Application of derivatives

Rate change

Differentiation or derivatives of a function is applied for many real life problems .The following problems are solved with the help of derivatives:-

1) To find rate of change of quantities
2) To find tangent and normal to the curve at a point
3) To find intervals where a function is increasing or decreasing
4) To find points of maxima /minima of a function

If y=f(x) then

$\frac{dy}{dx}$ *or* $f'(x)$ *represents change of* y with respect to x

If x=f(t) and y=g(t) then $\frac{dy/dt}{\frac{dx}{dt}}$ is the rate of change with respect to t, provided $dx/dt \neq 0$

Let the edge of a cube =x

Then volume V=x^3 ,surface area A=6x^2

Differentiating volume with respect to t

$\frac{dv}{dt}$=$\frac{d}{dx}$(x^3)*$\frac{dx}{dt} = 3x^2 * \frac{dx}{dt}$ = 8 cm^3/sec.

$\frac{dA}{dt} = \frac{d}{dx}(6x^2) * dx/dt$=12x*dx/dt

When x=12cm. dx/dt=8/3x^2=8/3*12^2=1/54

So dA/dt=12*12(1/54)=(8/3)cm^2/sec

The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Area of a circle = πr^2

Rate of change of radius = $\frac{dr}{dt} = 3$ cm/s

Rate of change of area = $\frac{dA}{dt} = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt}$

$= 2\pi r \cdot \frac{dr}{dt}$

when $r = 10$ cm. $\frac{dA}{dt} = 2\pi \times 10 \times 3$ cm²/sec.

$= 60\pi$ cm²/sec.

A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

$$A = \pi r^2 \qquad \frac{dr}{dt} = 5 \text{ cm/sec.}$$

$$\frac{dA}{dt} = \frac{d}{dr}(\pi r^2) \cdot \frac{dr}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

when $r = 8$ cm.

$$\frac{dA}{dt} = 2\pi \times 8 \times 5 \text{ cm}^2\text{/sec.}$$

$$= 80\pi \text{ cm}^2\text{/sec.}$$

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$$\frac{dx}{dt} = -5 \text{ cm./min.} \qquad \frac{dy}{dt} = 4 \text{ cm./min.}$$

$x = 8$ cm.

$y = 6$ cm.

Perimeter $P = 2x + 2y$

$$\frac{dP}{dt} = \frac{d}{dt}(2x) + \frac{d}{dt}(2y) = \frac{d}{dx}(2x)\cdot\frac{dx}{dt} + \frac{d}{dy}(2y)\cdot\frac{dy}{dt}$$

$$= 2\cdot\frac{dx}{dt} + 2\frac{dy}{dt} = 2(-5) + 2(4)$$

$$= -10 + 8 = \boxed{-2 \text{ cm./min}}$$

Area $= xy$.

$$\frac{dA}{dt} = x\cdot\frac{dy}{dt} + \frac{dx}{dt}\cdot y$$

$$= 8(4) + (-5)(6) = 32 - 30 = \boxed{2 \text{ cm}^2/\text{min}}$$

A balloon, which always remains
spherical on inflation, is being inflated by
pumping
in 900 cubic centimetres of gas per
second. Find the rate at which the radius
of
the balloon increases when the radius is
15 cm

Volume of a spherical balloon $= \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 900\ cm^3/sec. = \frac{d}{dr}\cdot\left(\frac{4}{3}\pi r^3\right)\cdot\frac{dr}{dt}$$

$$= \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 900.$$

So, $\frac{dr}{dt} = \frac{900}{4\pi r^2}$ when $r = 15$ cm.

$$\frac{dr}{dt} = \frac{900}{4\pi \times 15^2} = \frac{900}{900\pi} = \frac{1}{\pi}\ cm./sec.$$