RAY OPTICS - I

1. Refraction of Light
2. Laws of Refraction
3. Principle of Reversibility of Light
4. Refraction through a Parallel Slab
5. Refraction through a Compound Slab
6. Apparent Depth of a Liquid
7. Total Internal Reflection
8. Refraction at Spherical Surfaces - Introduction
9. Assumptions and Sign Conventions
10. Refraction at Convex and Concave Surfaces
11. Lens Maker's Formula
12. First and Second Principal Focus
13. Thin Lens Equation (Gaussian Form)
14. Linear Magnification

Refraction of Light:

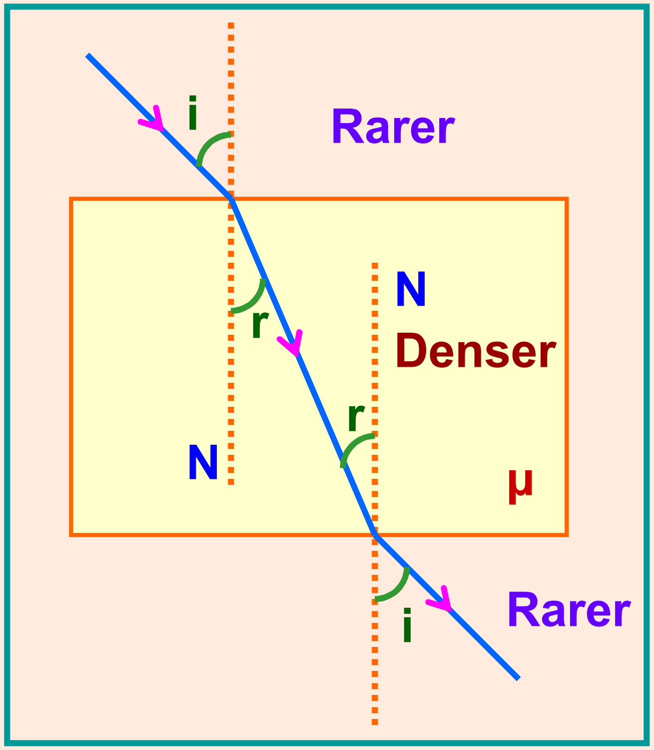
Refraction is the phenomenon of change in the path of light as it travels from one medium to another (when the ray of light is incident obliquely).

It can also be defined as the phenomenon of change in speed of light from one medium to another.

Laws of Refraction:

I Law: The incident ray, the normal to the refracting surface at the point of incidence and the refracted ray all lie in the same plane.

II Law: For a given pair of media and for light of a given wavelength, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. (Snell's Law)



$$\mu = \frac{\sin i}{\sin r}$$

(The constant μ is called refractive index of the medium, i is the angle of incidence and r is the angle of refraction.)

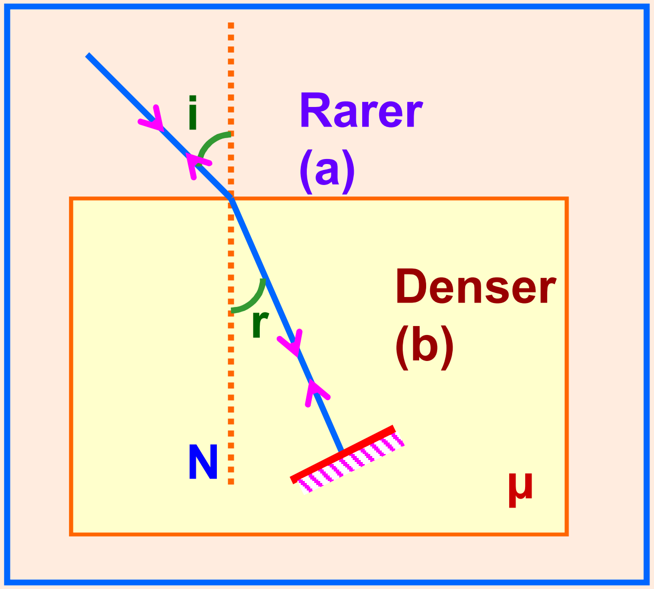
TIPS:

1. μ of optically rarer medium is lower and that of a denser medium is higher.
2. μ of denser medium w.r.t. rarer medium is more than 1 and that of rarer medium w.r.t. denser medium is less than 1. ($\mu_{air} = \mu_{vacuum} = 1$)
3. In refraction, the velocity and wavelength of light change.
4. In refraction, the frequency and phase of light do not change.
5. ${}_a\mu_m = c_a / c_m$ and ${}_a\mu_m = \lambda_a / \lambda_m$

Principle of Reversibility of Light:

$${}_a\mu_b = \frac{\sin i}{\sin r} \qquad {}_b\mu_a = \frac{\sin r}{\sin i}$$

$$\boxed{{}_a\mu_b \times {}_b\mu_a = 1} \quad \text{or} \quad \boxed{{}_a\mu_b = 1 / {}_b\mu_a}$$



If a ray of light, after suffering any number of reflections and/or refractions has its path reversed at any stage, it travels back to the source along the same path in the opposite direction.

A natural consequence of the principle of reversibility is that the image and object positions can be interchanged. These positions are called conjugate positions.

