mar-16 A.P)

17. Find the value of 'k', if the points $(4, k)$, $(2, 3)$ are conjugate with respect to $x^2 + y^2 = 17$

18. Find the number of common tangents that can be drawn to the circles

   $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 16 = 0$

19. Find the value of a if $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle and also radius. (Mar- 13)

20. State and necessary and sufficient condition for $lx + my + n = 0$ be a normal the circle

   $x^2 + y^2 + 2gx + 2fo + c = 0$

21. Show that $A(3, -1)$ lies on the circle $x^2 + y^2 - 2x + 4y = 0$ also find the other end of the diameter through A

22. Find the value of k, if the points $(4, 2)$ and $(k, -3)$ are conjugate with respect to the circle

   $x^2 + y^2 - 5x + 8y + 6 = 0$

23. Find the chord of contact of $(2, 5)$ with respect to circle $x^2 + y^2 - 5x + 4y - 2 = 0$

24. Find the equation of the normal to the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ at $(3, 2)$ also find the other point where the normal meets the circle.

25. Obtain the parametric equation of the circle represented by $x^2 + y^2 = 4$

26. Find the equation of the circle which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through $(-2, 14)$.

27. Find the value 'a' if $2x^2 + ay^2 - 3x + 2y - 1 = 0$ represents a circle and also find its radius.
   (Mar-16 AP)

28. If the length of a tangent from $(5, 4)$ to the circle $x^2 + y^2 + 2ky = 0$ is 'I', then find 'k'

   (Mar-16 AP)(May-15 T.S)

29. Find the power of the point $P(-1, 1)$ with respect to the circle $x^2 + y^2 - 6x + 4y - 12 = 0$

   (Mar- 16 TS)

30. Find the value of k, if the points $(l, 3)$ and $(2, k)$ are conjugate with respect to the circle $x^2 + y^2 = 35$.
   (Mar- 16 A.P)

31. If the circle $x^2 + y^2 - 4x + 6y + a = 0$ has radius 4, then find a.

32. Find the equation of the polar of $(l, -2)$ with respect to circle $x^2 + y^2 - 10x - 10^y + 25 = 0$ (Mar- 15 TS)

33. Find the length of the tangent from $(-2, 5)$ to the circle $x^2 + y^2 - 25 = 0$ (May 16 TS)

34. Find the length of the chord formed by $x^2 + y^2 = a^2$ on the line $x \cos \alpha + y \sin \alpha = P$ (May 16 TS)