

CISTEM ACADEMY

Mathematics

Time: 3 hrs

Max. Marks: 90

Note:

- Please check that this question paper contains 15 printed pages.
 - Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
 - Please check that this question paper contains 34 questions.
 - Please write down the Serial Number of the question before attempting it.
 - 15 minutes time has been allotted to read this question paper..The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.
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General Instructions:

- (i) All questions are compulsory.
- (ii) The question paper consists of 34 questions divided into four sections -A, B, C and D.
- (iii) Section A contains 8 questions of 1 mark each, which are multiple choice type questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 10 questions of 4 marks each.
- (iv) Use of calculators is not permitted.

SECTION A

Question numbers 1 to 8 carry 1 mark each. For each of the question numbers 1 to 8, four alternative choices have been provided, of which only one is correct. Select the correct choice.

1. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4, is

(A) $\frac{4}{15}$

(B) $\frac{2}{15}$

(C) $\frac{1}{5}$

(D) $\frac{1}{3}$

2. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30° . The distance of the car from the tower (in metres) is
- (A) $50\sqrt{3}$
- (B) $150\sqrt{3}$
- (C) $150\sqrt{2}$
- (D) 75
3. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. The value of $\angle APB$ is
- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°
4. If k , $2k - 1$ and $2k + 1$ are three consecutive terms of an A.P., the value of k is
- (A) 2
- (B) 3
- (C) -3
- (D) 5
5. A chord of a circle of radius 10 em subtends a right angle at its centre. The length of the chord (in em) is
- (A) $5\sqrt{2}$
- (B) $10\sqrt{2}$
- (C) $\frac{5}{\sqrt{2}}$
- (D) $10\sqrt{3}$

6. ABCD is a rectangle whose three vertices are B (4, 0), C(4, 3) and nco 3). The length of one of its diagonals is
(A) 5
(B) 4
(C) 3
(D) 25
7. In a right triangle ABC, right-angled at B, BC = 12 cm and AB = 5 cm. The radius of the circle inscribed in the triangle (in cm) is
(A) 4
(B) 3
(C) 2
(D) 1
8. In a family of 3 children, the probability of having at least one boy is
(A) $\frac{7}{8}$
(B) $\frac{1}{8}$
(C) $\frac{5}{8}$
(D) $\frac{3}{4}$

SECTION B

Question numbers 9 to 14 carry 2 marks each.

9. In Figure 1, common tangents AB and CD to the two circles with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.

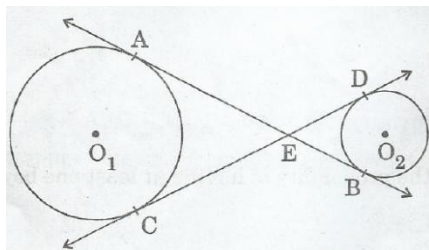


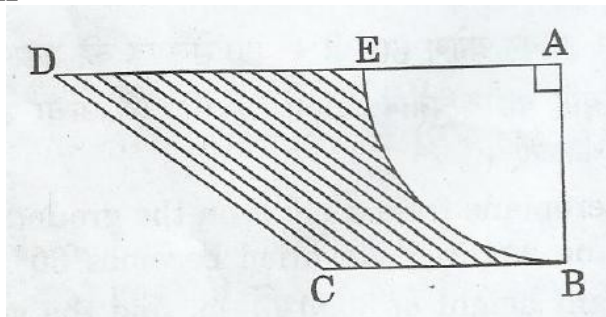
Figure 1

10. The incircle of an isosceles triangle ABC, in which $AB = AC$, touches the sides BC, CA and AB at D, E and F respectively. Prove that $BD = DC$.
11. Two different dice are tossed together. Find the probability
(i) that the number on each die is even.
(ii) that the sum of numbers appearing on the two dice is 5.
12. If the total surface area of a solid hemisphere is 462 cm^2 , find its volume.
Take $\pi = \frac{22}{7}$
13. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.
14. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

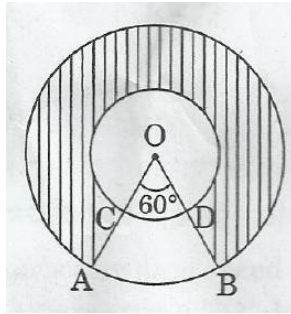
SECTION C

Question numbers 15 to 24 carry 3 marks each.

15. The angle of elevation of an aeroplane from a point on the ground is 60° . After a flight of 30 seconds the angle of elevation becomes 30° . If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.
16. The largest possible sphere is carved out of a wooden solid cube of side 7 em. Find the volume of the wood left. Use $\pi = \frac{22}{7}$
17. Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 4 km/h. How much area will it irrigate in 10 minutes, if 8 cm of standing water is needed for irrigation?
18. In Figure 2, ABCD is a trapezium of area 24.5 sq. cm. In it, $AD \parallel BC$, $\angle DAB = 90^\circ$, $AD = 10$ cm and $BC = 4$ cm. If ABE is a quadrant of a circle, find the area of the shaded region. Take $\pi = \frac{22}{7}$



19. Find the ratio in which the line segment joining the points A(3,- 3) and B(- 2, 7) is divided by x-axis. Also find the coordinates of the point of division.
20. In Figure 3, two concentric circles with centre O, have radii 21cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. Use $\pi = \frac{22}{7}$



21. Solve for x:
 $\frac{16}{x} - \frac{15}{x+1}; x \neq 0, -1$
22. The sum of the 2nd and the 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find the AP.
23. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.
24. Prove that the diagonals of a rectangle ABCD, with vertices A(2, -1), B(5, -1), C(5, 6) and D(2, 6), are equal and bisect each other.

SECTION D

Question numbers 25 to 34 carry 4 marks each.

25. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
26. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.
27. A container open at the top, is in the form of a frustum of a cone of height 24 cm with radii of its lower and upper circular ends, as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container. at the rate of 21 per litre.

Use $\pi = \frac{22}{7}$

28. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45° . If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60° , then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]
29. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
30. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?
31. Solve for x:
$$\frac{x-3}{x-4} + \frac{5}{6} = \frac{10}{3}; x \neq 4, 6$$
32. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is
- (i) of red colour
 - (ii) a queen
 - (iii) an ace
 - (iv) a face card
33. A(4, - 6), B(3, - 2) and C(5, 2) are the vertices of a ΔABC and AD is its median. Prove that the median AD divides ΔABC into two triangles of equal areas.
34. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.