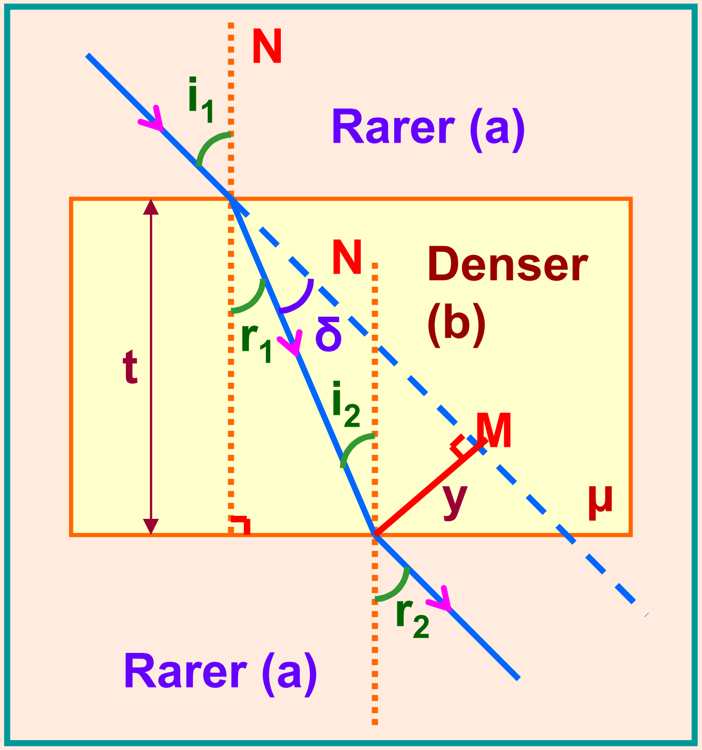
Refraction through a Parallel Slab:

$$ {}_a\mu_b = \frac{\sin i_1}{\sin r_1} \qquad {}_b\mu_a = \frac{\sin i_2}{\sin r_2} $$

But ${}_a\mu_b \times {}_b\mu_a = 1$

$$ \therefore \frac{\sin i_1}{\sin r_1} \times \frac{\sin i_2}{\sin r_2} = 1 $$

It implies that $i_1 = r_2$ and $i_2 = r_1$ since $i_1 \neq r_1$ and $i_2 \neq r_2$.



Lateral Shift:

$$ \boxed{y = \frac{t \sin \delta}{\cos r_1}} \quad \text{or} \quad \boxed{y = \frac{t \sin(i_1 - r_1)}{\cos r_1}} $$

Special Case:

If i_1 is very small, then r_1 is also very small.
i.e. $\sin(i_1 - r_1) = i_1 - r_1$ and $\cos r_1 = 1$

$$ \therefore \boxed{y = t\,(i_1 - r_1)} \quad \text{or} \quad \boxed{y = t\, i_1(1 - 1/{}_a\mu_b)} $$

Refraction through a Compound Slab:

$$ {}_a\mu_b = \frac{\sin i_1}{\sin r_1} $$

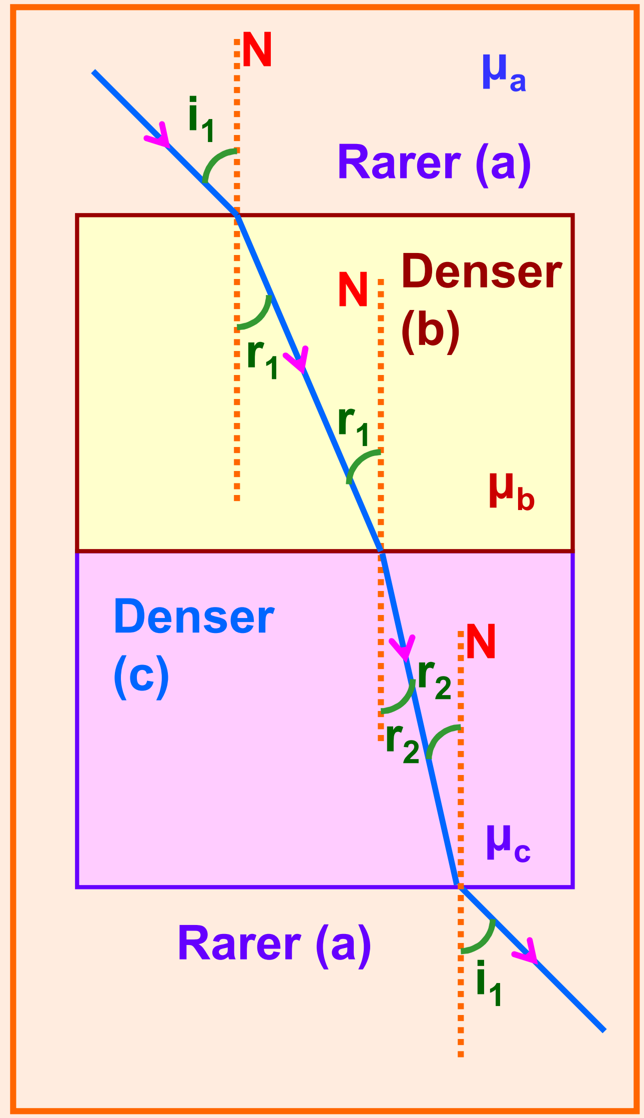
$$ {}_b\mu_c = \frac{\sin r_1}{\sin r_2} $$

$$ {}_c\mu_a = \frac{\sin r_2}{\sin i_1} $$

$$ \boxed{{}_a\mu_b \times {}_b\mu_c \times {}_c\mu_a = 1} $$

or $\boxed{{}_a\mu_b \times {}_b\mu_c = {}_a\mu_c}$

or $\boxed{{}_b\mu_c = {}_a\mu_c / {}_a\mu_b}$

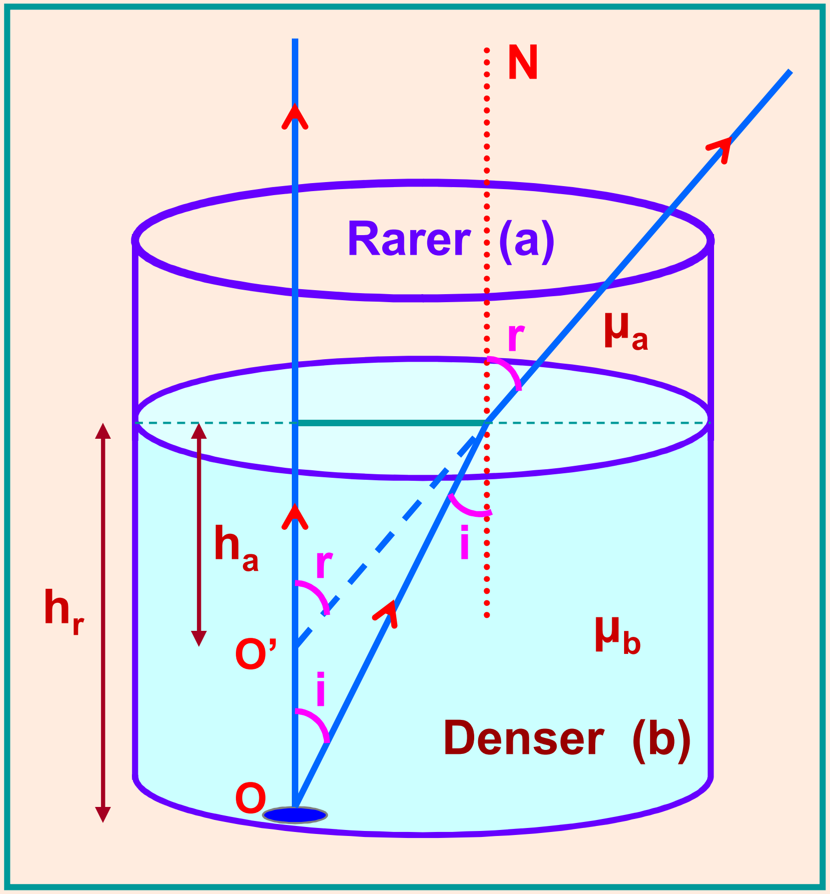


$\mu_c > \mu_b$

Apparent Depth of a Liquid:

$$_b\mu_a = \frac{\sin i}{\sin r} \quad \text{or} \quad _a\mu_b = \frac{\sin r}{\sin i}$$

$$_a\mu_b = \frac{h_r}{h_a} = \frac{\text{Real depth}}{\text{Apparent depth}}$$



Apparent Depth of a Number of Immiscible Liquids:

$$h_a = \sum_{i=1}^{n} h_i / \mu_i$$

Apparent Shift:

Apparent shift $= h_r - h_a = h_r - (h_r / \mu)$

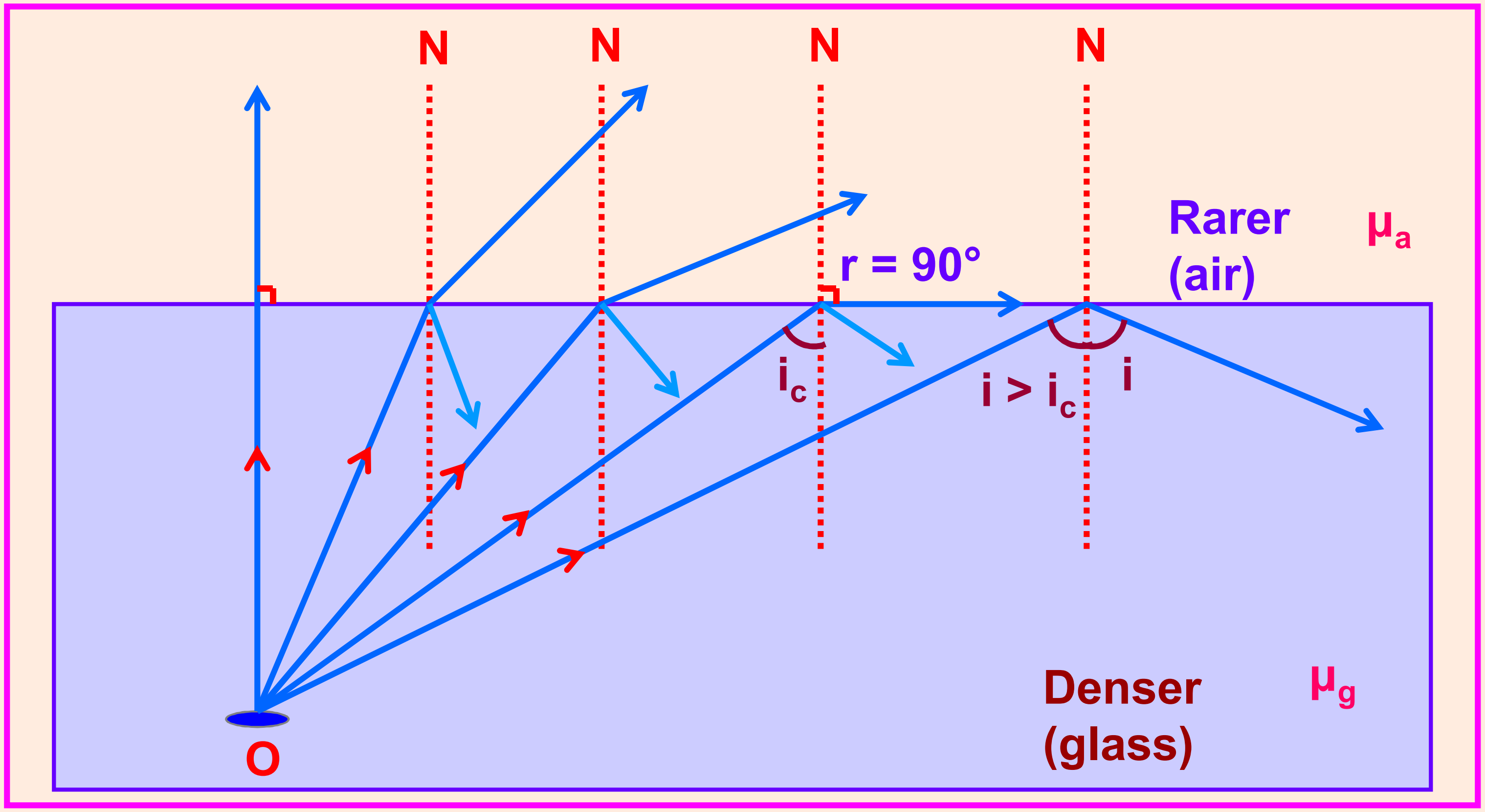
$= h_r \, [\, 1 - 1/\mu]$

TIPS:

1. If the observer is in rarer medium and the object is in denser medium then $h_a < h_r$. (To a bird, the fish appears to be nearer than actual depth.)
2. If the observer is in denser medium and the object is in rarer medium then $h_a > h_r$. (To a fish, the bird appears to be farther than actual height.)

Total Internal Reflection:

Total Internal Reflection (TIR) is the phenomenon of complete reflection of light back into the same medium for angles of incidence greater than the critical angle of that medium.



Conditions for TIR:

1. **The incident ray must be in optically denser medium.**
2. **The angle of incidence in the denser medium must be greater than the critical angle for the pair of media in contact.**

Relation between Critical Angle and Refractive Index:

Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90°.

$$ {}_{g}\mu_{a} = \frac{\sin i}{\sin r} = \frac{\sin i_c}{\sin 90°} = \sin i_c $$

or $$ {}_{a}\mu_{g} = \frac{1}{{}_{g}\mu_{a}} \quad \therefore \quad \boxed{{}_{a}\mu_{g} = \frac{1}{\sin i_c}} \quad \text{or} \quad \boxed{\sin i_c = \frac{1}{{}_{a}\mu_{g}}} \quad \text{Also} \quad \boxed{\sin i_c = \frac{\lambda_g}{\lambda_a}} $$

Red colour has maximum value of critical angle and Violet colour has minimum value of critical angle since,

$$ \boxed{\sin i_c = \frac{1}{{}_{a}\mu_{g}} = \frac{1}{a + (b/\lambda^2)}} $$

Applications of T I R:

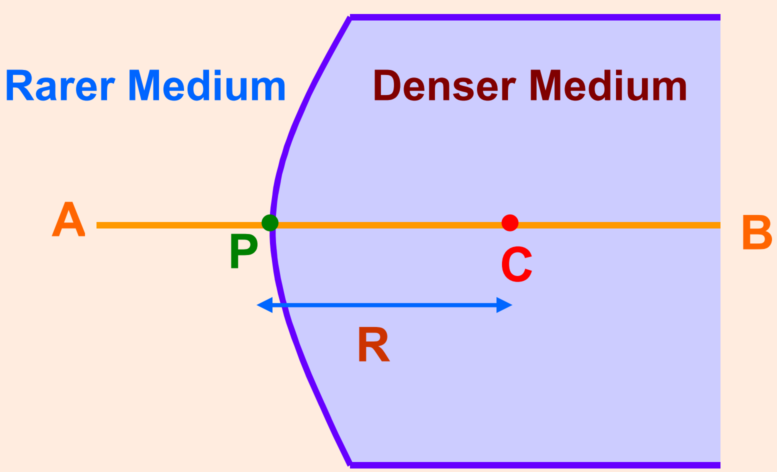
1. Mirage formation
2. Looming
3. Totally reflecting Prisms
4. Optical Fibres
5. Sparkling of Diamonds

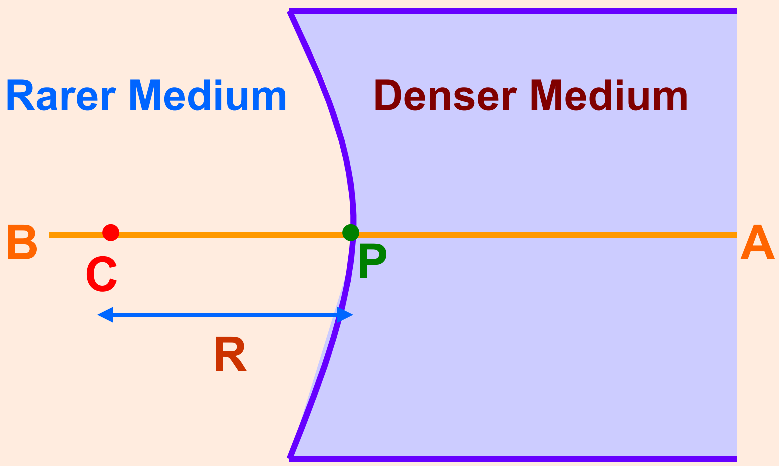
Spherical Refracting Surfaces:

A spherical refracting surface is a part of a sphere of refracting material.

A refracting surface which is convex towards the rarer medium is called convex refracting surface.

A refracting surface which is concave towards the rarer medium is called concave refracting surface.





APCB – Principal Axis
C – Centre of Curvature
P – Pole
R – Radius of Curvature

Assumptions:

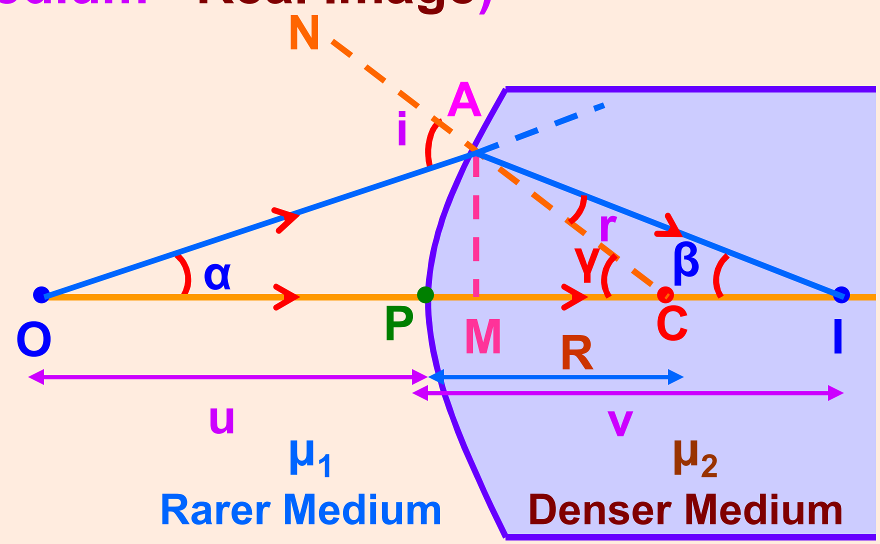
1. Object is the point object lying on the principal axis.
2. The incident and the refracted rays make small angles with the principal axis.
3. The aperture (diameter of the curved surface) is small.

New Cartesian Sign Conventions:

1. The incident ray is taken from left to right.
2. All the distances are measured from the pole of the refracting surface.
3. The distances measured along the direction of the incident ray are taken positive and against the incident ray are taken negative.
4. The vertical distances measured from principal axis in the upward direction are taken positive and in the downward direction are taken negative.

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Real Image)



$i = \alpha + \gamma$

$\gamma = r + \beta$ or $r = \gamma - \beta$

$\tan \alpha = \frac{MA}{MO}$ or $\alpha = \frac{MA}{MO}$

$\tan \beta = \frac{MA}{MI}$ or $\beta = \frac{MA}{MI}$

$\tan \gamma = \frac{MA}{MC}$ or $\gamma = \frac{MA}{MC}$

According to Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \frac{i}{r} = \frac{\mu_2}{\mu_1} \quad \text{or} \quad \boxed{\mu_1 i = \mu_2 r}$$

Substituting for i, r, α, β and γ, replacing M by P and rearranging,

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

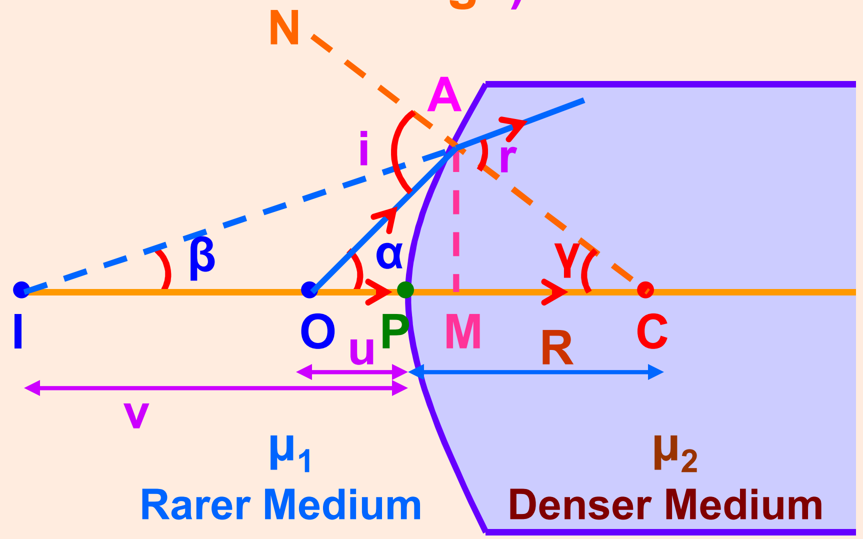
Applying sign conventions with values,
PO = - u, PI = + v and PC = + R

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

Refraction at Convex Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

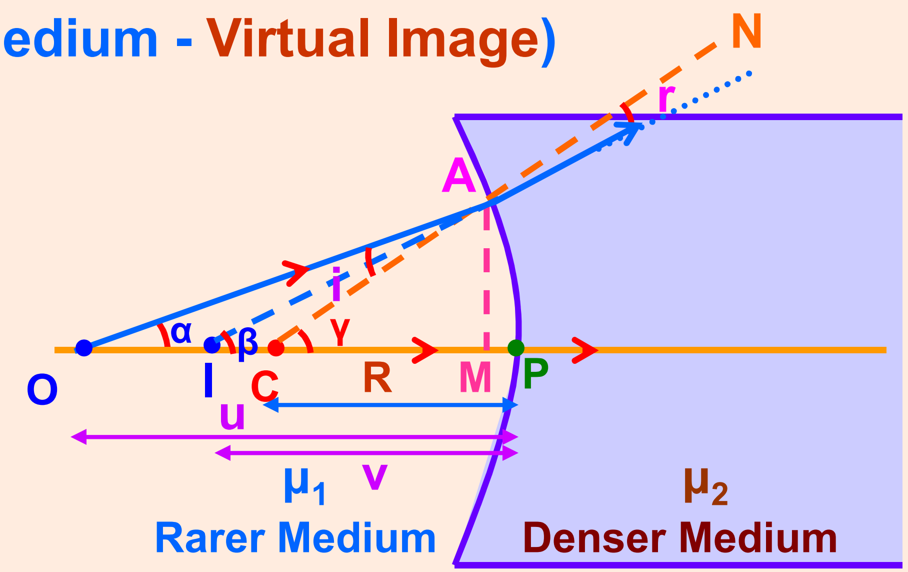
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Concave Surface:

(From Rarer Medium to Denser Medium - Virtual Image)

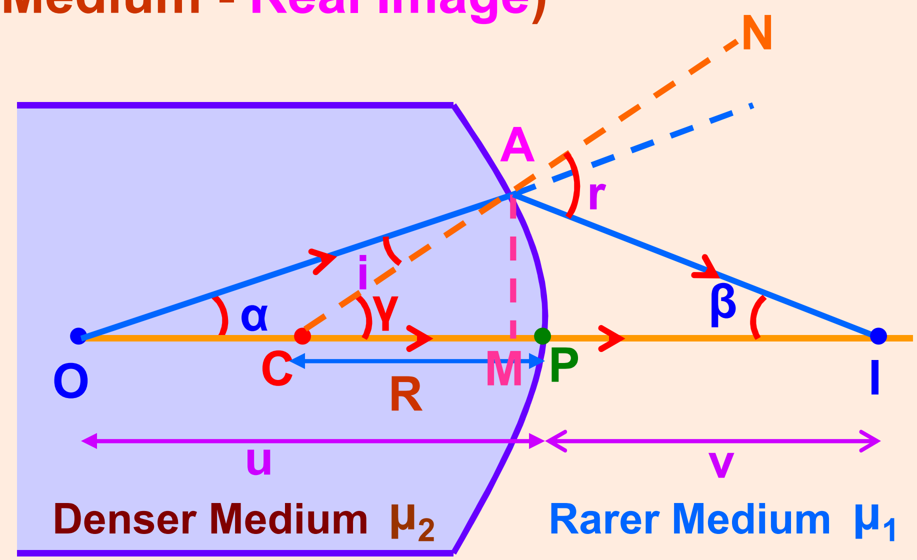
$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Real Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$



Refraction at Convex Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Refraction at Concave Surface:

(From Denser Medium to Rarer Medium - Virtual Image)

$$\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Note:

1. Expression for '**object in rarer medium**' is same for whether it is real or virtual image or convex or concave surface.

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

2. Expression for '**object in denser medium**' is same for whether it is real or virtual image or convex or concave surface.

$$\boxed{\frac{\mu_2}{-u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

3. However the values of u, v, R, etc. must be taken with proper sign conventions while solving the numerical problems.

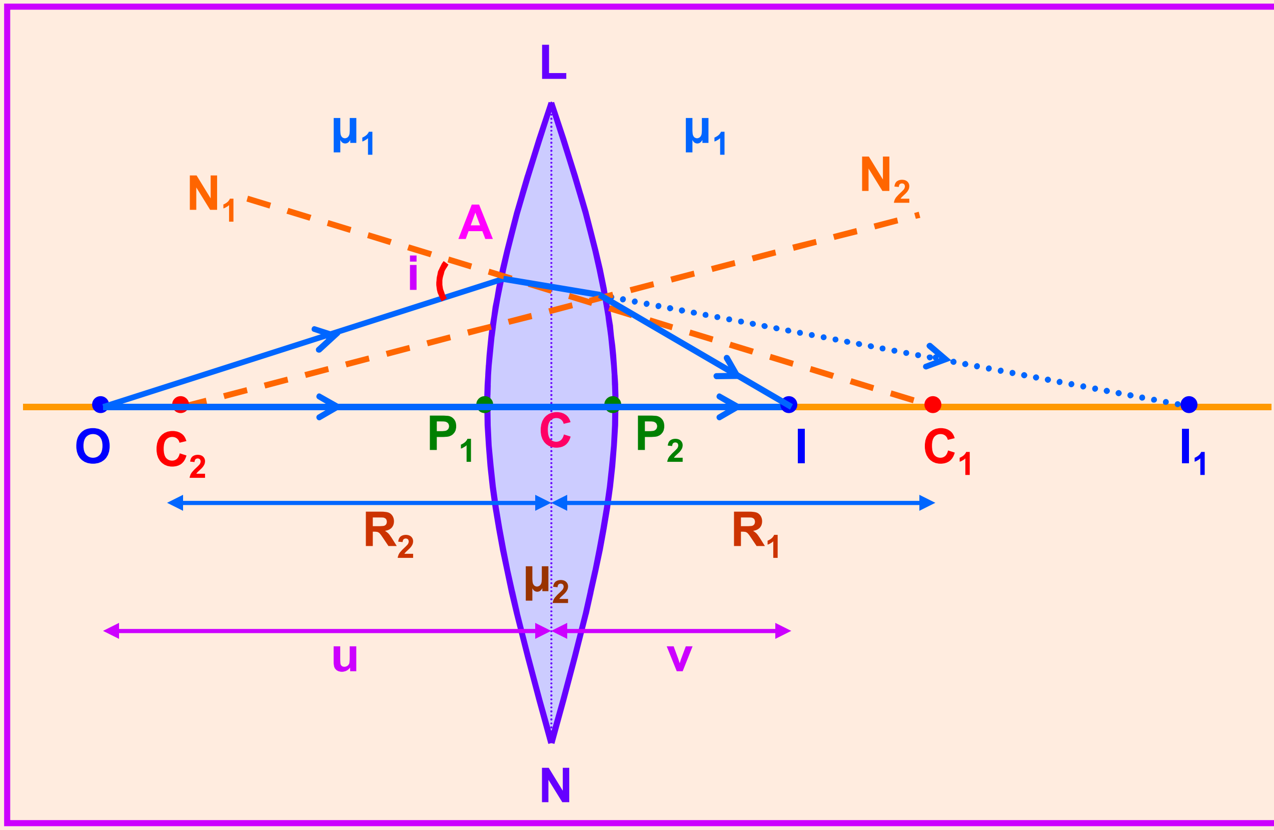
4. The refractive indices μ_1 and μ_2 get interchanged in the expressions.

Lens Maker's Formula:

For refraction at LP_1N,

$$\frac{\mu_1}{CO} + \frac{\mu_2}{CI_1} = \frac{\mu_2 - \mu_1}{CC_1}$$

(as if the image is formed in the denser medium)



For refraction at LP_2N,

$$\frac{\mu_2}{-CI_1} + \frac{\mu_1}{CI} = \frac{-(\mu_1 - \mu_2)}{CC_2}$$

(as if the object is in the denser medium and the image is formed in the rarer medium)

Combining the refractions at both the surfaces,

$$\frac{\mu_1}{CO} + \frac{\mu_1}{CI} = (\mu_2 - \mu_1)\left(\frac{1}{CC_1} + \frac{1}{CC_2}\right)$$

Substituting the values with sign conventions,

$$\frac{1}{-u} + \frac{1}{v} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $\mu_2 / \mu_1 = \mu$

$$\frac{1}{-u} + \frac{1}{v} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{-u} + \frac{1}{v} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the object is kept at infinity, the image is formed at the principal focus.

i.e. $u = -\infty$, $v = +f$.

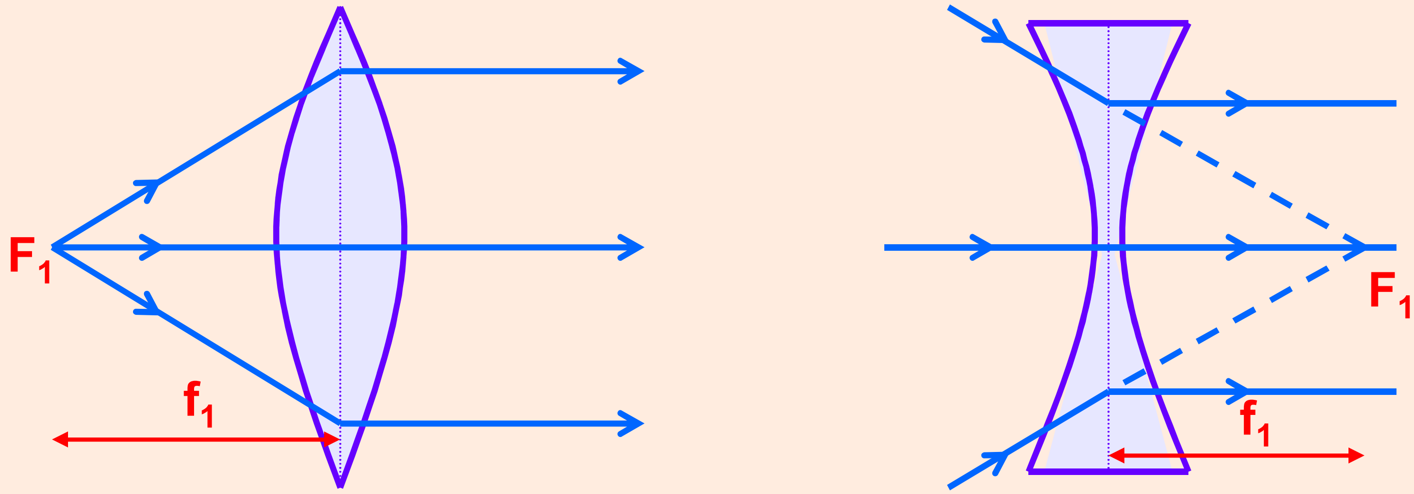
So, $$\boxed{\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

This equation is called 'Lens Maker's Formula'.

Also, from the above equations we get, $$\boxed{\frac{1}{-u} + \frac{1}{v} = \frac{1}{f}}$$

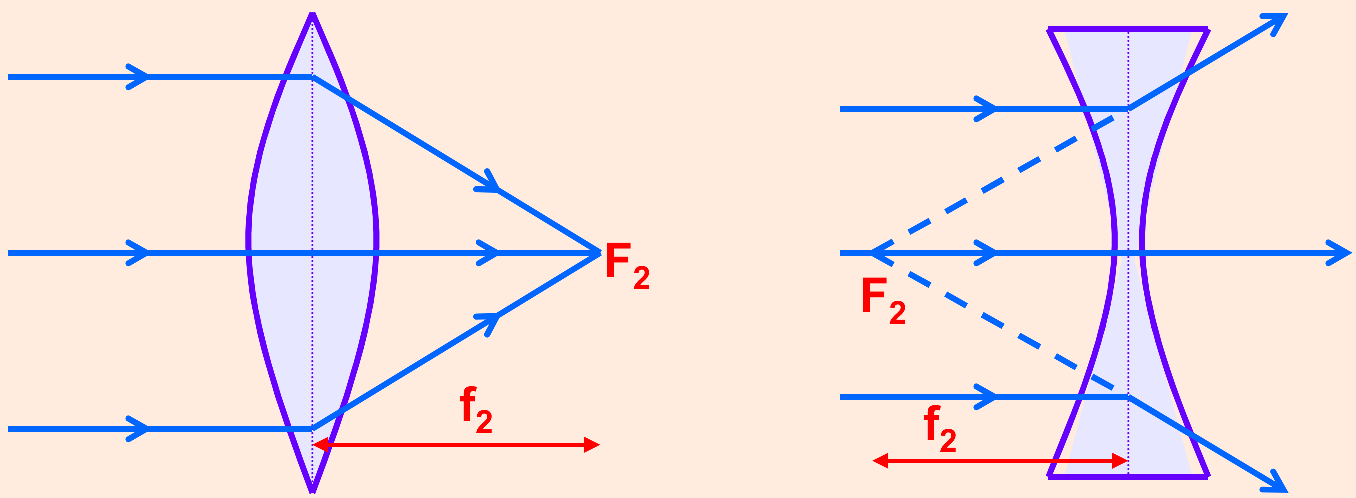
First Principal Focus:

First Principal Focus is the point on the principal axis of the lens at which if an object is placed, the image would be formed at infinity.



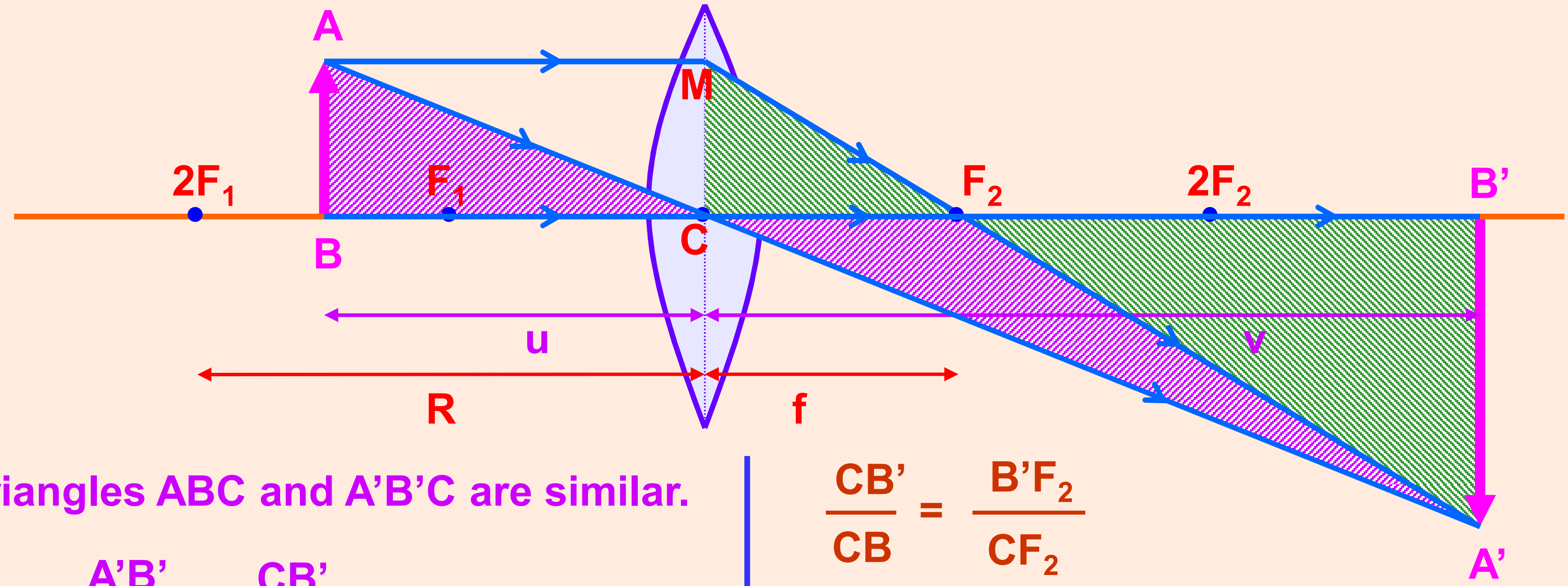
Second Principal Focus:

Second Principal Focus is the point on the principal axis of the lens at which the image is formed when the object is kept at infinity.



Thin Lens Formula (Gaussian Form of Lens Equation):

For Convex Lens:



Triangles ABC and A'B'C are similar.

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

Triangles MCF_2 and $A'B'F_2$ are similar.

$$\frac{A'B'}{MC} = \frac{B'F_2}{CF_2}$$

or $$\frac{A'B'}{AB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{B'F_2}{CF_2}$$

$$\frac{CB'}{CB} = \frac{CB' - CF_2}{CF_2}$$

According to new Cartesian sign conventions,

$CB = - u$, $CB' = + v$ and $CF_2 = + f$.

$$\therefore \boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

Linear Magnification:

Linear magnification produced by a lens is defined as the ratio of the size of the image to the size of the object.

$$m = \frac{I}{O}$$

$$\frac{A'B'}{AB} = \frac{CB'}{CB}$$

According to new Cartesian sign conventions,

$A'B' = + I$, $AB = - O$, $CB' = + v$ and $CB = - u$.

$$\frac{+ I}{- O} = \frac{+ v}{- u} \quad \text{or} \quad \boxed{m = \frac{I}{O} = \frac{v}{u}}$$

Magnification in terms of v and f:

$$\boxed{m = \frac{f - v}{f}}$$

Magnification in terms of u and f:

$$\boxed{m = \frac{f}{f - u}}$$

Power of a Lens:

Power of a lens is its ability to bend a ray of light falling on it and is reciprocal of its focal length. When f is in metre, power is measured in Dioptre (D).

$$\boxed{P = \frac{1}{f}}$$

End of Ray Optics - I